RELATIVE PRICE DISTORTIONS AND INFLATION PERSISTENCE*

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Sticky-price models often suggest that relative price distortion is a major cost of inflation. We provide an intuition for this: Even at low rates, inflation strongly affects price dispersion which in turn has an impact on the economy qualitatively similar to, and of the order of magnitude of, a negative shift in productivity. The utility cost of price dispersion is quantified and its impact on optimal monetary policy discussed. Price dispersion is incorporated into a linearized model. Strikingly, a contractionary nominal shock has a persistent, negative hump-shaped impact on inflation, but may have a positive hump-shaped impact on output.

JEL Classification:E52; E61; E63.

This paper investigates the macroeconomic implications of relative price distortions as this is where many, though not all, sticky-price models locate the costs of inflation[2]. First, we quantify how costly price dispersion is in a standard macroeconomic model with imperfect competition and price rigidity as in Calvo (1983). Despite being very costly in welfare terms, price dispersion is generally considered to be a term of second-order importance in linearized models. That is why many economists conclude that the direct impact of price dispersion on welfare is small (e.g., Canzoneri, Cumby and Diba, 2004). However, in economies with, say, trend inflation of $2 - 3\%$, no indexation and a degree of nominal price inertia, price dispersion, viewed through the lens of our simple model, will be an important (first-order) variable. The key margins which are distorted by price dispersion are identified and we develop what we think is a useful intuition on the costs of dispersion which has not been identified hitherto: Price dispersion impacts on the economy like a *negative* productivity shock, and so inflation surprises are far from costless in this set-up.

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[2] For example, in the sticky-price model of Rotemberg (1982) all firms charge the same price, even though that price differs from the price that would have been charged had price changes not been costly. So, there is no dispersion of prices *across* firms which is the focus of this paper.

1. The Analysis in More Detail

In the basic sticky-price model that we develop, private consumption is not maximized in the presence of relative price distortion ('price dispersion', for short), for a given amount of nominal expenditure. The reflection on the supply-side of the economy of that reduction in consumption is that labour is allocated away from 'high-price' firms to 'low-price' firms. Due to diminishing returns, average labour productivity is lower than it would be were all firms facing the same level of demand. In a sense, then, at the aggregate level the economy uses too much labour to produce a given level of output. Given increasing disutility of labour, there is upward pressure on the equilibrium real wage and hence the economy incurs higher total costs of production compared with an economy with no price dispersion.

In short, for a given output level, the economy with price dispersion behaves in a manner *qualitatively* similar to a low productivity economy, needing to employ more labour input to meet demand. We demonstrate this argument formally in Section 3 by forming a Ramsey policy problem which allows one easily to inspect the general equilibrium impact of price dispersion. Section 4 then shows that price dispersion also has an impact on outcomes *quantitatively* of the order of magnitude of a negative shift in productivity. It is observed that price dispersion is itself sensitive even to relatively low rates of inflation and increases sharply in the level of inflation. In Section 5 we then enquire, following Lucas (1987), what the consumption-equivalent impact is of a given level of price dispersion and confirm that it is indeed very costly. Of course, unlike productivity, price dispersion is not exogenous and so Section 6 analyzes the impact that price dispersion has on optimal monetary policy. We recover a result like Yun's (2005), demonstrating that in the presence of price dispersion, disinflation may be the optimal policy. Typically, linearized models do not come to that conclusion as price dispersion is absent from these models. It is explained why even a full second-order approximation to our model's equations would not recover Yun's or our result and would continue to conclude that the impact of price dispersion on welfare is quantitatively very small.

In order to analyze the impact of price dispersion on dynamics, in Sections 7 and 8 we develop a linearized model around a non-indexed, inflationary steady-state[3]; as a result, price dispersion is of first-order significance. We simply take as given that trend inflation is positive. The impact of a persistent, negative nominal shock appears similar to a persistent, positive productivity shock, which is consistent with the analysis in Sections 3 and 4. However, there is a marked difference between the models with and without price dispersion: We find that inflation follows a hump-shaped response following *both* nominal and real shocks in the model with price dispersion; its maximal response is not in the period following the shock. Interest rates also respond more gradually following shocks in the model with price dispersion. Underlying these results is the fact that any shock which decreases price dispersion will impart upward momentum to output and downward momentum to inflation and, because price dispersion is a persistent process,

[3]Some recent contributions have incorporated indexation of some prices as a means to impart peristence into inflation. However, as Blanchard and Gali (2005) note, there is probably little empirical justification for this assumption in low inflation economies. Indexing in this manner also implies price dispersion is a second-order term.

this momentum will itself be persistent. Section 9 offers some conclusions.

1.1. *Related Literature*

The observation that inflation and price dispersion is costly has been emphasized in a number of recent contributions. Ascari (2004) argues that increasing trend inflation reduces steady state output to an implausibly large degree in the New Keynesian (Calvo contracts) model. We pursue in depth the role of price dispersion as the source of the problem. Schmitt-Grohe and Uribe (2005) point to the link between trend inflation and the importance of price dispersion, as we do, but their discussion on the distortive effects is brief and most of their analysis relies on simulations (as their interest is not really in the costs of price dispersion, *per se*). Our paper is perhaps closest in spirit to Amano, Ambler and Rebei (2007) who also identify clearly through numerical simulations the cost of price dispersion. However, we pursue the issue analytically of the underlying intuition why price dispersion is costly in the Calvo-Yun type framework and provide detailed analyses of the welfare impact of price dispersion. Finally, Ascari and Ropele (2006) develop a linearized model where price dispersion is a first-order important variable and analyze optimal monetary policy under differing degrees of commitment. Some of our findings are similar but, unlike them, we show that in fact a negative monetary shock can have a persistent and hump-shaped impact on inflation, whilst having a positive impact on output. The key to understanding why the model behaves in the way it does, we argue, is in the intuition we develop earlier in the paper as to the similarity between the effects of price dispersion and productivity; our Proposition 3 is key.

2. The Model

This section presents a standard sticky-price model. The model is developed somewhat briskly as many of the details are familiar[4]. There are a large number of identical agents in the economy who evaluate their utility in accordance with the following criterion:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \frac{\lambda_t}{1+v} \left(\int_i N_t(i) di \right)^{1+v} \right). \tag{1}$$

E_t denotes the expectations operator at time t, β is the discount factor, C_t is consumption and $N_t(i)$ is the quantity of labour supplied to firm i. $N_t = \int_i N_t(i)di$. $v \geq 0$ reflects the labour supply elasticity while λ_t is a 'preference' parameter.

Consumption is defined over a basket of goods, $C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$, and the price-level is known to be $P_t = \left[\int_0^1 p_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$, where $p_t(i)$ denotes the nominal price of the final good produced by firm i. All firms pay the same real wage for the same labour, so $w_t(i) = w_t$, $\forall i$. All households provide the same share of labour to all firms and so we

[4]More detailed derivations of the model and other results in this paper are contained in the working paper version available as CDMA Working Paper 0611 at http://www.st-andrews.ac.uk/cdma/papers.html#WP

may write the agent's flow budget constraint as

$$\int_0^1 p_t(i)c_t(i)di + B_t = [1 + i_{t-1}]\, B_{t-1} + W_t N_t(1-\tau_t^h) + \Pi_t. \tag{2}$$

As all agents are identical, the only financial assets traded in equilibrium will be those issued by the fiscal authority. B_t denotes the nominal value at the end of date t of government bond holdings, $1+i_t$ is the nominal interest rate on this 'riskless' one-period nominal asset, W_t is the nominal wage in period t, and Π_t is profits remitted to the individual. The tax rate applied to labour income is denoted by τ_t^h. We also impose the following familiar restriction on the equilibrium plan of the representative agent:

$$\lim_{J\to\infty} E_t \prod_{j=0}^{J} R_{t+j-1} B_{t+J} \geq 0, \qquad R_t \equiv (1+i_t)^{-1}. \tag{3}$$

Hence, the necessary conditions for an optimum include:

$$N_t = \left[w_t \left(1-\tau_t^h\right)(\lambda_t C_t)^{-1}\right]^{1/v}; \tag{4}$$

and

$$E_t \left\{ \frac{\beta C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right\} = \frac{1}{1+i_t}. \tag{5}$$

The complete markets assumption implies a unique stochastic discount factor, $Q_{t,t+k} = \beta \frac{C_t P_t}{C_{t+k} P_{t+k}}$, where, $E_t \{Q_{t,t+k}\} = E_t \prod_{j=0}^{k} \frac{1}{1+i_{t+j}}$.

2.1. *Representative Firm: Factor Demand*

Labour is the only factor of production. Firms are monopolistic competitors who produce their distinctive goods according to the following technology

$$Y_t(i) = A_t \left[N_t(i)\right]^{1/\phi}, \tag{6}$$

where $N_t(i)$ denotes the amount of labour hired by firm i in period t, A_t is a stochastic productivity shock and $\phi > 1$.

The demand for output determines the demand for labour. Hence, using $Y_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} Y_t$, where Y_t denotes aggregate demand, we find that

$$N_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta\phi} \left(\frac{Y_t}{A_t}\right)^{\phi}. \tag{7}$$

It follows that the total amount of labour demanded will be

$$N_t = \int N_t(i)di = N_t^* \Delta_t \left\langle -\theta\phi \right\rangle. \tag{8}$$

We define $\Delta_t \langle -\theta\phi \rangle \equiv \Delta_t$ as our measure of price dispersion:

$$\Delta_t = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\theta\phi} di. \tag{9}$$

From an empirical point of view that is not a natural measure of price dispersion and so section 4 relates this measure to the coefficient of variation for prices. In this simple set-up, as confirmed below, were all firms given the chance to re-price at any instant in time, they would all choose the same price. In that case, given output, the labour supply would be

$$N_t^* = \left(A_t^{-1} Y_t \right)^{\phi}. \tag{10}$$

If one substitutes (10) into (8) one receives

$$N_t = \left(A_t^{-\phi} \Delta_t \right) Y_t^{\phi}, \tag{11}$$

which corresponds to the amount of labour employed to produce quantity Y_t should prices not be equal across industries. Finally, it follows that the equilibrium wage may be written as

$$w_t = \lambda_t \frac{1}{1-\tau_t^h} C_t \Delta_t^v \left(\frac{Y_t}{A_t} \right)^{\phi v}. \tag{12}$$

In short, (11) and (12) indicate that equilibrium labour input and real wage are higher in the presence of price dispersion, for given demand, than would otherwise be the case.

2.2. *Representative Firm: Price Setting*

The Calvo (1983) approach to modelling price-stickiness is adopted. This is a convenient and familiar approach to modelling sticky prices but the same basic issues that we are interested in would seem to arise in any model where price dispersion is present. Each period a measure, $1-\alpha$, of firms is allowed to adjust prices. Those firms choose the nominal price which maximizes their expected profit given that they may have to charge the same price in $k-$periods time with probability α^k.

Importantly, we are assuming that firms are cost-takers and that they do not anticipate the change in equilibrium wages in reaction to their price setting decision, evident from (12). The price setting problem can then be characterized as follows:

$$\max_{p_t'} E_t \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} \left(Y_{t+k} \left(\frac{p_t'}{P_{t+k}} \right)^{1-\theta} - w_{t+k} A_{t+k}^{-\phi} Y_{t+k}^{\phi} \left(\frac{p_t'}{P_{t+k}} \right)^{-\theta\phi} \right), \tag{13}$$

where p_t' is the price chosen by firms which update prices. There is no need to index this nominal price on i as it is clear that this will be a function solely of variables that affect all firms symmetrically. The first order condition with respect to p_t' implies

$$\left(\frac{p_t'}{P_t} \right)^{1+\theta(\phi-1)} = \left(\frac{\theta}{\theta-1} \right) \frac{\sum_{k=0}^{\infty} (\alpha\beta)^k E_t C_{t+k}^{-1} \left[\phi w_{t+k} A_{t+k}^{-\phi} Y_{t+k}^{\phi} (P_t/P_{t+k})^{-\theta\phi} \right]}{\sum_{k=0}^{\infty} (\alpha\beta)^k E_t C_{t+k}^{-1} \left[Y_{t+k} (P_t/P_{t+k})^{1-\theta} \right]}. \tag{14}$$

The price index then evolves according to the law of motion

$$P_t = \left[(1-\alpha)\, p_t'^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{1/(1-\theta)}. \tag{15}$$

Because the relative prices of the firms that do not change their prices in period t fall by the rate of inflation, we may derive a law of motion for our measure of price dispersion,

$$\Delta_t = \alpha \Delta_{t-1} \pi_t^{\theta\phi} + (1-\alpha) \left(\frac{p_t'}{P_t}\right)^{-\theta\phi}. \tag{16}$$

2.3. *Fiscal Authorities*

The government purchases goods in the same proportions as do private agents. These purchases yield no utility to agents nor do they boost the productive potential of the economy. Further, government expenditure is assumed exogenous and stochastic. For now, we assume that government raises revenue solely through taxes on labour income. We assume that the government can borrow by issuing a one period risk-free nominal bond. The nominal value of government debt evolves according to the law of motion,

$$B_t = (1 + i_{t-1}) B_{t-1} - S_t. \tag{17}$$

B_t and i_t were defined above, and S_t is the (primary) budget surplus,

$$S_t = \tau_t^h W_t N_t - G_t P_t.$$

It is assumed that the expected path of government surpluses satisfies an intertemporal solvency condition, by design, for all feasible paths of the model's endogenous variables. There is a sequence of intertemporal constraints for all t of the following sort,

$$(1 + i_{t-1})\, B_{t-1} = E_t \sum_{k=0}^{\infty} Q_{t,t+k} \left(S_{t+k}\right), \tag{18}$$

which one may simplify as

$$(1 + i_{t-1}) \frac{b_{t-1}}{C_t \pi_t} = E_t \sum_{k=0}^{\infty} \beta^k \frac{1}{C_{t+k}} \left(\tau_{t+k}^h w_{t+k} N_{t+k} - G_{t+k}\right), \tag{19}$$

and where b_{t-1} is a measure of the real value of debt inherited from the previous period, $b_{t-1} = B_{t-1}/P_{t-1}$, while π_t is inflation, $\pi_t = P_t/P_{t-1}$.

Associated with this sequence, is a sequence of transversality conditions. This sequence is ultimately related to the incompleteness of (government debt) markets (see Hahn, 1971). Finally, there is an economy-wide resource constraint such that total output is equal to (private plus government) consumption:

$$Y_t = C_t + G_t. \tag{20}$$

2.4. *A Policy Problem*

The policy problem is now formulated as a search for the best macroeconomic policy for a monopolistically competitive equilibrium defined as follows:

Definition 1 *A monopolistically competitive equilibrium is defined as a set of plans,* $\left\{C_{t+k}, Y_{t+k}, N_{t+k}, w_{t+k}, \Delta_{t+k}, B_{t+k}, p'_{t+k}, P_{t+k}, \right\}_{k=0}^{\infty}$*, given initial conditions,* $\{b_{t-1}, i_{t-1}, \Delta_{t-1}, P_{t-1}\}$*, and expected dynamics of future policy variables,* $\{E_t P_{t+k}, E_t \tau_{t+k}, \}_{k=0}^{\infty}$*, and exogenous shocks,* $\{E_t A_{t+k}, E_t G_{t+k}, E_t \lambda_{t+k}\}_{k=0}^{\infty}$*, and satisfying conditions (11), (12), (14), (15), (16), (19) and (20).*

We are now able to set out the Ramsey problem in Proposition 2:

Proposition 2 *The Ramsey plan is a choice of state contingent paths for the endogenous variables* $\left\{P_{t+k}, C_{t+k}, \Delta_{t+k}, \tau^h_{t+k}, \right\}_{k=0}^{\infty}$ *from date t onwards given* $\{E_t A_{t+k}, E_t G_{t+k}, E_t \lambda_{t+k}, b_{t-1}, i_{t-1}, \Delta_{t-1}, P$ *so as to maximize social welfare function (21) subject to constraints (22)-(24):*

$$\max E_t \sum_{k=0}^{\infty} \beta^k \left\{ \log\left(C_{t+k}\right) - \lambda_{t+k} \Delta_{t+k}^{v+1} \frac{\left[A_t^{-1}\left(C_{t+k} + G_{t+k}\right)\right]^{(v+1)\phi}}{v+1} \right\}; \tag{21}$$

subject to:

- *Solvency Constraint*

$$\begin{aligned} & \left(1 + i_{t-1}\right) \frac{b_{t-1}}{C_t \pi_t} \\ = \;& E_t \sum_{k=0}^{\infty} \beta^k \left\{ \frac{\tau^h_{t+k}}{1 - \tau^h_{t+k}} \lambda_{t+k} \Delta_{t+k}^{v+1} \left[A_{t+k}^{-1}\left(C_{t+k} + G_{t+k}\right)\right]^{(v+1)\phi} - \frac{G_{t+k}}{C_{t+k}} \right\}; \end{aligned} \tag{22}$$

- *Phillips Curve*

$$\begin{aligned} & \left(\frac{1 - \alpha \pi_t^{\theta - 1}}{1 - \alpha}\right)^{\frac{-\theta + \theta\phi + 1}{1 - \theta}} E_t \sum_{k=0}^{\infty} (\beta\alpha)^k \frac{C_{t+k} + G_{t+k}}{C_{t+k}} \left(\frac{P_t}{P_{t+k}}\right)^{1-\theta} \\ = \;& \frac{\theta\phi}{1-\theta} E_t \sum_{k=0}^{\infty} (\beta\alpha)^k \frac{\lambda_{t+k}}{1 - \tau^h_{t+k}} \Delta^v_{t+k} \left[A_{t+k}^{-1}\left(C_{t+k} + G_{t+k}\right)\right]^{(v+1)\phi} \left(\frac{P_t}{P_{t+k}}\right)^{-\theta\phi}; \end{aligned} \tag{23}$$

- *Law of Motion of Prices*

$$\Delta_t = \alpha \Delta_{t-1} \pi_t^{\theta\phi} + (1 - \alpha) \left(\frac{1 - \alpha \pi_t^{\theta-1}}{1 - \alpha}\right)^{\frac{\theta\phi}{\theta - 1}}. \tag{24}$$

Proof. See Appendix. ■

The foregoing formulation of the policy problem brings out very clearly the general equilibrium impact of price dispersion and the sense in which its impact is like a drag on the level of factor productivity. Note, first, that $\Delta_t \geq 1$.[5] Hence, the following change of variables, $A_t^R := A_t \Delta_t^{-\frac{1}{\phi}}$, demonstrates, that any degree of price dispersion greater than

[5]See Schmitt-Grohe and Uribe (2005). In addition, the working paper version of our paper contains an explicit proof of this assertion using the Jensen inequality.

unity impacts in the utility function and the solvency constraint exactly like a *downward* shift in the level of productivity. This change of variables does not quite work in the Phillips curve where price dispersion enters as Δ_t^v (as opposed to Δ_t^{v+1} in the utility function and solvency constraint). One may be tempted to conclude that this simply points to the fact that optimal monetary policy ought to ensure that price dispersion is minimized, or set to unity (i.e., perfect price-level stability). However, in an appendix available from the authors, it is demonstrated that this analogy between price dispersion and productivity shocks goes through when one incorporates nominal wage stickiness in the manner of Erceg, Henderson and Levin (2000). That is important since in the presence of more than one source of nominal rigidity some systematic deviation from price stability will in general be optimal. Additionally, if one derives a log-linear approximation to this model economy around a non-zero inflation steady state then one finds that in general a policy of ensuring perfect price stability will not be part of a Ramsey program[6]. Section 6 pursues this issue further.

We also note, in passing, that price dispersion also bears a close similarity to a preference shift into leisure. Using the following change of variables, $\lambda_t^R := \lambda_t \Delta_t^{v+1}$, one observes that the problem facing the policymaker is almost identical to that facing a policymaker in an economy with a higher preference for leisure. Again, this change of variables does not quite work in the Phillips relation; here the price dispersion term enters in a less quantitatively significant way: $\Delta_{t+k}^v,\ \forall k$, as opposed to $\Delta_{t+k}^{v+1},\ \forall k$. We prefer to emphasize the similarity between price dispersion and productivity since in the presence of nominal wage rigidity the wage dispersion term is naturally 'paired' with the preference shifter while the price dispersion term is naturally linked, as above, with productivity.

Section 6 returns to the implications for price dispersion of this policy problem. First, we investigate the quantitative impact of price dispersion in the model.

3. The Costs of Price Dispersion

Proposition 3 establishes that rising price dispersion, ceteris paribus, makes the economy behave like a high-cost economy:

Proposition 3 *At the economy-wide level, for a given output level,*
(i) *the labour input employed;*
(ii) *the aggregate production costs;*
(iii) *the disutility from labour,*
all increase in price dispersion.

Proof. The proof of (i) follows immediately from (11).

Not surprisingly, total production costs are increasing in labour employed. Combining (11) with (12) we can calculate total production costs

$$TC_t := w_t N_t = \mu_t \lambda_t \frac{1}{1-\tau_t^h} C_t \left(A_t^{-\phi} Y_t^{\phi} \Delta_t \right)^{1+v}. \tag{25}$$

It follows immediately that $[\partial TC_t / \partial \Delta_t] > 0$.

[6] Anderson et al. (2008) demonstrate that optimal policy under discretion also results in a trend inflation.

Finally, the higher is employment the less time households have for leisure. The aggregate disutility from labour, for a given level of output, is given in (26) and it is clear that this also is increasing in price dispersion:

$$\lambda_t \frac{1}{1+v} N_t^{1+v} = \lambda_t \frac{1}{1+v} \left(A_t^{-\phi} Y_t^{\phi} \Delta_t \right)^{1+v}. \tag{26}$$

■

The implications of this proposition will be useful in interpreting the impulse responses that we report in Section 8.

4. Price Dispersion and Productivity shocks: Some Back-of-the-Envelope Calculations

One may use the law of motion (24) to make some inference on the impact of price dispersion. We do this by mapping a given average level of inflation, via its impact on price dispersion, into an equivalent decrease in productivity using the change of variable deduced above. This is shown in the bottom line of Table 1[7].

Column I corresponds to a benchmark economy, while column II shows that a higher level of competition, θ, makes price dispersion more costly, as does, respectively, the degree of concavity of the production function, ϕ, (column III), inflation, π_t, (column IV) and the degree of price stickiness, α (column V). The final row in the figure, under the maintained assumptions, maps a given degree of price dispersion into an equivalent percentage decrease in productivity. These numbers, and those in subsequent tables, are in terms of annualized percentage decreases. It is striking that a steady-state inflation rate of 2.5% maps into an almost equivalent (2.4%) decrease in factor productivity in the base case (column I).

Table 1

Approximate productivity equivalent cost of inflation

	I	II	III	IV	V
θ	7	10	7	7	7
ϕ	1.38	1.38	1.6	1.38	1.38
π	2.5%	2.5%	2.5%	5%	2.5%
α	0.5	0.5	0.5	0.5	0.6
Δ	1.034	1.09	1.06	1.28	1.08
$1-\Delta^{-1/\phi}$	-2.4%	-5.8%	-3.6%	-16.6%	-5.4%

However, an obvious question follows from this simple analysis: How large is price dispersion in the data? Unfortunately, so far as we are aware, there is little direct empirical guidance on this issue, although there is some general evidence on price dispersion. For instance, Baye, Morgan and Scholten (2004) calculate the coefficient of variation (*cvar*) for

[7] The values for the parameters in column I in this table and in the subsequent tables correspond to those we used in conducting the simulations reported in Section 8. These appear to be in line with much of the literature.

online products in the USA. They find it equals 10% on average. And it may well be the case that the coefficient of variation could be significantly larger in European countries. Gatti and Kattuman (2003), for example, find that the coefficient of variation for online products in the Netherlands is 12.6%, although they also report that the coefficient of variation for online bookstores can be up to 30%. One can, in fact, map these numbers into the above approximate productivity equivalent measure, making no assumptions about trend inflation. Recall the definition of the coefficient of variation:

$$\begin{aligned} cvar &= \frac{\left[\int p^2(i)di - \left(\int p(i)di\right)^2\right]^{1/2}}{\int p(i)di} = \\ &= \frac{\left[\int \left(\frac{p(i)}{P}\right)^2 di - \left(\int \frac{p(i)}{P} di\right)^2\right]^{1/2}}{\int \frac{p(i)}{P} di}. \end{aligned}$$

The appendix shows how one can relate this measure to the model's measure of price dispersion to arrive at the following expression:

$$\Delta \simeq 1 + \frac{1}{2}\frac{\theta\phi}{\theta+1}\left(\theta\phi - \theta + 1\right)\frac{cvar^2}{1 - \frac{1}{2}\frac{\theta}{\theta+1}\left(cvar^2+1\right)}. \tag{27}$$

Applying formula (27) one can estimate the effect of price dispersion in terms of productivity permitting the coefficient of variation to go from 5% to 20% (recall the studies above suggest a range of something like 10% to 30%). The results are reported in Table 2.

Table 2

Approximate productivity equivalent cost of price dispersion

	I	*II*	*III*	*IV*	*V*
$cvar$	0.05	0.1	0.1	0.1	0.2
θ	7	7	10	7	7
ϕ	1.38	1.38	1.38	1.6	1.38
Δ	1.01	1.04	1.06	1.07	1.16
$1-\Delta^{-1/\phi}$	-0.7%	-2.7%	-3.8%	-3.9%	-10.3%

Interestingly, column *II* in Table 2 corresponds quite closely to column *I* in Table 1, in terms of the ultimate productivity-equivalent impact, suggesting that a coefficient of variation of 10%, or a little lower, may be a realistic number. And we emphasize, *no* assumption has been made about inflation in constructing Table 2. Taken together, the complementary evidence in Tables 1 and 2 indicates that an empirically plausible level of price dispersion is potentially very costly in welfare terms. In the spirit of Lucas (1987), we now ask how costly in terms of utility is a given degree of price dispersion.

5. The Consumption Equivalent Cost of Price Dispersion

Two economies are compared. One corresponds to an environment where all firms charge the same price, whilst the other incorporates what we hope is a reasonable level of price dispersion.

Let Φ represent the percentage point amount by which consumption would need to be higher every period, to achieve the same level of utility as in the case when all firms charge the same price, $\Delta_{t+k}^{v+1} = 1$[8]. To calculate this welfare equivalent one sets

$$U_t\left(\Phi, \Delta_{t+k}\right) = E_t \sum_{t=0}^{\infty} \beta^{t+k} \left[\log(C_{t+k}) + \log \Phi - \lambda_{t+k} \Delta_{t+k}^{v+1} \frac{\left(A_{t+k}^{-1} Y_{t+k}\right)^{(v+1)\phi}}{v+1} \right]$$

such that $U_t\left(\Phi, \Delta_{t+k}\right) = U_t(1, 0)$. Table 3 provides details of the calculations based on this expression[9]. The required change in consumption appears far from negligible. Indeed, even on relatively moderate assumptions that number does not fall below 0.5%, and may rise substantially above it; column *II*, assuming a coefficient of variation of prices of 10%, implies a consumption equivalent of 2.2%.

Table 3

Welfare loss from price dispersion: consumption equivalent

	I	*II*	*III*	*IV*	*V*
cvar	0.05	0.1	0.1	0.1	0.2
θ	7	7	10	7	7
ϕ	1.38	1.3	1.38	1.6	1.38
Δ_t	1.01	1.04	1.06	1.07	1.16
g	0.15	0.15	0.15	0.15	0.15
τ	0.25	0.25	0.3	0.25	0.25
$1+\nu$	2.8	2.8	2.8	2.8	2.8
$\Delta^{\nu+1}-1$	2.8%	11.5%	16.4%	19.4%	52.3%
$\frac{1-\tau}{1-g}\frac{1-\theta}{\theta\phi(1+\nu)}$	0.2	0.2	0.19	0.17	0.2
$\Phi\%$	0.5%	2.2%	3.1%	3.3%	10.2%

6. Optimal Discretionary Policy under Price Dispersion

This section returns to the problem of Section 2.4. Following Tack Yun (2005), an economy is considered where there is an initial degree of price dispersion, $\Delta_{t-1} > 1$ and access to lump-sum taxation (the full problem is set out and solved in an appendix of the working paper version). Yun (2005) considers an economy with linear production, $\phi = 1$, whilst the more general case of concave production technology is considered here. The next proposition shows that optimization over price dispersion implies *negative* inflation in a transition period. Lump sum taxes are employed to meet the solvency requirement

[8] Of course, Lucas (1987) considered the mean-variance trade-off in consumption; our thought experiment is trading off mean consumption and mean price dispersion.

[9] In the working paper version we derive an analytical expression for $\log \Phi$.

attached to the policy program. The price setting constraint in this case can be supported by payroll subsidies, τ_{t+k}. Yun (2005) shows that with competitive labour markets the optimal subsidy rate should correct for the distortion associated with imperfect competition, $\tau^h_{t+k} = -\frac{1}{\theta-1}$.

Proposition 4 *(Tack Yun, 2005) Given initial price dispersion, the optimal policy corresponds to negative inflation.*

Proof. One can easily recover this result by writing the first-order condition for the law of motion (24)

$$\frac{\partial \Delta_t}{\partial \pi_t} = \theta\phi\alpha \left[\Delta_{t-1}\pi_t^{\theta\phi-1} - \left(\frac{1-\alpha\pi_t^{\theta-1}}{1-\alpha} \right)^{\frac{\theta\phi}{\theta-1}-1} \pi_t^{\theta-2} \right] = 0. \tag{28}$$

We can simplify (28), which gives us the optimal rate of inflation

$$\pi_t = \left[(1-\alpha)\, \Delta_{t-1}^{\frac{\theta-1}{\theta\phi+1-\theta}} + \alpha \right]^{\frac{1}{1-\theta}}. \tag{29}$$

Clearly, this implies that $\pi_t < 1$ iff $\Delta_{t-1} > 1$. Finally, this optimal path for inflation is feasible[10]. ■

Substituting the expression for optimal inflation (29) into the law of motion (24) one obtains the optimal level of price dispersion for next period which implies the following dynamic relation between inflation and price dispersion:

$$\pi_t^{\theta\phi+1-\theta} = \frac{\Delta_t}{\Delta_{t-1}}. \tag{30}$$

It is important to note that one still cannot recover an optimal stabilization policy for price dispersion should one adopt a second-order approximation around a zero-inflation steady state. The logarithmic second-order approximation to the law of motion is given by

$$\widehat{\Delta_t} = \alpha\widehat{\Delta_{t-1}} + \frac{1}{2}\frac{\alpha}{1-\alpha}\theta\phi\left(\theta\phi+1-\theta\right)\widehat{\pi}_t^2 + O\left(\left\|\xi^3\right\|\right) \tag{31}$$

and the policy that minimizes price dispersion implies immediate inflation stabilization: $\widehat{\pi}_t = 0$.

The usual linear-quadratic approach drops the law of motion (31) as one of "second-order importance", and therefore does not allow one to investigate the dynamics of price dispersion at all. As we noted in the introduction, this assumption lies at the heart of the usual conclusion in the literature that the direct impact of price dispersion on welfare is close to negligible.

[10] The formal demonstration is in an appendix to the working paper version.

7. Reincorporating Price Dispersion into Linearized Models

The reason why price dispersion is generally excluded from linearized models is because the linearization takes place around a steady state in which there is no price dispersion[11]. The previous sections have tried to indicate that price dispersion can be significant even at relatively low rates of inflation. In the remainder of the paper, a log-linear version of our model is developed in which price dispersion is no longer of second-order importance. Crucially, the model is linearized around an inflationary steady state in which there remains some price dispersion.

First, consider price adjustment in the Calvo-Yun set-up. Each period firms who are unable to reprice adjust their price for steady state inflation, $\overline{\pi}$. Other firms are allowed to adjust prices in a more sophisticated way, optimally choosing their price. The aggregate price-level, (15), implies

$$\left(\frac{1-\alpha\left(\pi_t/\overline{\pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{1}{1-\theta}} = \left(\frac{p_t'}{P_t}\right). \tag{32}$$

Thus, the dynamics of price dispersion can be shown to be given by:

$$\begin{aligned}\Delta_t &= \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^{-\theta\phi} di \\ &= \alpha\left(\pi_t/\overline{\pi}\right)^{\theta\phi}\Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\left(\pi_t/\overline{\pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{\theta-1}},\end{aligned} \tag{33}$$

where $\overline{\pi}$ is steady-state inflation. The steady state value of Δ is given by

$$\Delta = \alpha\left(\overline{\pi}/\overline{\pi}\right)^{\theta\varphi}\Delta + (1-\alpha)\left(\frac{1-\alpha\left(\overline{\pi}/\overline{\pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\varphi}{\theta-1}}, \tag{34}$$

which implies that $\Delta = 1$ in steady state. Hence, this is also consistent with the case in which steady-state inflation is zero. Linearizing this expression around this steady state results in

$$\widehat{\Delta}_t = \alpha^k\widehat{\Delta}_{t-k} + O\left(\left\|\xi^2\right\|\right) \simeq O\left(\left\|\xi^2\right\|\right).$$

Now consider the approximation to the law of motion around a steady state with positive inflation and no indexation. This seems a reasonable approach given that one observes little or no indexation in low inflation economies and that most monetary authorities, to put it mildly, do not seem to wish to achieve zero inflation. One finds that[12]

$$\widehat{\Delta}_t = \alpha\pi^{\theta\phi}\widehat{\Delta}_{t-1} + \alpha\theta\phi\frac{\left(\pi^{\theta\phi}-\pi^{\theta-1}\right)}{\left(1-\alpha\pi^{\theta-1}\right)}\widehat{\pi}_t + O\left(\left\|\xi^2\right\|\right). \tag{35}$$

[11] Of course, price dispersion is not entirely absent in L-Q approximate models. That is because price dispersion is the source of the inflation stabilization objective in quadratic approximations to the representative agent's utility function. See Woodford (2003).

[12] The steady-state is characterized by $\Delta = \frac{(1-\alpha)}{(1-\alpha\pi^{\theta\phi})}\left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}\right)^{\frac{\theta\phi}{\theta-1}}$.

Thus price dispersion is not a second-order term any longer and an approximate log-linear model will include price dispersion terms. The law of motion (35) has to be part of the linear system of the model's equations. An implication is that the inflation rate which reduces price dispersion is necessarily below trend inflation, so that one recovers a version of Yun's (2005) result (although we do not pursue that issue in this paper). Also, price dispersion and inflation will now *directly* affect production costs in the same way as a negative productivity shock.

8. The log-linear model

The equations of the full linearized model are in the appendix. The only real algebraic complication is with the Phillips relation. First, recall equation (14). Now, let X_t denote the discounted expected marginal revenue for a firm which charges average price P_t and let Z_t represent the discounted marginal costs. With a little algebra, one may write equation (14) in the following form:

$$\begin{aligned} \frac{\theta\phi}{\theta-1}Z_t &= \left(\frac{1-\alpha\pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{1-\theta+\theta\phi}{1-\theta}} X_t; \\ X_t &= \frac{Y_t}{C_t} + \alpha\beta E_t \pi_{t+1}^{\theta-1} X_{t+1}; \\ Z_t &= \frac{w_t}{C_t}\left(\frac{Y_t}{A_t}\right)^{\phi} + \alpha\beta E_t \pi_{t+1}^{\theta\phi} Z_{t+1}. \end{aligned} \tag{36}$$

Hence, linearizing these expressions results in a 'Phillips bloc':

$$\widehat{Z}_t = \widehat{X}_t + (\theta\phi + 1 - \theta)\frac{\alpha\pi^{\theta-1}}{1-\alpha\pi^{\theta-1}}\widehat{\pi}_t; \tag{37}$$

$$\widehat{X}_t = \left(1-\alpha\beta\pi^{\theta-1}\right)\left(\widehat{Y}_t - \widehat{C}_t\right) + \alpha\beta\pi^{\theta-1}E_t\left[(\theta-1)\widehat{\pi}_{t+1} + \widehat{X}_{t+1}\right]; \tag{38}$$

$$\widehat{Z}_t = \left(1-\alpha\beta\pi^{\theta\phi}\right)\left[\widehat{w}_t - \widehat{C}_t + \phi\left(\widehat{Y}_t - \widehat{A}_t\right)\right] + \alpha\beta\pi^{\theta\phi}E_t\left(Z_{t+1} + \theta\phi\widehat{\pi}_{t+1}\right). \tag{39}$$

When $\pi = 1$, one recovers a standard New Keynesian Phillips relation:

$$\widehat{\pi}_t = \frac{(1-\alpha\beta)}{(\theta\phi+1-\theta)}\frac{1-\alpha}{\alpha}\left(\widehat{w}_t - \widehat{Y}_t + \phi\left(\widehat{Y}_t - \widehat{A}_t\right)\right) + E_t\beta\pi_{t+1}. \tag{40}$$

It is worth emphasizing that the relative price dispersion term operates through the wage to impact on current and future marginal costs. Recalling the expressions for the real wage (12) and labour demand (11) one recovers

$$\widehat{w}_t - \widehat{C}_t + \phi\left(\widehat{Y}_t - \widehat{A}_t\right) = v\widehat{\Delta_t} + \phi(v+1)\left(\widehat{Y}_t - \widehat{A}_t\right) - \widehat{s}_t + \widehat{\lambda}_t, \tag{41}$$

where we define $\widehat{s}_t = \log(\frac{1-\tau_t}{1-\bar{\tau}})$. So, the dynamic equation for marginal cost Z_t may be written as

$$\widehat{Z}_t = \left(1-\alpha\beta\pi^{\theta\phi}\right)\left(v\widehat{\Delta_t} + \phi(v+1)\left(\widehat{Y}_t - \widehat{A}_t\right) - \widehat{s}_t + \widehat{\lambda}_t\right) + \alpha\beta\pi^{\theta\phi}E_t\left(Z_{t+1} + \theta\phi\widehat{\pi}_{t+1}\right).$$

(42)

From equation (42) one clearly sees that price dispersion affects the Phillips curve in the opposite direction to the productivity shock, although its coefficient is about half the size. It also affects the Phillips curve the same way as a tax and cost push shock[13].

Monetary policy may be taken to follow a simple Taylor-type rule:

$$\begin{aligned} \widehat{i}_t &= i_t^* + (\psi_\pi + 1)\widehat{\pi}_t + \psi_y \widehat{Y}_t; \\ i_t^* &= \rho i_{t-1}^* + \widehat{m}_t. \end{aligned}$$

Here, $\widehat{m}_t$ is a white-noise, serially uncorrelated shock; i_t^* is an exogenous stochastic process as in Woodford (2001) which reflects many potential factors such as shifts in the natural rate of output, preference shocks, and such like, and we assume $\rho = 0.9$, consistent with the analysis in Rudebusch (2002)[14]. There is some debate about which output gap monetary authorities actually do react to, so in what follows we simply set $\psi_y = 0$; in effect we assume a simple Wicksell-Woodford reaction function[15]. Had we set $\psi_y = 0.5$, none of our conclusions below would be altered.[16] Finally, we assume that fiscal authorities respond to lagged debt in the following way, $\widehat{s}_t = -\xi \widehat{b}_{t-1}$ and that productivity follows an AR(1) process with white-noise shock, $\widehat{A}_{t+1} = \rho_A \widehat{A}_t + \varepsilon_{t+1}^A$.

Consider a shock to the interest rate target. Each graph in Figure 1 compares the model with price dispersion (the solid line) to the model with no price dispersion (broken line). Following the shock, inflation falls in both model economies but by more in the no-price-dispersion (npd) case. More interestingly, it follows a hump-shaped path in the economy with price dispersion (pd), and appears to be more persistent. That hump-shaped pattern shows up in the path of interest rates (not shown), suggesting a form of interest rate smoothing.

The impact of this shock on price dispersion is persistent and long-lasting. Although Proposition 3 took as given the level of output, it provides insight as to the implications of this fall in price dispersion. Producers anticipate a persistent decline in price dispersion and thus a period of lower than average production costs. This means that firms increase production (so that equilibrium production costs actually rise) as seen in the middle figure. Consequently, labour input rises as does output (lower panel).

The rise in output in the pd economy is again hump-shaped and is a rather striking finding. The reduction in price dispersion, from a distorted steady-state, acts like a positive productivity shock, so long as the change in the target rate is sufficiently persistent.

[13]Our formulation of the Phillips curve may appear similar to Ascari and Ropele (2007). However, in their formulation the impact of $\widehat{\Delta_t}$ appears to have been omitted. That term is absent in Bhakshi et al. (2007) because they focus on firm-specific labour. Hence, there is no direct impact of price dispersion on the equilibrium real wage.

[14]In fact, Rudebusch's results suggest that a value for ρ slightly higher than 0.9 is plausible.

[15]See Woodford (2003) chapter 4.

[16]Our baseline parameter settings are as follows: Preference parameters: $\upsilon = 1.8$, $\lambda = 1$, $\beta = 0.96$. Technology parameters: $\phi = 1.38$, $\theta = 7$, $\alpha = 0.5$. Fiscal policy in a steady state: $b/Y = 0.4$, $g = 0.15$. Monetary policy parameters: $\psi_\pi = 0.5$, $\xi = 0.1$. Persistence of stochastic shocks: $\rho_A = 0.9$.

The model is linearized around two steady states, one where steady-state price dispersion is zero, and inflation is zero, and another where inflation is 2.5% and there is price dispersion in steady state: i.e., $\pi = 1$, or $\pi = 1.025$.

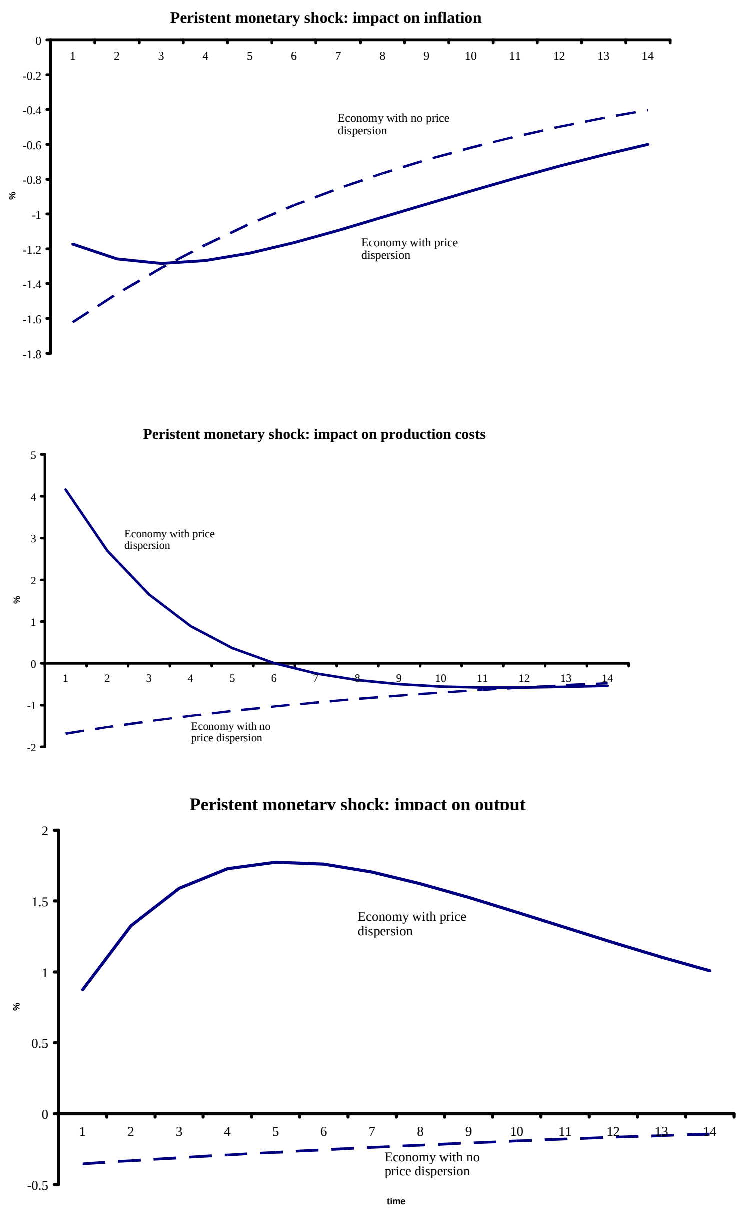

Figure 1. Impulse Responces to Monetary Shock

And it is this increase in output (and hence demand) that accounts for the smaller initial fall in inflation in the pd economy. This result is somewhat reminiscent of the disinflationary booms found by Ball (1994), Ireland (1997) and Nicolae and Nolan (2006). It is worth stressing, however, that our result is distinct in the sense that both economies (i.e., the pd and the npd economies) will display the behaviour identified by Ball for a future anticipated tightening in monetary policy; the channel we have identified is over and above that identified by Ball.

Less persistent shocks to the target rate, ceteris paribus, tend to make inflation persistence less pronounced, although the hump-shaped pattern to interest rates may still be present.

Following a productivity shock[17], inflation and interest rates again follow the hump-shaped path back to base. The deviation of price dispersion is again persistent, whilst output responds maximally in the first period in both model economies. More generally, Table 4 confirms unsurprisingly that productivity and price dispersion are strongly negatively correlated and affect the model's endogenous variables in opposite directions:

Table 4
Correlation coefficients

	Y	Interest rate	π	Debt	Tax	Δ	N	wage
Δ	-0.98	0.98	0.98	0.98	-0.98	1.00	0.62	-0.68
A	0.92	-0.90	-0.90	-0.86	0.86	-0.86	-0.19	0.90

We conclude that the expected impact of a nominal shock looks to be highly dependent on both the persistence of that shock and on the steady state from which the economy is perturbed. If that steady state is distorted by what appears to be an empirically plausible amount of relative price dispersion (here we assumed an economy with a trend inflation of 2.5% and no indexation) then one may obtain some surprising results. By incorporating price dispersion, one can account for a persistent and gradual response in inflation to two familiar types of shocks. However, the response of output to a persistent, contractionary 'nominal' shock is striking and further work is required to understand this and reconcile it with how one typically thinks the economy responds to such a shock.

9. Conclusion

This paper investigated the impact of price dispersion in a simple economic model. Price dispersion impacts the economy like a negative productivity shock. Some issues of approximation around an inflationary steady state were clarified and a Phillips block of equations, with an intuitive interpretation, was derived. The impact of price dispersion on welfare and dynamics is substantial; it made the economy evolve in a more sluggish manner than the model with no price dispersion. Notably, inflation followed a hump-shaped path following either a real or a persistent nominal shock, and so any observed persistence in the

[17] See the working paper version for details.

policy rate was ultimately due to the persistence in the nominal shock, and not 'sluggish' policy decisions. These sorts of issues have been of concern to quantitative theorists recently; see the insightful discussion in Mash (2004). However, the expansionary impact on output of a persistent nominal contraction may be a challenge for the positive properties of the set-up. A number of research questions appear important. It would be especially interesting to know how dispersed are actual prices through time, how that changes with inflation and the persistence of monetary shocks. To slow the response of output in our set-up one may think of incorporating sticky wages, as that may stop production costs from falling so quickly following a monetary contraction. Incorporating learning may also be useful in this regard[18].

[18]Nicolae and Nolan (2006) showed in a related, but simpler, model to the one presented here that one could 'avoid' disinflationary booms by incorporating a period of learning into the model.

10. Appendix

Proposition 2

Proof. The Ramsey plan is a policy plan $\left\{P_{t+k},\ \tau^h_{t+k},\right\}_{k=0}^{\infty}$ which is a monopolistically competitive equilibrium corresponding to Definition 1 and which maximizes (1). We recall that a monopolistically competitive equilibrium is a path for endogenous variables $\left\{C_{t+k},\ Y_{t+k},\ N_{t+k},\ w_{t+k},\ \Delta_{t+k},\ p'_{t+k},\ P_{t+k},\right\}_{k=0}^{\infty}$ satisfying conditions (11), (12), (14), (15), (16), (19) and (20). To obtain a simpler set-up one first substitutes for Y_{t+k}, N_{t+k}, and w_{t+k} using (11), (12) and (20). This results in revised expressions for social welfare (21), the solvency constraint (22) and the Phillips Curve (43),

$$\begin{aligned}&\left(p'_t/P_t\right)^{-\theta+\theta\phi+1} E_t \sum_{k=0}^{\infty} (\beta\alpha)^k \frac{C_{t+k}+G_{t+k}}{C_{t+k}} \left(\frac{P_t}{P_{t+k}}\right)^{1-\theta} \qquad (43)\\ =\ &\frac{\theta\phi}{1-\theta} E_t \sum_{k=0}^{\infty} (\beta\alpha)^k \frac{\lambda_{t+k}}{1-\tau^h_{t+k}} \Delta^v_{t+k} \left(A^{-1}_{t+k}\left(C_{t+k}+G_{t+k}\right)\right)^{(v+1)\phi} \left(\frac{P_t}{P_{t+k}}\right)^{-\theta\phi}.\end{aligned}$$

Then, using (15) one may calculate the optimal relative price,

$$p'_t/P_t = \left(\frac{1-\alpha\pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{1}{1-\theta}}, \qquad (44)$$

which can be used in (16) to obtain the law of motion as in (24). Finally, one uses (44) in the transformed Phillips curve (43) to receive (23). ∎

The Coefficient of Variation and Price dispersion: Derivation of (27)

Recall that Δ_t is our measure of price dispersion. Define $\Delta_t \langle x\rangle := \Delta_t = \int_0^1 \left(\frac{p_t(i)}{P_t}\right)^x di.$ For any x (45) is true up to second order:

$$\Delta_t \langle x\rangle = 1 + x\int \left(\widehat{p_t}(i) - \widehat{P_t}\right) di + \frac{1}{2}x^2 \int \left(\widehat{p_t}(i) - \widehat{P_t}\right)^2 di + O\left(\left\|\xi^3\right\|\right). \qquad (45)$$

Furthermore, from the definition of average price one knows that $\Delta_t \langle 1-\theta\rangle = 1$, which together with (45) gives

$$\int \left(\widehat{p_t}(i) - \widehat{P_t}\right) di = \frac{\theta-1}{2} \int \left(\widehat{p_t}(i) - \widehat{P_t}\right)^2 di + O\left(\left\|\xi^3\right\|\right). \qquad (46)$$

Combining (46) and (45) and setting $x = -\theta\phi$ one finds

$$\Delta_t \langle -\theta\phi\rangle \simeq 1 + \frac{1}{2}\theta\phi\left(\theta\left(\phi-1\right)-1\right) \int \left(\widehat{p_t}(i) - \widehat{P_t}\right)^2 di. \qquad (47)$$

The coefficient of variation is the ratio of standard deviation to mean,

$$cvar = \frac{\sqrt{\Delta_t\langle 2\rangle - \Delta_t^2\langle 1\rangle}}{\Delta_t\langle 1\rangle}. \qquad (48)$$

One may express $\Delta_t \langle 2 \rangle$ and $\Delta_t \langle 1 \rangle$ using relation (47). Combining the resulting pair of equations one receives

$$\Delta_t \langle 1 \rangle \simeq 1 + \frac{1}{2} \frac{\theta}{\theta + 1} \left(\Delta_t \langle 2 \rangle - 1 \right). \tag{49}$$

Expressions (48) and (49) help us to relate $\Delta_t \langle 2 \rangle$ and the coefficient of variation, $cvar$:

$$\frac{cvar^2}{1 - \frac{1}{2} \frac{\theta}{\theta+1} \left(cvar^2 + 1 \right)} = \Delta_t \langle 2 \rangle - 1 \tag{50}$$

Finally, one can combine (47) and $\Delta_t \langle 2 \rangle$ to receive

$$\Delta_t \langle -\theta\phi \rangle \simeq 1 + \frac{1}{2} \frac{\theta\phi}{\theta + 1} \left(\theta\phi - \theta + 1 \right) \left(\Delta_t \langle 2 \rangle - 1 \right). \tag{51}$$

Now, using (50) in (51) one receives the final expression, (27), used in the main text.

11. The log-linear model

The expression for the real wage is obtained using (12)

$$\widehat{\lambda}_t + v\widehat{N}_t + \widehat{C}_t = \widehat{w}_t + \widehat{s}_t, \tag{52}$$

where we define $\widehat{s}_t = \log(\frac{1-\tau_t}{1-\bar{\tau}})$.

2. The log-linear form of labour demand is derived from (11):

$$\widehat{N}_t = \widehat{\Delta_t} + \phi \left(\widehat{Y}_t - \widehat{A}_t \right). \tag{53}$$

3. Market clearing is derived using (20):

$$\widehat{Y}_t = (1 - g)\widehat{C}_t + g\widehat{G}_t. \tag{54}$$

4. The log-linear form of the Phillips relation was discussed in the text.
5. Approximating equation (5) yields

$$E_t\widehat{C}_{t+1} + E_t\widehat{\pi}_{t+1} = \widehat{C}_t + \widehat{i}_t, \tag{55}$$

where $\widehat{i_t}$ is the gross nominal interest rate, $\widehat{i_t} = \log\left(\frac{\beta}{\pi}(1 + i_t)\right)$.

6. We log linearize $E_t b_t \pi_{t+1} \frac{1}{1+i_{t-1}} = b_{t-1} - \tau_t^h w_t N_t + G_t$ to yield

$$\frac{b}{C}\beta \left(\widehat{b}_t + E_t\widehat{\pi}_{t+1} - \widehat{i_{t-1}} \right) = \frac{b}{C}\widehat{b}_{t-1} - \tau \frac{wN}{C} \left(\frac{\tau - 1}{\tau} \widehat{s}_t + \widehat{w}_t + \widehat{N}_t \right) + \frac{g}{1-g}\widehat{G}_t. \tag{56}$$

7. The log-linear dynamics of price dispersion is

$$\widehat{\Delta}_{t+1}\pi^{-\theta\phi} - \alpha\theta\phi \frac{\left(\pi^{\theta\phi} - \pi^{\theta-1}\right)}{\left(1 - \alpha\pi^{\theta-1}\right)} \widehat{\pi}_{t+1} = \alpha\widehat{\Delta}_t. \tag{57}$$

To close the system we need to specify the actions of the fiscal and monetary authorities which we did in the main text.

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