Revisiting Overborrowing and its Policy Implications[*]

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First Draft: April 27, 2010

Abstract

This paper analyzes quantitatively the extent to which there is overborrowing (i.e., inefficient borrowing) in a business cycle model for emerging market economies with production and an occasionally binding credit constraint. The main finding of the analysis is that overborrowing is not a robust feature of this class of model economies: it depends on the structure of the economies and their parametrization. Specifically, we find underborrowing in a production economy with our baseline calibration, but overborrowing with more impatient agents and more volatile shocks. Endowment economies display overborrowing regardless of parameter values, but they do not allow for policy intervention when the constraint binds (in crisis times). Quantitatively, the welfare gains from implementing the constrained-efficient allocation are always larger near crisis times than in normal ones. In production economies, they are one order of magnitude larger than in endowment economies both in crisis and normal times. This suggests that the scope for economy-wide macro-prudential policy interventions (e.g. prudential taxation of capital flows and capital controls) is weaker in this class of models.

1 Introduction

Economies with imperfect financial market access may experience crises that cause significant economic dislocation. These crises are characterized by the sudden stop of domestic or international credit flows and are associated with large declines in consumption, output, relative prices, and asset prices.[1]

An important question for emerging-market economies is whether the likelihood and the severity of these crises is affected by excessive borrowing in normal times, when access to financial markets is unconstrained and plentiful. This question is important because the policy implications of alternative answers are very different. If there is excessive or inefficient borrowing in good times (i.e., "overborrowing"), policy should be geared primarily toward addressing the ex ante inefficiency that causes it; for example, by imposing a tax on capital flows or other forms of capital controls and prudential regulations to reduce the incentives to borrowing excessively.[2] In this case, policy should be less concerned about mitigating the consequences of the crisis, when one occurs, and instead strengthen the ex ante incentives to borrow efficiently in good times. In contrast, if there is no overborrowing in good times, policy should be geared primarily toward designing efficient ex post interventions mechanisms in bad times (such as bailout interventions financed nationally or multilaterally), trying to minimize the costs of the inevitable crises associated with imperfect access to financial markets.[3]

A fast growing literature has examined this issue in related work. In an early contribution Fernandez-Arias and Lombardo (1998) and Uribe (2007) have examined the possibility of overborrowing in economies subject to exogenous (either individual or aggregate) debt limits. Lorenzoni (2008) and Korinek (2010) have explored the possibility of overborrowing qualitatively in models in which the debt limit is endogenous. Uribe (2007) and Bianchi (2009) have examined the issue quantitatively with contrasting results. While Uribe (2007) finds no overborrowing, Bianchi (2009) finds that overborrowing is quantitatively relevant and has significant welfare implications. In endowment economies, Korinek (2010) and Bianchi (2009) suggest that only macro prudential policies have scope for preventing and mitigating crises. In contrast, Benigno, Chen, Otrok, and Rebucci (2009), based on a model with production similar to the one used in this paper, find that it is optimal (in Ramsey

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[1]The recent global crisis that stemmed from the US sub-prime mortgage market is the most vivid example of a financial sudden stop, but the long sequence of emerging market crashes since the mid-1990s is an equally important illustration of how disruptive financial "sudden stops" can be.

[2]See for instance the recent introduction of a tax on international portfolio flows by Brazil, or Chile's earlier experiance with capital controls on foreign inflows.

[3]See Caballero (2010) for a detailed discussion of alternative modalities of ex post interventions.

sense) to intervene mostly ex post, once a sudden stop actually occurs.

This paper analyzes quantitatively the extent to which there is overborrowing in a business cycle model for emerging market economies with production and an occasionally binding credit constraint. We investigate overborrowing in small open economy model with production and imperfect access to international capital markets. Our occasionally binding credit constraint is embedded in a standard two-sector (tradable and non-tradable good) small open economy in which financial markets are not only incomplete but also imperfect, as in Mendoza (2002). In this model, for simplicity, production takes place only in the non-tradable sector of the economy. The asset menu is restricted to a one period risk-free bond paying off the exogenously given foreign interest rate. In addition to asset market incompleteness, we assume that access to foreign financing is constrained to a fraction of households' total income. Thus, foreign borrowing is denominated in units of the tradable good but it is leveraged on income generated at different relative prices (i.e. the relative price of non-tradable good). The specification of the borrowing constraint thus captures "liability dollarization" a key feature of emerging market capital structure (e.g., Krugman, 1999).[4] As it is well known, however, pecuniary externalities like the one at work in our model can arise in much more general circumstances: namely, whenever a relative price enters the specification of a financial friction in a multiple good economy. (See Arnott, Greenwald, and Stiglitz, 1994 for a detailed discussion and a survey of the theoretical literature.)

There are two defining features of this environment that are shared with most of the related literature. First the international borrowing constraint binds only occasionally: the crisis, defined as the event in which the constraint binds, is an endogenous event that depends on agents' decisions, the policy regime, and the state of the economy. Second, potential scope for policy intervention arises in this environment because of the existence of a pecuniary externality that stems from individual agents failing to internalize the aggregate implications of their borrowing decision on the relative price of non-tradable goods, that in turns affect the value of the collateral.[5]

To investigate overborrowing quantitatively we compare the competitive equilibrium (CE) with the constrained efficient allocation chosen by a welfare maximizing social planner (SP), solving them with global solution methods. That is we solve for decision rules for all

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[4]The latest wave of crises in emerging Europe and the corporate sector problems in Mexico and Brazil in the fourth quarter of 2008 represent striking evidence of the importance of such feature.

[5]Benigno et al. (2009), among others, show that CE allocation of this economy is not constrained-inefficient in the sense of Kehoe and Levine (1993). Benigno et al. (2009) also discuss how efficiency can be restored with a distortionary tax on non-tradable consumption in a deterministic two period version of the model used here.

endogenous variables across both states of the world when the constraint binds and when it does not. Such an approach enforces that the behavior away from the crisis periods is based on full knowledge of what the equilibrium will be when the economy enters the crisis state. This solution method, while computationally costly, is critical for understanding the interaction between different states of the world.[6]

We find that overborrowing is a quantitative matter: it depends on both the model specification and the values of the model parameters. Specifically, in our production model, there is a divergence between the CE and the SP allocations both when the constraint binds and when it does not, with under- or over-borrowing in normal times (i.e. when the constraint does not bind) depending on the parametrization of the economy. In the baseline calibration, we find underborrowing in normal times. In an alternative calibration, with more impatient agents and more volatile shocks, we find overborrowing in normal times. In both cases, however, in crisis times (i.e. when the constraints binds), there is inefficient under-borrowing. That is, in crisis times, agents in the CE always reduce their consumption of tradable more than in the SP allocation.

In general, the main difference between the CE and the SP allocations is that the social planner takes into account the effects of his consumption choices on aggregate prices, and thus on the value of the collateral (i.e. this is what the literature refers to as "pecuniary externality"). The implications of this pecuniary externality depend on the structure of the economy. In general, indeed, even in normal times, the possibility that the constraint might bind in the future increases the current marginal utility of tradable consumption (i.e. increases the private marginal value of saving). But the *social* marginal value of saving (from the perspective of the social planner) is higher than the *private* value (from the perspective of individual agents) because of the effect of the pecuniary externality. All else equal, this mechanism implies higher saving in the SP allocation compared to the CE allocation, and it generates overborrowing in the endowment economies studied by Bianchi (2009) and Korinek (2010). But in a production economy an opposite force arises. The relatively higher marginal utility of tradable consumption from the social planner's perspective generates a higher social marginal benefit of supplying one more unit of labor compared to the private one in normal times. A relatively higher production and consumption of non tradable goods can then lead to a relatively higher borrowing and tradable consumption in the SP compared to the CE, and thus generates the possibility of underborrowing. The relative strength of these two effects depends on the parametrization of the economy: for example,

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[6]The technical challenge in solving such a model is that the constraint binds only occasionally and changes location in the state space of the model depending on the realization of both the exogenous and the endogenous state variable.

the second channel dominates the first one in our baseline calibration , but we find that the first channel dominates the second when agents are more impatient and shocks are more volatile, inducing overborrowing rather underborrowing like in our baseline calibration. Overborrowing always arises in the endowment economies we study because the second effect is not present. Also, in the endowment case, the planner cannot manipulate the value of the collateral when the constraint binds as it cannot alter the production possibilities of the economy: thus CE and SP allocation must always coincide once the crisis occurs in an endowment economy.[7]

From a qualitative point of view, the findings of our analysis suggest that there is a clear cut rationale only for ex post interventions to address the economic dislocation associated with the sudden stop. The findings also suggest that the design of economy-wide ex ante policy interventions is not robust: indeed, different structures of the economies or calibrations may imply that the type of intervention might differ depending on the existence of either under- or over-borrowing.[8]

We then measure quantitatively the gap between the CE and the SP allocations. To do so we determine the percentage of consumption that agents are willing to forgo to move from one allocation to the other in every state and date. We find that, in production economies, the overall welfare gains from implementing the SP allocation are one order of magnitude larger than in endowment economies. In addition, welfare gains are always larger near crisis times than in normal ones in both production and endowment economies.

In terms of policy implications, the findings of the analysis in the paper are consistent with the importance of bailouts financed nationally or multilaterally to help mitigating the effects of crises when they occur. In contrast, the analysis suggests that the case for economy-wide macro-prudential policy interventions tools such as taxes on capital flows and capital controls (as opposed to interventions specifically targeting the financial system) is very weak in the context of our modelling approach.

There are important caveats to these policy conclusions. Moral hazard and time-consistency considerations, as well as the economic cost of the distortions introduced by ex ante intervention policies, are not present in the class of models analyzed in the paper. As a result, the case for ex-post (ex ante) policy intervention may be over (under) stated by our analysis. Considering moral hazard would weaken (strengthen) the case for ex post (ex ante) interventions. In addition, Chari and Kehoe (2009), show that the lack of cred-ibility of efficient ex post intervention policies call for an ex ante prudential intervention

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[7]The equivalence between SP and CE allocation arises in states of the world in which the crisis occurs (i.e. the constraint it binding) for both allocations.

[8]Implementation issues are not discussed in this paper.

geared toward containing the excesses induced by the time-inconsistency of the optimal ex post intervention. This would further weaken (strengthen) the case for ex post (ex ante) interventions.

Second, while it is well known that bail outs can induce moral hazard, it is less well understood that prudential regulations and capital controls can hamper long run growth. Nikolov (2009), for instance, studies the private choice of leverage ratio in a model with heterogenous firm productivity, based on a stochastic version of Kiyotaki and Moore (1997). He finds that mandating tighter economy-wide leverage ratios than those chosen by private agents in a competitive equilibrium does reduce aggregate volatility, but at the cost of lowering average growth with welfare reducing consequences. As a result, in his model, the aggregate leverage ratio of the competitive equilibrium is constrained-efficient. This strengthen (weaken) the case for ex post (ex ante) interventions.

The rest of the paper is organized as follows. Section 2 discusses the pecuniary externality that may give rise to under or over-borrowing. Section 3 describes the model we use. Section 4 discusses its parametrization and solution. Section 5 illustrates the working of the model and its basic properties, and reports the main quantitative results of the paper, comparing the CE and the SP equilibria under alternative model specifications and parameter values. Section 6 discusses the policy implication of the analysis. Section 7 concludes.

2   Overborrowing and pecuniary externalities

Before turning to the presentation of the model we discuss the source of the externality that may give rise to over- or under-borrowing and hence scope for policy intervention. Overborrowing has been discussed extensively in the literature so our discussion of the pecuniary externality than may give rise to it is in the form of a review of the literature closely related to our work.

In an early contribution, Fernandez-Arias and Lombardo (1998) investigate analytically whether an economy with an aggregate debt limit tends to overborrow relative to an economy in which the debt limit is imposed at the level of the individual agent. They find that agents fail to internalize the debt limit, and the economy overborrows. Uribe (2007) investigates overborrowing quantitatively and finds that the amount borrowed is independent on foreign lenders basing their decisions on individual as opposed to aggregate variables.

The models used in these early analyses are similar. The key difference among the two environments is that in Uribe (2007), when the constraint is binding, the domestic interest rate adjusts and induces agents to internalize the credit limit, while Fernandez-Arias and Lombardo (1998) assume that the domestic interest rate is equal to the world interest rate

and agents fail to internalize the debt ceiling in their deterministic model. In both papers, however, there are two common ingredients. First, the debt ceiling is exogenously specified.[9] Second, the economy is a one good economy, in which the pecuniary externality we focus on cannot arise (See Benigno et al, 2009, Section 2 for details on this).

More recent work has considered richer environments in which there are multiple goods and the borrowing limit is endogenous. In this environments, the interaction between the borrowing constraint and the dependence of the borrowing limit on a relative price generates a pecuniary externality that is not internalized in the competitive equilibrium allocation and might give rise to constrained-inefficient borrowing. The social planner, on the other hand, takes into account the way in which such relative price is determined in the competitive allocation when choosing its optimal plan and chooses accordingly a constrained-efficient amount of borrowing (again, see Benigno et al (2009) for more details). For instance, in a closed economy model, Lorenzoni (2008) shows that entrepreneurs do not take into account the effects of asset prices on the amount that they can borrow so that in the competitive equilibrium, under certain specific assumptions, financial contracts result in excessive borrowing. Similar analysis has been conducted by Korinek (2010) and Bianchi (2009) in a small open economy setting similar to our baseline model, but without production, in which the amount that individuals can borrow depends on the income generated in both sectors of the economy and their relative price. Both authors conclude that there is overborrowing, qualitatively (Korinek, 2010) and quantitatively with potentially significant welfare consequences (Bianchi, 2009). The policy implications of these analysis is to recommend a prudential taxation to capital flows to align the competitive allocation of the economy to the one efficiently chosen by the social planner.

In related work, Nikolov (2009), in a stochastic version of the Kiyotaki and Moore (1997) model, finds that these pecuniary externalities do not necessarily induce sizable divergence between the CE and the SP when a leverage ratio is a variable of choice. This is because, interestingly, in Nikolov's (2009) model, there is not only production but also firm heterogeneity. Thus, in this environment, there is a trade off between the lower volatility and the lower average growth associated with mandating a lower aggregate leverage ratio than that privately chosen in the CE of the economy. So mandating lower regulatory leverage ratios may impose significant efficiency costs that are welfare reducing in this set up.

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[9]Uribe (2007) considers one extension in which the constraint is endogenous in the sense we explained in the previous section. In this case, he finds a small overborrowing.

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3 Model

The model that we propose is a simple two-sector (tradable and non-tradable) small production open economy in which financial markets are not only incomplete but also imperfect as in Mendoza (2002), and in which production occurs only in the non-tradable sector.

3.1 Households

There is a continuum of households $j \in [0, 1]$ that maximize the utility function

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{1}{1-\rho} \left(C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta} \right)^{1-\rho} \right\}, \tag{1}$$

with C_j denoting the individual consumption basket and H_j the individual supply of labor. For simplicity we omit the j subscript for the remainder of this section, but it is understood that all choices are made at the individual level. The elasticity of labor supply is δ, while ρ is the coefficient of relative risk aversion. In (1), the preference specification follows from Greenwood, Hercowitz and Huffman (1988): in the context of a one-good economy this specification eliminates the wealth effect from the labor supply choice. Here, in multi-good economy, the sectoral allocation of consumption will affect the labor supply decision through relative prices. The consumption basket, C_t, is a composite of tradable and non-tradable goods:

$$C_t \equiv \left[\omega^{\frac{1}{\kappa}} \left(C_t^T \right)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} \left(C_t^N \right)^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}. \tag{2}$$

The parameter κ is the elasticity of intratemporal substitution between consumption of tradable and nontradable goods, while ω is the relative weight of the two goods in the utility function.

We normalize the price of traded goods to 1. The relative price of the nontradable good is denoted P^N. The aggregate price index is then given by

$$P_t = \left[\omega + (1-\omega) \left(P_t^N \right)^{1-\kappa} \right]^{\frac{1}{1-\kappa}},$$

where we note that there is a one to one link between the aggregate price index P and the relative price P^N. Households maximize utility subject to their budget constraint, which is expressed in units of tradeable consumption. The constraint each household faces is:

$$C_t^T + P_t^N C_t^N = \pi_t + W_t H_t - B_{t+1} + (1+i) B_t, \tag{3}$$

where W_t is the wage in units of tradable goods, B_{t+1} denotes the foreign borrowing at the end of period t with gross real return $1 + i$. Households receive profits, π_t, from owning the representative firm. Their labor income is given by $W_t H_t$.

International financial markets are incomplete and access to them is also imperfect. The asset menu includes only a one-period bond denominated in units of tradable consumption. In addition, we assume that the amount that each individual can borrow internationally is limited by a fraction of his current total income:

$$B_{t+1} \geq -\frac{1-\phi}{\phi}\left[\pi_t + W_t H_t\right].\tag{4}$$

This constraint captures the effects of liability dollarization since foreign borrowing is denominated in units of tradables while the income that can be pledged as collateral is generated also in the non-tradable sector. This constraint is also endogenous as it depends on the current realization of profits and wage income. We don't derive explicitly the credit constraint as the outcome of an optimal credit contract between lenders and borrowers. However, we can interpret this constraint as the outcome of the interaction between lenders and borrowers in which the lenders is not willing to permit borrowing beyond a certain limit.[10] This limit depends on the parameter ϕ that measures the tightness of the borrowing constraint and it depends on current gross income that could be used as a proxy of future income.[11]

Households maximize (1) subject to (3) and (4) by choosing C_t^N, C_t^T, B_{t+1}, and H_t. The first order conditions of this problem are the following:

$$C_T : \left(C_{j,t} - \frac{H_{j,t}^\delta}{\delta}\right)^{-\rho} \omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t,\tag{5}$$

$$C_N : \left(C_{j,t} - \frac{H_{j,t}^\delta}{\delta}\right)^{-\rho} (1-\omega)^{\frac{1}{\kappa}} \left(C_t^N\right)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t P_t^N,\tag{6}$$

$$B_{t+1} : \mu_t = \lambda_t + \beta \left(1 + i\right) E_t\left[\mu_{t+1}\right],\tag{7}$$

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[10] As emphasized by Mendoza (2002), this form of liquidity constraint shares some features, namely the endogeneity of the risk premium, that would be the outcome of the interaction between a risk-averse borrower and a risk-neutral lender in a contracting framework as in Eaton and Gersovitz (1981). It is also consistent with anecdotal evidence on lending criteria and guidelines used in mortgage and consumer financing.

[11] As we discuss in Benigno et al. (2009), a constraint expressed in terms of future income which could be the outcome of the interaction between lenders and borrowers in a limited commitment environment would introduce further computational difficulties that we need to avoid for tractability.

and

$$H_t : \left(C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta} \right)^{-\rho} \left(H_{j,t}^{\delta-1} \right) = \mu_t W_t + \frac{1-\phi}{\phi} W_t \lambda_t. \tag{8}$$

Note here that, when the credit constraint is binding $(\lambda_t > 0)$, the Euler equation (7) incorporates effects that can be interpreted as arising from a country-specific risk premium on external financing. The extent of this affect is governed by the degree of risk aversion. Moreover in this framework there is an intertemporal effect coming from the possibility that the constraint might be binding in the future: this effect is embedded in the term $E_t \left[\mu_{t+1} \right]$, which implies that future consumption of tradeable goods would be lower compared to the unconstrained case when the constraint is expected to bind in the future.

From the previous conditions, we can combine (5) and (6) to obtain the intratemporal allocation of consumption and (5) with (8) to obtain the labor supply schedule.

$$\frac{(1-\omega)^{\frac{1}{\kappa}} \left(C_t^N \right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} \left(C_t^T \right)^{-\frac{1}{\kappa}}} = P_t^N \tag{9}$$

$$\left(H_{j,t}^{\delta-1} \right) = \left(\frac{\omega C}{C_T} \right)^{\frac{1}{\kappa}} W_t \left(1 + \frac{1-\phi}{\phi} \frac{\lambda_t}{\mu_t} \right). \tag{10}$$

Note here that

$$\left(\frac{\omega C}{C_T} \right)^{\frac{1}{\kappa}} = \left(1 + \left(\frac{1-\omega}{\omega} \right)^{\frac{1}{\kappa}} \left(\frac{C_t^N}{C_t^T} \right)^{\frac{\kappa-1}{\kappa}} \right)^{\frac{1}{\kappa-1}} = \left(1 + \left(\frac{1-\omega}{\omega} \right) (P_t^N)^{1-\kappa} \right)^{\frac{1}{\kappa-1}}.$$

So, if we were in a one good economy, there would be no effect coming from the marginal utility of consumption on the labour supply choice because of the GHH specification. For later use, at this point, it is also useful to note that an increase in P_N would lower $\left(\frac{\omega C}{C_T} \right)^{\frac{1}{\kappa}}$, and the labor supply curve becomes flatter as P_N increases. When the constraint is binding $(\lambda_t > 0)$, the marginal utility of supplying one more unit of labor is higher and this helps to relax the constraint. In this case, the labor supply becomes steeper and agents substitute leisure with labor to increase the value of their collateral for given wages and prices.

Importantly, labor supply is also affected by the possibility that the constraint may be binding in the future. If in period t the constraint is not binding but it may bind in period $t+1$, we have

$$\left(C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta} \right)^{-\rho} \left(H_{j,t}^{\delta-1} \right) = \mu_t W_t$$

and

$$\mu_t = \beta\left(1+i\right)E_t\left[\lambda_{t+1} + \beta\left(1+i\right)E_t\left[\mu_{t+2}\right]\right],$$

so that the marginal benefit of supplying one more unit of labor today is higher, the higher is the probability that the constraint will be binding in the future. This effect will induce agents to supply more labor for any given wage, and the labor supply curve will be flatter relative to the case in which there is no credit constraint. In equilibrium, this effect increases the level of non-tradable production and consumption and affects tradable consumption depending on the degree of substitutability between tradable and non-tradable goods. When goods are complements, the increases in nontradable consumption is associated with an increase in tradable consumption that reduces the amount agents save in the competitive equilibrium. The opposite would occur if goods were substitute.

3.2   Firms

The firms are endowed with a stochastic stream of tradable goods, $\exp(\varepsilon_t^T)Y^T$, where ε_t^T is a stochastic process, and produce non-tradable goods, Y^N. We assume that ε^T follows an autoregressive process of the first order (AR(1)). We abstract from other sources of macroeconomic uncertainty, such as shocks to the technology for producing non-tradables and the world interest rate for simplicity.

Firms produce non-tradables goods, Y_t^N, with a variable labor input and a Cobb-Douglas technology

$$Y_t^N = AH_t^{1-\alpha},$$

where A is a scaling factor. The firm's problem is static and current-period profits (π_t) are:

$$\pi_t = \exp\left(\varepsilon_t^T\right)Y^T + P_t^N AH_t^{1-\alpha} - W_t H_t.$$

The first order condition for labor demand is:

$$W_t = \left(1-\alpha\right)P_t^N AH_t^{-\alpha}, \tag{11}$$

so that the value of the marginal product of labor is set equal to the real wage (W_t). For the case in which we have constant return to scale $(\alpha = 0)$ we obtain:

$$W_t = P_t^N A,$$

so that the real wage in terms of the relative price of non-tradable is constant (as long as we

don't have any shock to productivity of non-tradable), and equilibrium labor is determined by the supply side while the wage rate is determined by the demand side of the labor market.

3.3 Aggregation and equilibrium

To gain insight on the working of the model, we focus on the labor market equilibrium condition when firms have constant returns to scale technology ($\alpha = 0$). Combining (11) with (10) we obtain:

$$\left(H_{j,t}^{\delta-1}\right) = \left(\frac{\omega C}{C_T}\right)^{\frac{1}{\kappa}} P_t^N A \left(1 + \frac{1-\phi}{\phi}\frac{\lambda_t}{\mu_t}\right).$$

When the international borrowing constraint is not binding ($\lambda_t = 0$), a shock that triggers a decrease in P_t^N will reduce the supply of labor, and the amount of non-tradable goods produced. Indeed, in this case, the equilibrium in the labor market would become:

$$\left(H_{j,t}^{\delta-1}\right) = \left(1 + \left(\frac{1-\omega}{\omega}\right)\left(P_t^N\right)^{1-\kappa}\right)^{\frac{1}{\kappa-1}} P_t^N A.$$

To determine the good market equilibrium, combine the household budget constraint and the firm's profits with the equilibrium condition in the nontradable good market to obtain the current account equation of our small open economy:

$$C_t^T = Y_t^T - B_{t+1} + (1+i)B_t. \tag{12}$$

Nontradable good market equilibrium condition implies that

$$C_t^N = Y_t^N = AH_t.$$

Finally, using the definitions of firm profits and wages, the credit constraint implies that the amount that the country as a whole can borrow is constrained by a fraction of the value of its GDP:

$$B_{t+1} \geq -\frac{1-\phi}{\phi}\left[\exp\left(\varepsilon_t^T\right)Y^T + P_t^N Y^N\right], \tag{13}$$

so that (12) and (13) determines the evolution of the foreign borrowing.

3.4 Social Planner Problem

Let's now consider the social planner's problem. The planner maximizes (1) subject to the resource constraints, the international borrowing constraint from an aggregate perspective, and the pricing rule of the competitive equilibrium allocation. In particular, by noting that the relative price is determined by the competitive rule (9), we can rewrite (13) as:

$$B_{t+1} \geqslant -\frac{1-\phi}{\phi} \left[\exp\left(\varepsilon_t^T\right) Y^T + \left(\frac{(1-\omega)\left(C_t^T\right)}{\omega} \right)^{\frac{1}{\kappa}} (AH_t)^{\frac{\kappa-1}{\kappa}} \right].$$

The planner chooses the optimal path of C_t^T, C_t^N, B_{t+1} and H_t, and the first order conditions for its problem are given by:

$$C_T : \left(C_{j,t} - \frac{H_{j,t}^\delta}{\delta} \right)^{-\rho} \omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_{1,t} - \lambda_t \frac{1-\phi}{\phi} \frac{1}{\kappa} \frac{(1-\omega)}{\omega} \left(\frac{(1-\omega)\left(C_t^T\right)}{\omega} \right)^{\frac{1}{\kappa}-1} (AH_t)^{\frac{\kappa-1}{\kappa}},$$

(14)

$$C_N : \left(C_{j,t} - \frac{H_{j,t}^\delta}{\delta} \right)^{-\rho} (1-\omega)^{\frac{1}{\kappa}} \left(C_t^N\right)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_{2,t}, \tag{15}$$

$$B_{t+1} : \mu_{1,t} = \lambda_t + \beta\left(1+i\right) E_t \left[\mu_{1,t+1}\right] \tag{16}$$

and

$$H_t : \left(C_t - \frac{H_t^\delta}{\delta} \right)^{-\rho} \left(H_t^{\delta-1}\right) = \mu_{2,t} A + \frac{1-\phi}{\phi} \lambda_t \left(\frac{(1-\omega)\left(C_t^T\right)}{\omega} \right)^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa} A\left(AH_t\right)^{-\frac{1}{\kappa}}. \tag{17}$$

There are two main differences between the competitive equilibrium first order conditions and the those of the planner's problem introduced by the presence of the occasionally binding financial friction. First, equation (14) shows that, in choosing tradable consumption, the planner takes into account the effects that a change in tradable consumption has on the value of the collateral (see also Korinek, 2010 and Bianchi, 2009). This is what is usually referred as the price externality in the related literature and it occurs when the constraint is binding (i.e. $\lambda_t > 0$). As noted above, however, even if the constraint is not binding today, the possibility that it might bind in the future can affect the marginal value of tradable consumption today (i.e. the marginal value of saving). Indeed, as Bianchi (2009) notes, the Euler equation from the planner perspective becomes

$$\mu_{1,t} = \beta\left(1+i\right) E_t \left[\lambda_{t+1} + \beta\left(1+i\right) E_t \left[\mu_{1,t+2}\right] \right]$$

where $E_t\left[\mu_{1,t+2}\right]$ is given by (14) and takes into account the future effect of the pecuniary externality. This crucially implies that, at the same allocation, the marginal social value of saving (the marginal value in the SP allocation), through this effect, will be higher than the private value (in the CE allocation). Thus, the decentralized equilibrium might display overborrowing.

In the production economy that we study, the presence of the occasionally binding financial friction has an additional effect. In particular, we can rewrite the labor supply equation by using (15) and the equilibrium condition in the non-tradable good market as:

$$\left(H_t^{\delta-1}\right) = \left(\frac{(1-\omega)C_t}{AH_t}\right)^{\frac{1}{\kappa}} A \left\{1 + \frac{1-\phi}{\phi} \frac{\lambda_t}{\mu_{2,t}} \left(\frac{(1-\omega)\left(C_t^T\right)}{\omega}\right)^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa} \left(AH_t\right)^{-\frac{1}{\kappa}}\right\}.$$

This expression shows that, when the constraint is binding, the marginal utility of supplying one extra unit of labor is affected by the degree of substitutability between tradables and non-tradables. If the goods are substitutes then, when the borrowing constraint is binding, it is worth supplying one more unit of labor as that helps in relaxing the constraint. On the other hand, when goods are complement, it is worth decreasing the amount of labor supplied. In both cases the planner tends to relax the international borrowing constraint by increasing the value in units of tradable of non-tradable production: in the case of complements this is achieved by an increase in prices that dominates the negative effect of lower non-tradable production and consumption. In the case of substitute, this is achieved by an increase of non-tradable production and consumption that dominates the effect of lower prices.

But, more importantly, changes in labor supply occur also when the constraint is expected to bind in the future. Indeed, in this case, taking the ratio of (15) and (14) we have:

$$\frac{(1-\omega)^{\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}} \frac{\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\left(C_t^T\right)^{-\frac{1}{\kappa}}} \mu_{1,t} = \mu_{2,t}. \tag{18}$$

This expression shows that relatively higher current marginal utility of tradable consumption in the SP (that arises because of the possibility that the constraint might bind in the future) implies that also a higher marginal utility of non-tradable consumption, which in turn increases the marginal utility of supplying one unit of labor today. As a result, in the SP allocation, labor supply and non-tradable production are relatively higher than in the CE than in the SP even when the constraint is not binding. When goods are complements this increase in nontradable consumption will be associated with a higher increase in tradable consumption (reducing the amount agents save) in the SP allocation relative to the CE

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allocation. When goods are substitute, instead, the amount the planner save will increase as agents substitute tradable consumption with non-tradable consumption.

Thus this mechanism could generate underborrowing in the CE allocation compared to the SP one. Underborrowing could occur both when goods are complements or substitutes. It depends on the strength of the labor supply effect and on the relative adjustment in tradable consumption in the CE versus the SP allocation. For example, even when goods are substitute and tradable consumption would fall following the labor supply mechanism just emphasized, it might happen that the fall in tradable consumption is larger in the CE than in the SP one, thus implying that agents would underborrow.

4 Parameter values and solution method

In this section we discuss the parameter values chosen and briefly describe the global solution method that we use in the numerical computations.

4.1 Parameter values

The model is calibrated at quarterly frequency and the parameter values we use are reported in Table 1.[12] As in Benigno et al (2009) these values are set following the work of Mendoza (2002, 2010) and Keheo and Ruhl (2008) to the extent possible, but also to facilitate the convergence of the numerical solution procedure.

We set the world interest rate to $i = 0.0159$, which yields an annual real rate of interest of 6.5 percent; a value that is between the 5 percent of Kehoe and Ruhl (2008) and the 8.6 percent of Mendoza (2010). The elasticity of intratemporal substitution between tradables and nontradables follows from Ostry and Reinhart (1992) who estimates a value of $\kappa = 0.760$ for developing countries.[13] The value of δ is set to 2 implying a Frisch elasticity of labor of 2. For simplicity, the elasticity of intertemporal substitution is unitary ($\rho = 1$).

For simplicity, the labor share of production in the non-tradable sector is also assumed to be unitary ($\alpha = 0$). We then normalize steady-state tradable output to one (i.e., $Y_T = 1$) and set ω and A to obtain a steady-state ratio of tradable to non-tradables output of 0.75 (slightly higher than Mendoza, 2002) and a unitary relative price of non-tradables in steady state (i.e., $P^N = 1$).

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[12]When we calibrate the model at annual frequency, for robustness, the results are qualitatively the same. Some quantitative difference emerge due to the fact that the annual calibration allows for more foreign borrowing as a share of GDP in the stochastic steady state of the model for the same parameter values.

[13]There is considerable debate about the value of this parameter. The estimate we use is consistent with Kehoe and Ruhl (2005) who set this parameter to 0.5.

We set $\beta = 0.98$ (implying an annual value of 0.92237) to obtain a foreign borrowing to annualized GDP ratio of about 25 percent in the deterministic steady state.[14] The value of the credit constraint parameter (ϕ) determines the probability of a sudden stop. We set this parameter to 0.7, which makes the constraint binding in the deterministic steady state and yields a realistic probability of sudden stop, as typically defined in the empirical literature. In the competitive equilibrium, the unconditional probability of being in a sudden stop is about 2 percent per quarter (or 8.2 percent annually). For this calculation a sudden stop is defined as the event in which the constraint is strictly binding.

Finally, in our analysis, we focus on the behavior of the economy subject to only one stochastic shock, to the endowed tradeable output, which we model as an AR(1) process. Specifically, the shock process for tradable GDP is,

$$\varepsilon_t^T = \rho_\varepsilon \varepsilon_{t-1}^T + v_t, \tag{19}$$

where v_t is an iid N(0,σ_ε^2) innovation. The parameters of this process are set to $\rho_\varepsilon = 0.86$ and $\sigma_\varepsilon = 0.015$, which are the the first autocorrelation and the standard deviation of total GDP reported by Mendoza (2010).

As parametrized, as Benigno et al (2009) show, the model produces a sharp reversal in capital flows, a large drop in output and consumption, and a large real exchange rate depreciation (proxied by the fall in the relative price of non-tradable goods) that is typical of a sudden stop. In this sense, our model is quantitatively capturing the sudden stop phenomena we observe in the data.

4.2 Solution

The algorithm for the solution of the competitive equilibrium is the one proposed by Benigno et al (2009). Here we summarize their solution procedure and explain how we compute the solution to the social planner problem. A key ingredient is a transformation of the system of Kuhn-Tucker conditions into a standard system of nonlinear equations that is due to Garcia and Zangwill (1981). The transformed system can then be solved using standard nonlinear equation solution methods.

The equilibrium of the model can be represented as a recursive dynamic programming problem summarized by the following Bellman equation:

$$V(b, B, \varepsilon) = \max_{B'} \{u(C - z(H)) + \beta E[V(b', B', \varepsilon')|\varepsilon]\}. \tag{20}$$

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[14]For this calculation we added an elastic discount factor to the model to pin down foreign debt in steady state.

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where

$$u\left(C - z\left(H\right)\right) = \frac{1}{1-\rho}\left(C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta}\right)^{1-\rho}.$$

The value function, $V\left(b, B, \varepsilon\right)$, depends on three state variables: individual borrowing (b), aggregate borrowing (B), and the stochastic shock to the tradable endowment (ε). In equilibrium, individual and aggregate borrowing must coincide, but from the perspective of the representative agent of our model the borrowing constraint is imposed at the individual level, taking relative prices as given. Our solution explicitly accounts for this feature of the model specification by treating aggregate and individual debt separately in the value function.

A solution for the decentralized equilibrium defined above will be given by (i) a value function $V\left(B, \varepsilon\right)$ and (ii) a set of laws of motion (hereafter, also called decision rules or policy functions) for aggregate borrowing $(B = G_B^n\left(B, \varepsilon\right))$, aggregate employment $(H = G_H^n\left(B, \varepsilon\right))$, and the relative price of the non-tradable good basket $(P^N = G_{PN}^n\left(B, \varepsilon\right))$ that satisfy the Bellman equation above. Note that while the value function depends on both individual and aggregate borrowing, the decision rules for all other endogenous variables only depend on aggregate borrowing.

To solve for the social planning equilibrium we set up a dynamic programming problem. The programming problem is written as a optimization of the value function subject only to the resources constraints and the borrowing constraint. Thus, the planner chooses all quantities directly. Specifically the problem can be written as:

$$v\left(B, \varepsilon\right) = \max_{c_T, c_N, h, B'} \left\{u\left(C - z\left(H\right)\right) + \beta E_{\varepsilon'}\left[v\left(B', \varepsilon'\right)|\varepsilon\right]\right\}$$

subject to

$$C^T = \left(1 + r\right)B + \varepsilon - B'$$

$$C^N = AH$$

$$B' \geq -\frac{1-\varphi}{\varphi}\left(\varepsilon + P^N AH\right)$$

$$P^N = \frac{\left(1-\omega\right)^{\frac{1}{\kappa}}\left(C^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C^T\right)^{-\frac{1}{\kappa}}}$$

We compute a solution to this problem numerically. The shock is discretized into a Markov Chain with 11 states as in Floden (2008). The methods to solve the programming problem are standard (e.g. Johnson et al 1993). In particular, we use cubic splines to

approximate the value function and we then solve the maximization problem using a feasible sequential quadratic programming routine.

5   Quantifying overborrowing

In this section we discuss the basic properties of the competitive equilibrium allocation and we compare it with the social planner one to quantify overborrowing. We conduct this comparison under alternative model specifications and assumptions for key parameter values.

5.1   Competitive equilibrium

The properties of the competitive equilibrium are more fully explained in Benigno et al (2009). Here we review them only briefly. The policy function for B_t is plotted in Figure 1. In this figure, each solid line depicts the policy function for B_t conditional on a particular state of the tradable shock. This line is drawn assuming the same shock is received in each period. For illustrative purposes, we report the decision rule for the worst state (State 1), and progressively better ones, together with the 45 degree line (dashed line). If the first state is received perpetually then the policy function will meet the 45-degree line right at the point where the constraint becomes binding. The economy remains at this point thereafter. At this point the multiplier is still zero. If the economy is currently at the intersection between the decision rule for one of the better states and the 45-degree line and receives a worse shock, the constraint can bind strictly on impact as the economy jumps to the corresponding new decision rule. For example, if we are at the point where state 3 intersects the 45-degree line and we receive a worse shock we move up directly to a point where the constraint binds strictly (with positive multiplier). So the point on the support of the decision rule in which the constraint starts to bind strictly, depends on the particular exogenous state at which we evaluate the rule as well as the value of the endogenous state variable B_t.

Figure 2 contains the policy functions for other variables of the model as a function of the endogenous state, B_t. The policy functions are drawn for the continued realization of the worst shock. All variables (C_t^T, P_t^N, C_t^N, and H_t) follow a similar pattern. Before the constraint binds (before the kink in these rules) the economy behaves in a seemingly linear manner as this shock continues to realize. Far from the constraint, the continued realization of this shock leads to a decrease in both tradable and non-tradable consumption and an increase in debt (not reported in Figure 2) as agents will smooth the impact of the shock

by borrowing more from abroad. Once the constraint is reached, however, the decision rules are driven by the need to meet it. Agents cannot longer borrow their desired amount: consumption of tradable goods decreases lowering the relative price of non-tradable goods. A falling relative price of non tradable goods has two effects. The first one is to reduce the borrowing capacity by lowering the collateral value of non-tradable income and hence generating an amplification mechanism similar to Irving Fisher's debt deflation discussed by Mendoza (2010). This effect amplifies the fall in tradable consumption. The second effect comes from the production side of the economy. Indeed as the price of non-tradable goods falls, the wage in units of tradables declines inducing a reduction in labor supply despite the fact that, as the constraint binds, the marginal utility of supplying one more unit of labor is higher. This second channel, combined with the amplified response of tradable consumption and the relative price of non-tradable, produces a fall in employment and non-tradable production and consumption.

The foreign debt distribution in the stochastic steady state of the model illustrates the working of the borrowing constraint more intuitively. In Figure 3, we compare the ergodic distribution of foreign debt for two economies, one with and one without the occasionally binding borrowing constraint.[15] As we can see, the foreign debt distribution of the economy with the constraint is shifted to the far right of that of the unconstrained economy and it is truncated. That is, agents would like to borrow much more than they can in the constrained economy, and are aware of the state-contingent borrowing limit as well as the possibility to run into a sudden stop because of that. The precautionary saving motive of private agents then implies that the amount that is borrowed is on average lower than in the unconstrained economy. In the stochastic steady state of the economy, which averages over all possible equilibrium outcomes, there therefore is an endogenous debt limit beyond which agents do not want to go. The ergodic distribution of borrowing will be truncated at that point. Note however that this is not necessarily the point at which the borrowing constraint binds strictly in any particular time or state of the economy.

5.2 Comparing to the social planner equilibrium

We now compare the allocations in the competitive equilibrium (CE) with those chosen by the social planner (i.e., SP for brevity), under alternative model specifications and parameter assumptions.

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[15]To compute the ergodic distribution of the unconstrained economy we need a stationary model. To achieve stationarity we use an elastic discount factor in both the constrained and the unconstrained economy. However, the elastic discount factor is not present in the model with the constraint that we use to produce all other results.

Figure 4 plots the decision rule for B_t for the worst possible state of the exogenous state, ε_t, in our baseline model with endogenous labor supply. This Figure shows that there is a small underborrowing when the constraint is not binding and a much larger one when the constraint is binding (i.e., for each value of the endogenous state B_t, B_{t+1} is significantly smaller in the CE than in the SP throughout the support of the decision rule). This shows that, in the benchmark economy, there is theoretical scope for both ex ante and ex post policy interventions geared toward inducing more borrowing than private agents choose to take on, both before and after a sudden stop. This comparison would suggest that a welfare-improving policy in the CE allocation would require subsidizing rather than taxing foreign borrowing. This could be achieved by keeping the relative price of tradable consumption relatively more appreciated than in the CE with a subsidy to non tradable consumption as suggested by Benigno et al (2009) or providing a direct subsidy with a negative tax on foreign borrowing along the line discussed by Korinek (2010) and Bianchi (2009).

Figure 5 compares the behavior of the other endogenous variables for the worst value of the exogenous state ε_t, like in Figure 2. Consistent with the underborrowing finding in Figure 4, there is a wedge between the policy functions of the CE allocation and the SP, which is larger when the constraint binds. As we noticed earlier, when the constraint does not bind, there are two opposite forces at work in our production economy. On the one hand, the social planner would like to reduce current consumption of tradable because it keeps into account the amplification effects caused by the price externality that might occur in the future when the constraint binds. On the other hand, the increase in marginal utility of tradables causes an increase in the marginal utility of non-tradables and an increase in labor supply with higher non-tradable production and consumption. Under our baseline calibration, this second effect dominates the first one, causing tradables consumption to be higher and saving to be lower than in the CE allocation. The equilibrium relative price of non-tradables is also going to be higher in the SP compared to the CE. A policy intervention geared toward moving the CE closer to the SP would therefore have to induce more borrowing in normal times as well as more appreciated relative price on non tradables.

When the constraint binds the differences between the CE and the SP are even more marked. There are two key differences: first, the relative price of non-tradable increases in the SP, while it collapses in the CE (Figure 5) as the economy goes deeper into debt. Secondly in the SP allocation, we observe lower labor and non-tradables consumption than in the CE. These differences arise because of the way agents and the planner react to the constraint in the two equilibria. The planner limits the deflationary impact of meeting the borrowing constraint by increasing the value of the collateral through prices (i.e. by increasing P^N) rather than quantities (i.e. it reduces Y^N). As we discussed in Section 3,

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when goods are complement, there is a relative marginal benefit of supplying one less unit of labor in the SP than in the CE. The value of the collateral is higher in the SP than in the CE because, when goods are complement, the relative price of nontradables increases and offsets the negative impact of lower non-tradables production and consumption. The overall implications of the planner's allocation is to allow for higher borrowing capacity and as a consequence higher tradable consumption even when the constraint binds. In contrast, in the CE, when the constraint is binding, all else equal, agents supply more labor to relax the constraint by increasing their non-tradable labor income. However, they don't internalize the effect that higher labor supply has, all else equal, on the equilibrium relative price. Indeed a lower relative price will tighten the constraint even more and induce a lower level of tradable consumption. As a result tradable consumption falls much more and much faster than in the SP.

Figure 6, compares the ergodic distributions of borrowing in the CE and the SP allocation. It shows that the two allocations have a similar ergodic distribution of debt despite the differences in the decision rules conditional on the worst possible state.[16] Nonetheless, the mean debt-to-GDP of this distribution is slightly smaller in the CE than in the SP, as one would expect based on the discussion above. As Table 2 reports, the average level of debt as a ratio to annual GDP is -10.20 and -10.22 percent in the CE and the SP, respectively. This difference is very small, but statistically significant (standard errors not reported).

The probability of having the constraint strictly binding is higher in the SP than in the CE (Table 2). It is 2.3 percent of the simulated quarters in the SP and only 2.06 percent in the CE (or 9.2 and 8.2 percent per year, respectively). This difference can be interpreted in terms of precautionary saving behavior, and the decision rules we discussed above illustrate how the latter comes about in our benchmark production economy. The sudden stop is more costly in the CE than in the SP equilibrium in terms of total consumption in units of tradable goods, with a welfare gain from removing the constrained-inefficiency imposed by it of 0.03 percent of consumption at each state and date. Agents therefore try to borrow less and to face a sudden stop less frequently in the CE than the SP.

Consistent with the small differences in average debt and the probability of sudden stop we reported, the overall welfare gain of moving from the CE to the SP equilibrium is a mere 0.01 percent of consumption at each date and state. The result is partly driven by the fact that the largest differences in the behavior of these economies arise at the sudden stop, which in turn occurs only infrequently.[17] Thus, the welfare analysis shows that, while there

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[16]The reason is that the decision rules for the better states are much closer to each others when the constraint does not bind and the economy spends little time in the worst state.

[17]The intuition for this result is that welfare is state dependent in our economy. And given that the economy spends most of its time outside the sudden stop state, the overall welfare difference between the

is theoretical scope for both ex ante and ex post policy action in our benchmark production economy, second best policy interventions might have an overall limited ability to affect welfare significantly. And this would stresses the importance of first best policy actions geared toward the complete removal of the financial friction at the root of the problem.

Consider now the same economy under an alternative calibration in which agents are more impatient (i.e., the discount factor is lower at 0.91) and the shocks are less persistent but four times more volatile than in the baseline (i.e., ■ = 0.54 and ■ = 0.059, as for instance in Bianchi, 2009). Figure 7 reports the same decision rules as Figure 5, while Figure 8 compares the ergodic distributions of B_t in the CE and the SP allocations. As we can see from Figure 7, with more impatient agents and more volatile shocks, we now generate overborrowing in the CE equilibrium relative to the SP equilibrium when the constraint does not bind. Being more impatient implies that agents have relatively higher current consumption of tradables good: since marginal utility of current consumption is now smaller than in the previous case, the increase in current consumption (away from the constraint) dominates the negative effect of lower current consumption of tradables induced by the labor margin so that tradable consumption is higher in the CE allocation compared to the SP one. In equilibrium, as goods are complement, we observe higher consumption of tradables, higher consumption of nontradables and higher relative price of non-tradables in the SP allocation. Instead, when the constraint is binding, the decision rules of the CE behave in a similar manner as in the benchmark economy, relative to those of the SP.

This economy's behavior thus differs not only quantitatively but also qualitatively with respect to the benchmark economy. The important policy implication is that this alternative economy would require an ex ante policy intervention of opposite sign to that in the benchmark model to close the gap between the CE and the SP. However, when the constraint binds (after the kink in the decision rules), the difference compared to the benchmark calibration is only quantitative. This suggests that the sign of an ex post policy intervention would be the same in the two economies, although the intensity of that intervention would possibly be different because of different parameter values.

As Table 2 reports, average debt in the stochastic steady state of the economy with the alternative calibration is smaller than in the benchmark model (despite the higher degree of impatience assumed), and larger in the CE than in the SP (at -7.31 and -6.9 percent of annual GDP, respectively) because here there is overborrowing. Average debt is smaller in both the CE and the SP than in the benchmark economy because here the sudden stop is

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two allocations is very small. As shown by Mendoza (2002), the second moments of an economy with and without such constraints are quite similar. Yet, in our benchmark model, the welfare gain to move from the CE to the SP is relatively small even at the sudden stop.

much more costly (about 30 times more costly in both allocations), with a welfare gain of moving from the CE to the SP at the sudden stop of 0.9 percent of consumption at every date and state (and a overall welfare gain of 0.3 percent). As a result, private agents self insure more relatively to the benchmark economy. This also leads to a significantly smaller probability of sudden stop in the CE in this case (1.53 percent per quarter). In contrast, the SP faces sudden stops with about the same probability as in the benchmark economy (2.2 percent of the quarters).

Consider now an endowment economy under the baseline and the alternative calibration for the same two sets of parameter values used for the production economy. The only change compared to the benchmark economy presented in section 3 is that labor supply in the non-tradable sector is now exogenous. Figure 9, compares the decision rule and the ergodic distribution for foreign borrowing in the CE and the SP for both calibrations. Figure 10 compares the decision rule for borrowing, tradable consumption, and the relative price of non-tradables. As we can see from Panel A of Figure 9, for the baseline parameter values and the worst realization of the shock, once we shut off endogenous labor supply, there is essentially no difference in the decision rule for foreign borrowing between the CE and the SP allocation, either before or after the constraint binds. Nonetheless, we can see that in the ergodic distribution of the foreign borrowing (which averages over all possible realizations of the shock and points on the support of the decision rules) there is a very small overborrowing of about 0.10 percent of annual GDP (with average foreign borrowing reported in Table 2 at -10.25 and -10.14 percent of annual GDP in the CE and the SP, respectively). This shows that, in this case, as we discussed before, the distortion introduced by the credit constraint in the intertemporal margin leads households to undervalue current marginal utility of tradable consumption relative to future one for more favorable realizations of the exogenous state. The distortion, however, leads to a very small difference between the private and socially efficient level of foreign borrowing for the baseline parameter values.

Interestingly, the probabilities of sudden stops are 13.0 percent in the CE and 1.7 percent per quarter in the SP. In the CE, the probability of sudden stop is much higher in the endowment economy than in the production economy. This is because households cannot rely on the labor margin to supply more collateral when the constraint binds or is expected to bind in the future, but face about the same incentive to borrow. As a result average borrowing is slightly higher as a average share of total income and the probability of a sudden stop is much higher in the endowment economy than in the production economy. In contrast, in the social planner allocation for an endowment economy, in which there is no margin on which to act once the sudden stop is reached, there is less borrowing than in the production economy and a significantly lower probability of reaching the sudden stop, both

with respect to the CE equilibrium of the endowment economy and the SP equilibrium of the production economy. Note here that the sudden stop is more costly for the SP of the endowment economy than the SP of the production economy as tradable consumption falls by about 40 and 25 percent respectively (Figure 5 and Figure 10, Panel A). But the sudden stop cost is about the same in the CE and the SP equilibrium of the endowment economy because the SP cannot improve on the CE when the constraint binds in the endowment economy. Consistent with this observation, the welfare gains of moving from the CE to the SP in this endowment economy, either overall or at the sudden stop, are one order of magnitude smaller than in the production economies above, at only 0.001 and 0.003 percent of consumption at each date and state, respectively (Table 3).

In an endowment economy with more impatient agents and larger shocks, overborrowing is larger than in the endowment economy with base calibration, but precautionary saving is higher in both the CE and the SP equilibrium. Overborrowing, as measured by the difference in the average of the ergodic distribution of foreign borrowing, is about 0.30 percent of annual GDP, with average foreign borrowing of -7.40 and -7.10 in the CE and the SP, respectively (Table 2). This is also evident from Panel B of Figure 9, which shows that the decision rule for B_t conditional on the worst possible state displays clearer evidence of over-borrowing in the intermediate region of the state space. Because of the higher precautionary saving, the probabilities of sudden stops are also much smaller than in the endowment economy with base calibration (at 2.36 and 0.23 percent per quarter in the CE and the SP, compared to 13.66 and 1.70, respectively). The differences in the probability of sudden stop across calibrations and the higher precautionary saving in this economy is associated with a much more costly sudden stop dynamics with the alternative calibration with respect to the endowment economy with the base calibration. As we can see from Figure 10, Panel B, in fact, tradable consumption falls by about 75 percent with the alternative calibration compared to about 25 percent in the baseline one. A much higher cost of sudden stop leads to a large (overall and at the sudden stop) welfare gain of moving from the CE to the SP equilibrium in this economy despite the fact that the planner cannot ameliorate the CE allocation at the sudden stop, at 0.04 and 0.12 percent of consumption at all time and state, respectively. The planner incentive to curtail borrowing is particularly strong in this case.

6 Policy Implications

The quantitative analysis in the previous section has important policy implications. A recent literature reviewed in section 2 has focused on the theoretical and quantitative pos-

sibility of overborrowing, unambiguously recommending ex ante type of interventions such as a Tobin tax or other economy-wide prudential controls on international capital inflows, to curtail it.

While consistent with a theoretical second best view of the world, this clear cut policy prescription in practice warrants several qualifications. First, it is not possible to analyze the relative merits of both ex ante and ex post intervention strategies in models in which the planner knows it can intervene only ex ante. In an endowment economy there is no scope for ex post policy interventions by construction. As tradable consumption is pinned down by the constraint when this binds, in an endowment economy, neither private agents nor the planner can manipulate the collateral value of nontradables income to relax the borrowing constraint, and hence seeking to achieve a better allocation.

Second, overborrowing is clearly a quantitative matter, and there is no solid basis to conclude that it is a key and general feature of emerging economies. As we saw in the quantitative analysis conducted in the previous section, it is sufficient to change key parameter values that are not easily anchored to the data in simple models to find a small underborrowing rather then an overborrowing in production economies. It follows that both sets of policy instruments should be put in place to "hedge" the model and parameter uncertainty the policy makers face. By the well established standards of the DSGE methodology, such lack of robustness is sufficient to require a more cautious approach to economy-wide prudential controls on capital inflows, especially in light of the (at best mixed) historical experience with such policy tools.[18] By the standards of the DSGE methodology, the pros and cons of alternative policy regimes should be evaluated quantitatively in models that fit the data well, as it is now the case for standard monetary and fiscal stabilization policy issues. But rich models with occasionally binding financial frictions are not easily amenable to be analyzed quantitatively with the same ease with which the standard new Keynesian model has been investigated in the monetary policy literature. So there is a need to recognize that these models are in their infancy and not yet ready to provide clear cut policy recommendations. One important implication is that economy-wide capital controls alone as recommended in the literature (as recently implemented by Brazil) may not achieve constrained efficiency in more richly specified and parametrized economies.

Third, such interventions are distortionary and may hamper economic efficiency if inappropriately imposed. As Nikolov (2009) has pointed out, for instance, in this kind of model environment, there is a trade off between the higher volatility associated with mandating looser prudential controls (i.e., a higher leverage ratio in his model) and the lower average

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[18]See Ostry et al. (2010) for a thorough review of the existing literature, as well as newer empirical evidence, on the effectiveness of economy-wide capital controls.

growth associated with imposing tighter prudential controls (i.e., lower leverage ratios in his model). So mandating lower, economy-wide regulatory leverage ratios on prudential grounds may impose significant efficiency costs in terms of lower average growth.[19] This point is largely absent from the current policy debate, in part because it is difficult to evaluate such trade off quantitatively in the models available. Nonetheless, Nikolov's (2009) analysis clearly highlights the risk involved, consistent with the traditional debate in the literature on capital controls reviewed by Ostry et al. (2010).

Fourth, even when they are the appropriate economy-wide policy regime from a second-best welfare perspective, ex-ante economy-wide interventions do not eliminate sudden stops and financial crises completely. They just mitigate their severity, and possibly their likelihood as our analysis highlights. So even with prudential policies in place, we would still need to design policies that can be implemented in response to sudden stops in financial flows as Caballero (2010) stresses. This is brought out clearly by our analysis of the two production economies considered in which there are wedges between the CE and the SP allocations both before and after the constraint binds.

Finally, there are no moral hazard or time-consistency concerns in our set up. Using these models for positive purposes, to describe the cycle or the crisis, or both, the microfoundations of financial friction matter less. For policy analysis, they can make important differences. For instance, moral hazard considerations may surface in a microfounded specification of our constraint. And once moral hazard of ex post policies is considered, the implication may be reversed, and ex ante policies may become more clearly preferable. Similarly, time-inconsistency problems are absent from these models. As Chari an Keheo (2009) illustrate, time inconsistency of optimal ex post interventions may call for ex ante interventions. But considering two-sided commitment problems in models of default, such as in Cavallo, Chang, and Velasco (2010), the recommendation would be to introduce policies that facilitate creditors' commitment to borrowers to alleviate the consequences of the credit friction. In practice, this would means ex post policies that facilitate the smooth functioning of international capital markets such as the now defunct sovereign debt restructuring mechanism, the collective action clauses, and the international corporate debt restructuring mechanisms that are starting to be reconsidered in the international policy debate. That is, policy should be geared toward improving the stability of the of international markets, rather the climate in the receiving country.

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[19]Note this does not imply that specific sectors of the economy, such as the domestic financial system, would not benefit from such policy interventions.

7 Conclusions

In this paper we study overborrowing quantitatively in production and endowment small open economies subject to a borrowing constraint that occasionally binds giving rise to sudden stops in foreign capital inflows, endogenously. In this environment, there is a well known pecuniary externality that creates theoretical scope for policy intervention. We found that while the competitive equilibrium of these economies is never constrained efficient, differences in allocations with respect to a socially planned equilibrium are both a qualitative and quantitative matter. Differences between the two allocations can arise both when the constraint binds and when it is not in a production environment. But they arise only when the constraint is not binding in an endowment economy. Moreover, within production economies, whether or not there is overborrowing depends on the value of specific parameters. In the endowment economies we considered there is always overborrowing, but its quantitative relevance does depend heavily on parameter values. In particular, significant differences when the constraint does not bind can arise only for economies with relatively impatient agents and large shocks compared to the typical emerging market economy. As we discuss in the paper, this finding has important policy implications and suggests that both ex ante and ex post policy interventions are likely to be welfare improving, but the analysis in the paper suggests that the largest welfare payoffs are likely to stem from ex post interventions. Indeed, as the discussion of the implication of our analysis illustrates, a whole range of policies tools should be explored and pursued at the same time. And studying overborrowing with fully micro-funded financial frictions is an important area of future research.

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Table 1. Model Parameters

Structural parameters	Values
Elasticity of substitution between tradable and non-tradable goods	$\kappa = 0.760$
Intertemporal substitution and risk aversion	$\rho = 1$
Labor supply elasticity	$\delta = 2$
Credit constraint parameter	$\phi = 0.7$
Labor share in production	$\alpha = 0$
Relative weight of tradable and non-tradable goods	$\omega = 0.48568$
Discount factor	$\beta = 0.98$

Exogenous variables	Values
World real interest rate	$i = 0.0159$
Steady State Relative Price of Non-tradable	$P_N = 1$

Productivity process	
Persistence	$\rho_{\varepsilon^T} = 0.86$
Volatility	$\sigma_{\varepsilon^T} = 0.015$

Table 2. Average foreign borrowing and probability of a sudden stop

Annual average debt in the ergodic distribution	CE	SP
(Percent of annual GDP)		
Production, benchmark parameters	-10.20	-10.22
Production, alternative parameters	-7.31	-6.90
Endowment, benchmark parameters	-10.25	-10.14
Endowment, alternative parameters	-7.40	-7.10
Quarterly unconditional sudden stop probabilities		
(Percent per quarter)		
Production, benchmark parameters	2.06	2.30
Production, alternative parameters	1.53	2.20
Endowment, benchmark parameters	13.66	1.70
Endowment, alternative parameters	2.36	0.23

Table 3. Welfare gain of moving from the CE to the SP
(In percent of tradable consumption at each time and state)

	Overall	At the sudden stop
Production, benchmark parameters	0.01	0.03
Production, alternative parameters	0.30	0.90
Endowment, benchmark parameters	0.001	0.003
Endowment, alternative parameters	0.04	0.12

The welfare gains of moving from the CE to SP are calculated as the percent of total consumption that the agents are willing to forego at every date and state to move from one allocation to the other. That is the percent reduction in consumption at all future dates and states in the SP that equates expected utility in the CE with expected utility in the SP. This cost is calculated at each point on the state space. The "overall" welfare cost is calculated by weighting the cost in each state by the unconditional probability of being in that state. We also construct the welfare gain when near a sudden stop. This calculation is complicated by the fact that the sudden stop does not always occur in the same state. Our solution is to simulate the model for 100,000 periods and keep track of the state(s) in which the economy is before entering a sudden stop. We then average the gains over these states right before a sudden stop occurs.

Figure 1: Decision Rule For Foreign Borrowing (Competitive Equilibrium)
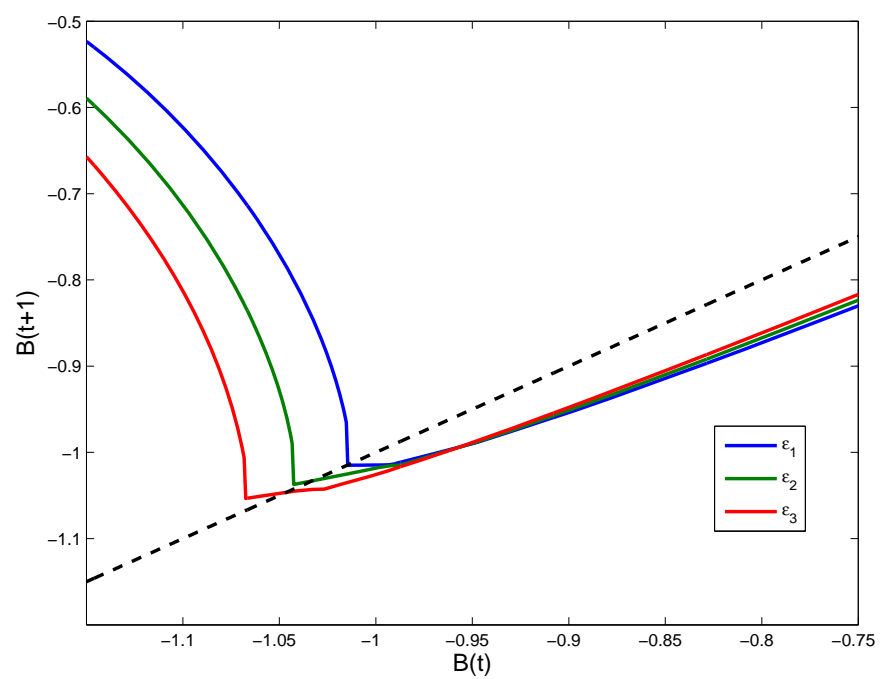
Figure 2: Decision Rules For Relative Price, Consumption, and Labor (Competitive Equilibrium)

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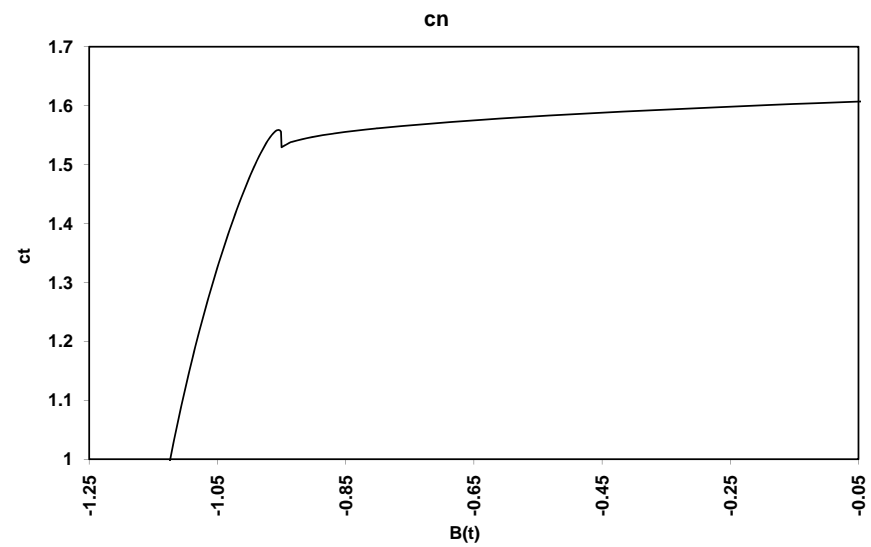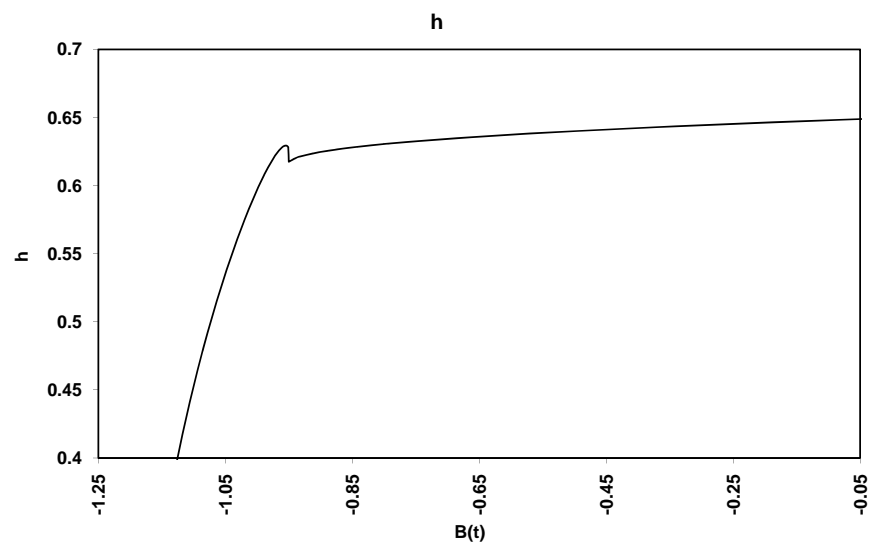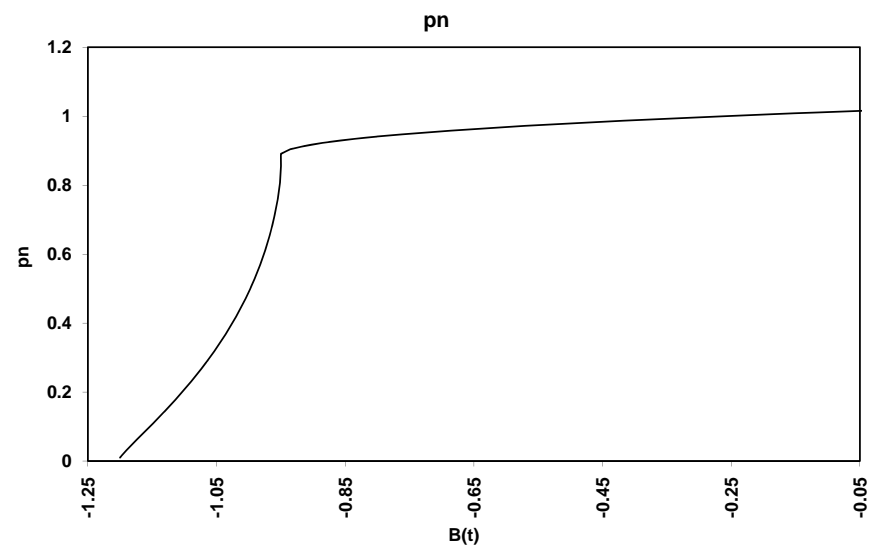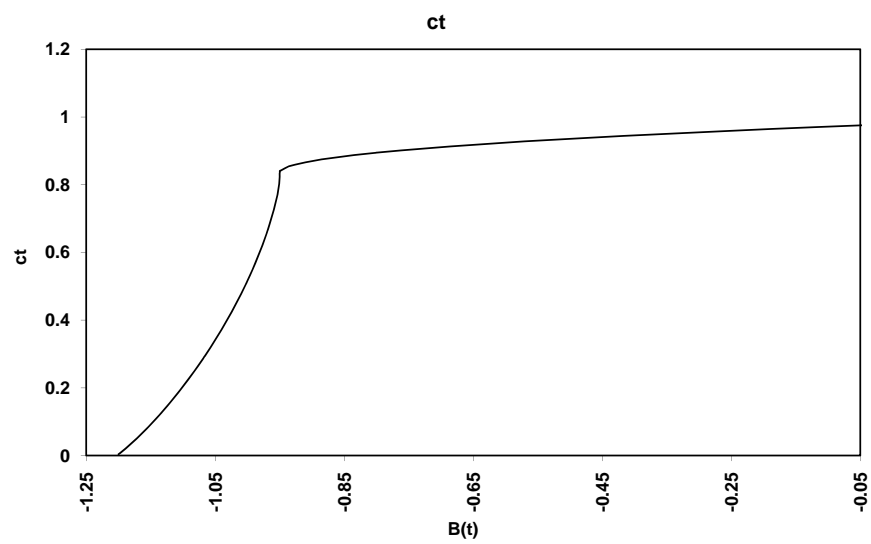
Figure 3. Ergodic Distribution For Foreign Borrowing
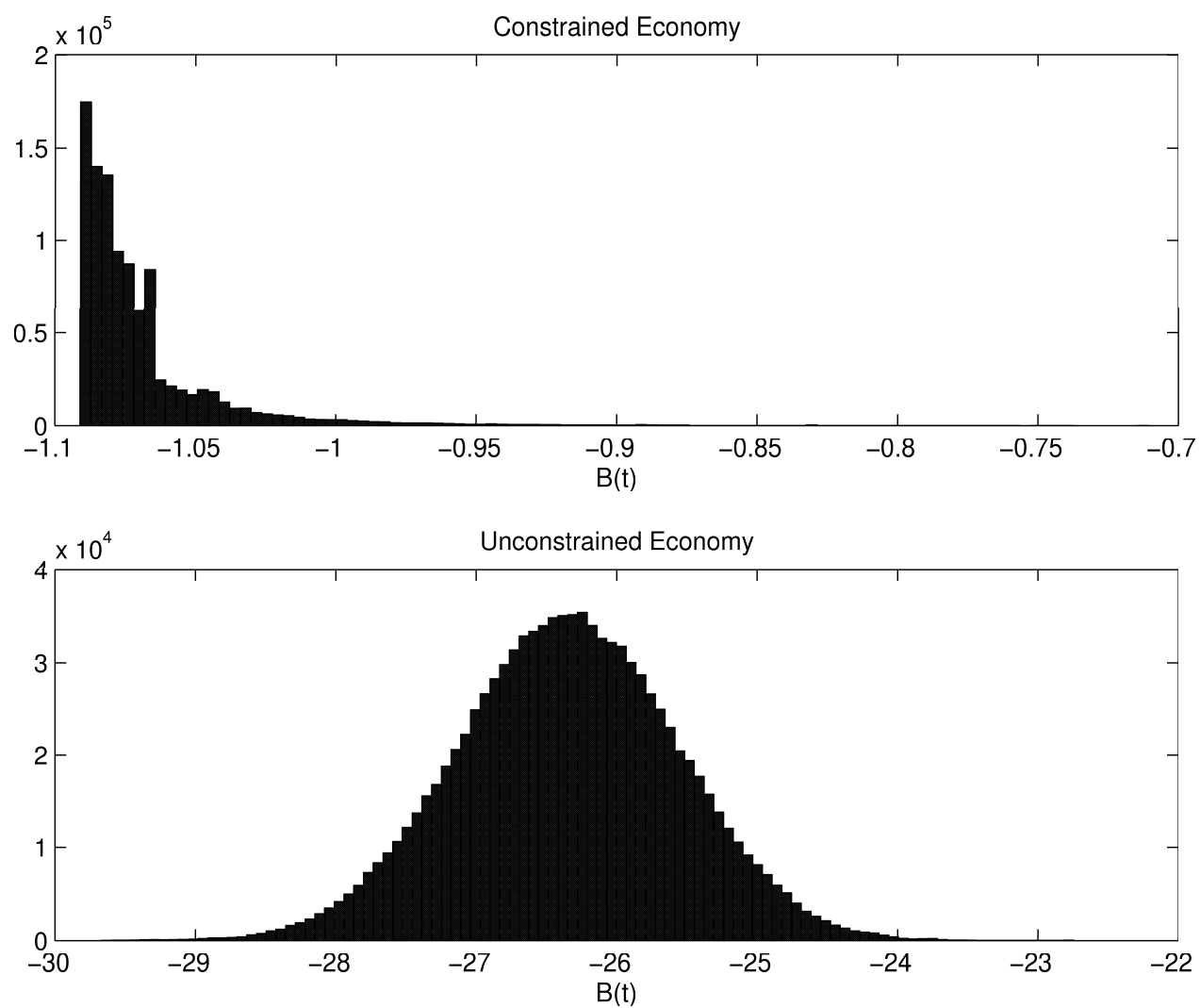
Figure 4: Decision Rule For Foreign Borrowing (Competitive Equilibrium and Social Planner)
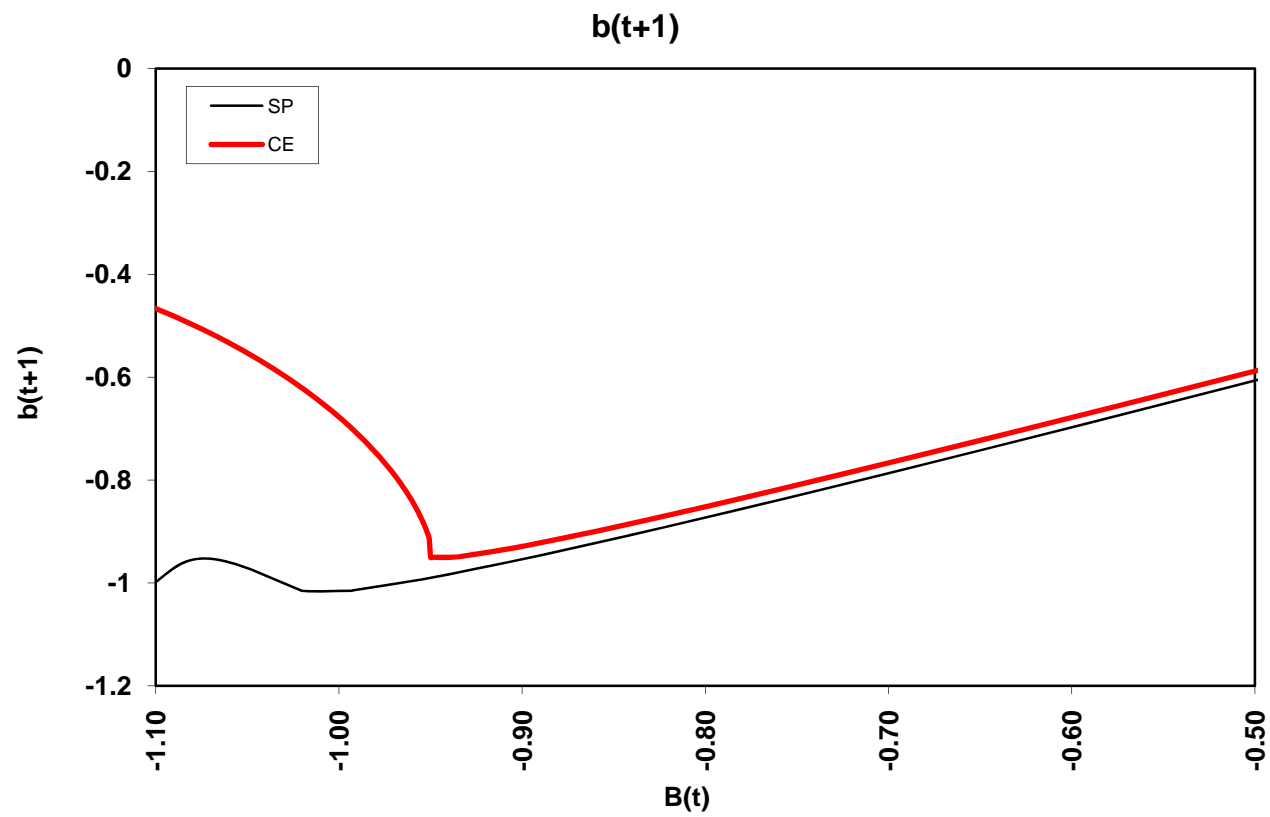
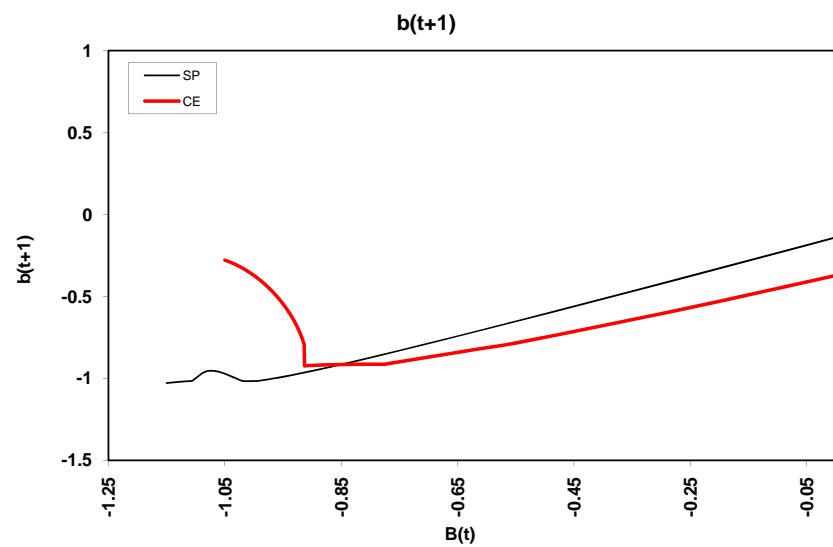Figure 5: Decision Rules For Relative Prices, Consumption, Labor (Competitive Equilibrium and Social Planner)
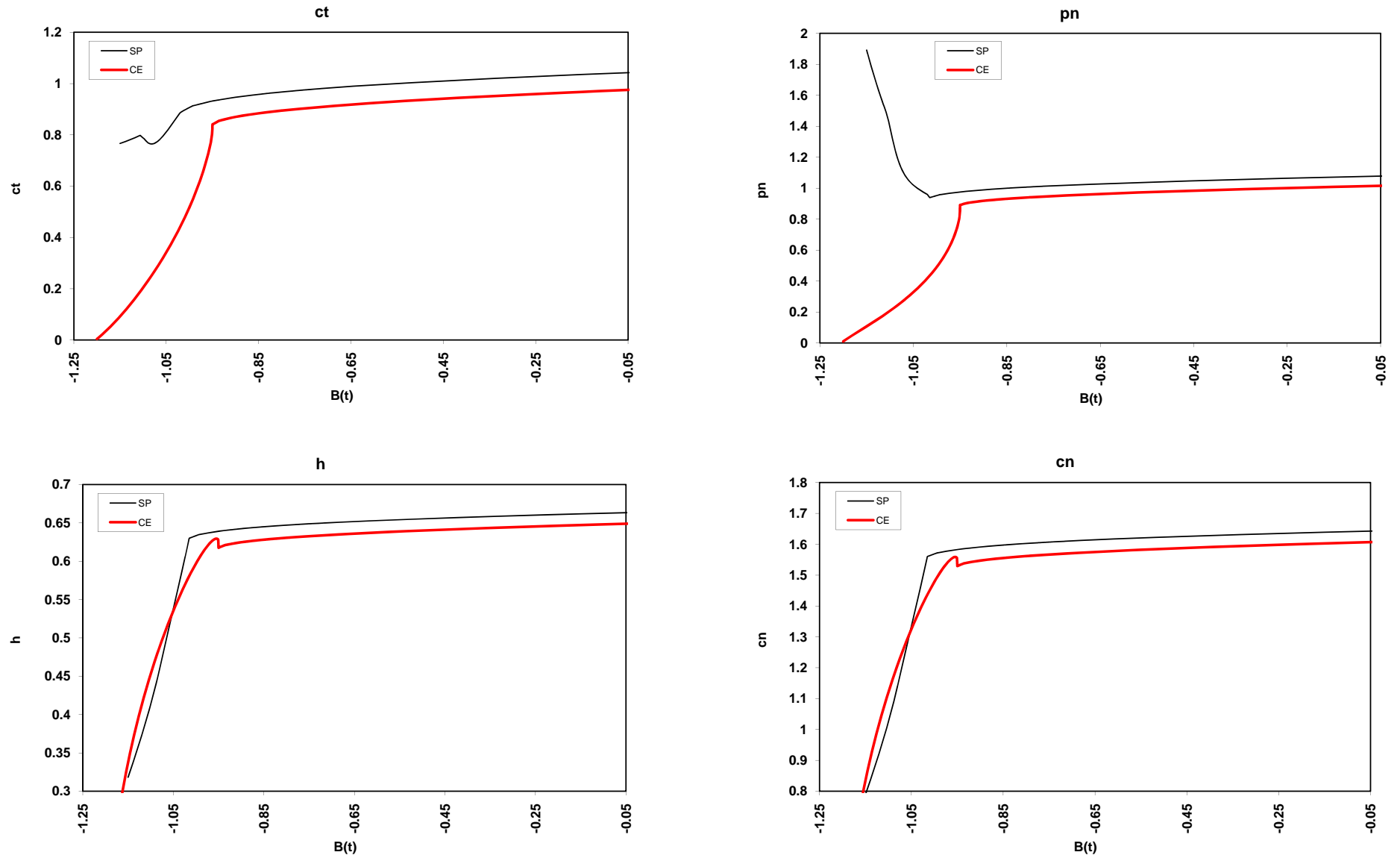
Figure 6: Ergodic Distribution For Foreign Borrowing (Competitive Equilibrium and Social Planner)
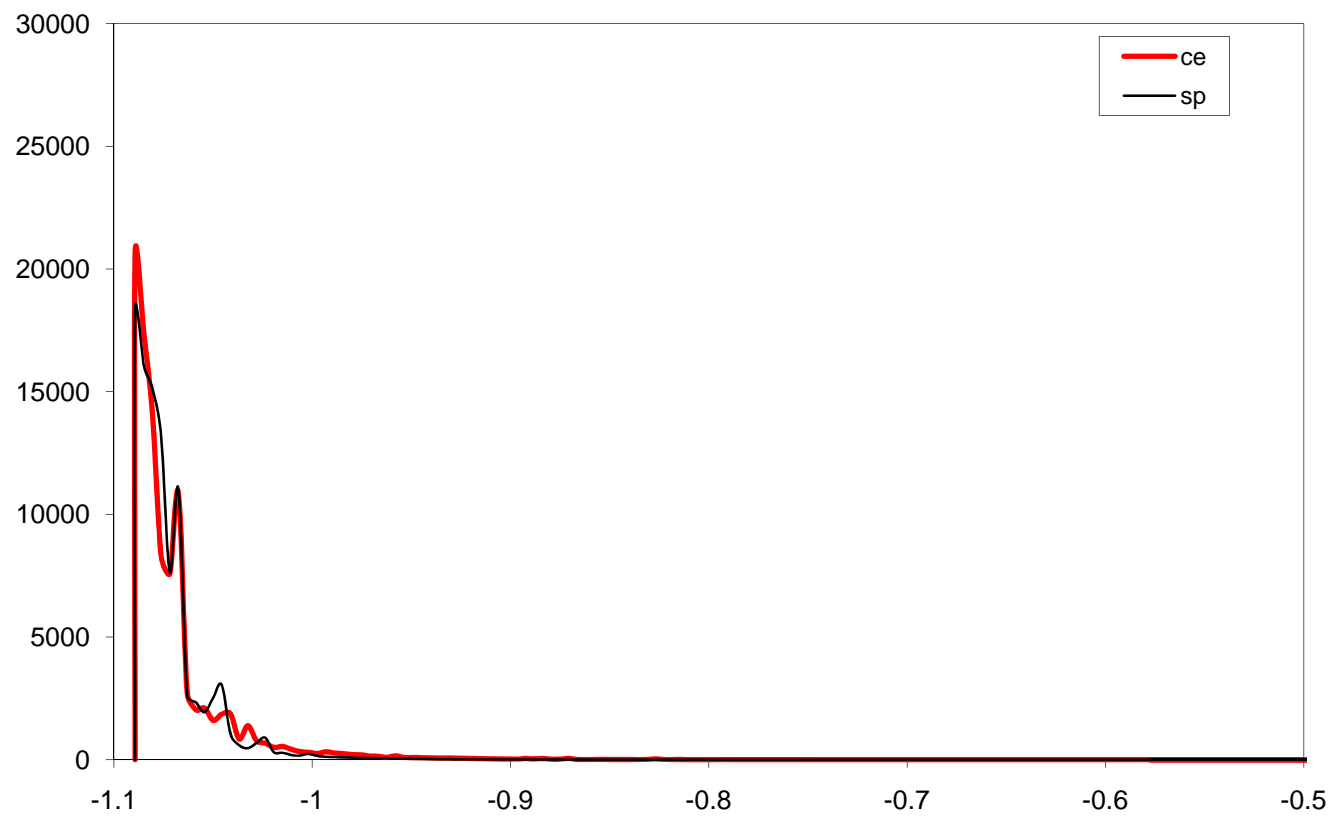
Figure 7: Alternative Calibration (More impatient agents and larger shocks)
Decision Rules For Foreign Borrowing, Relative Price, Tradable Consumption, and Labor (Competitive Equilibrium and Social Planner)
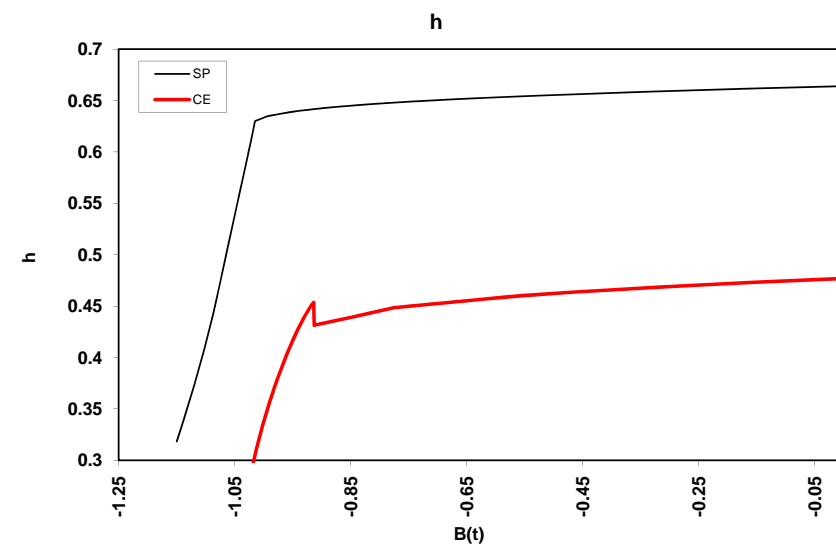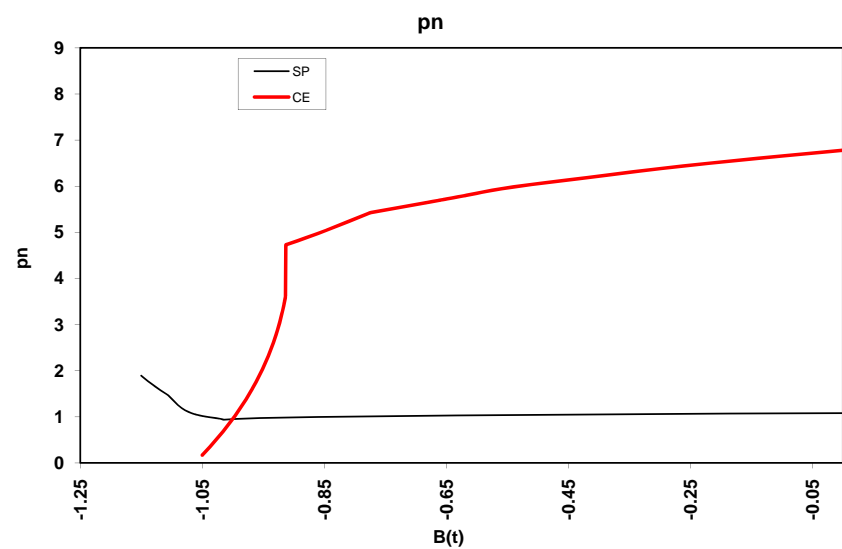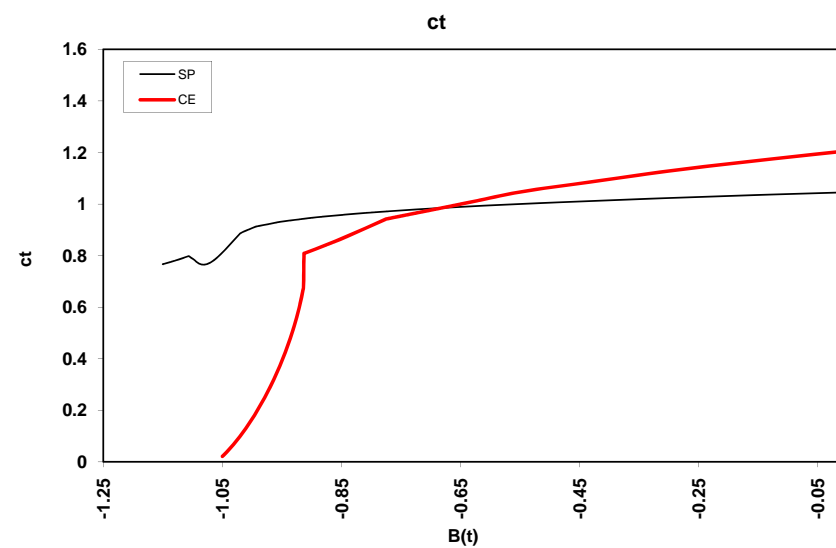
Figure 8: Alternative Calibration (More impatient agents and larger shocks)
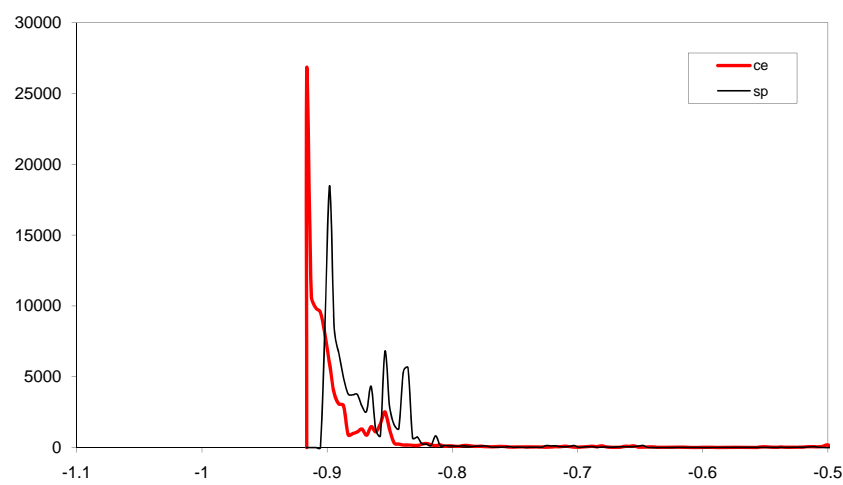Ergodic Distribution For Foreign Borrowing (Competitive Equilibrium and Social Planner)
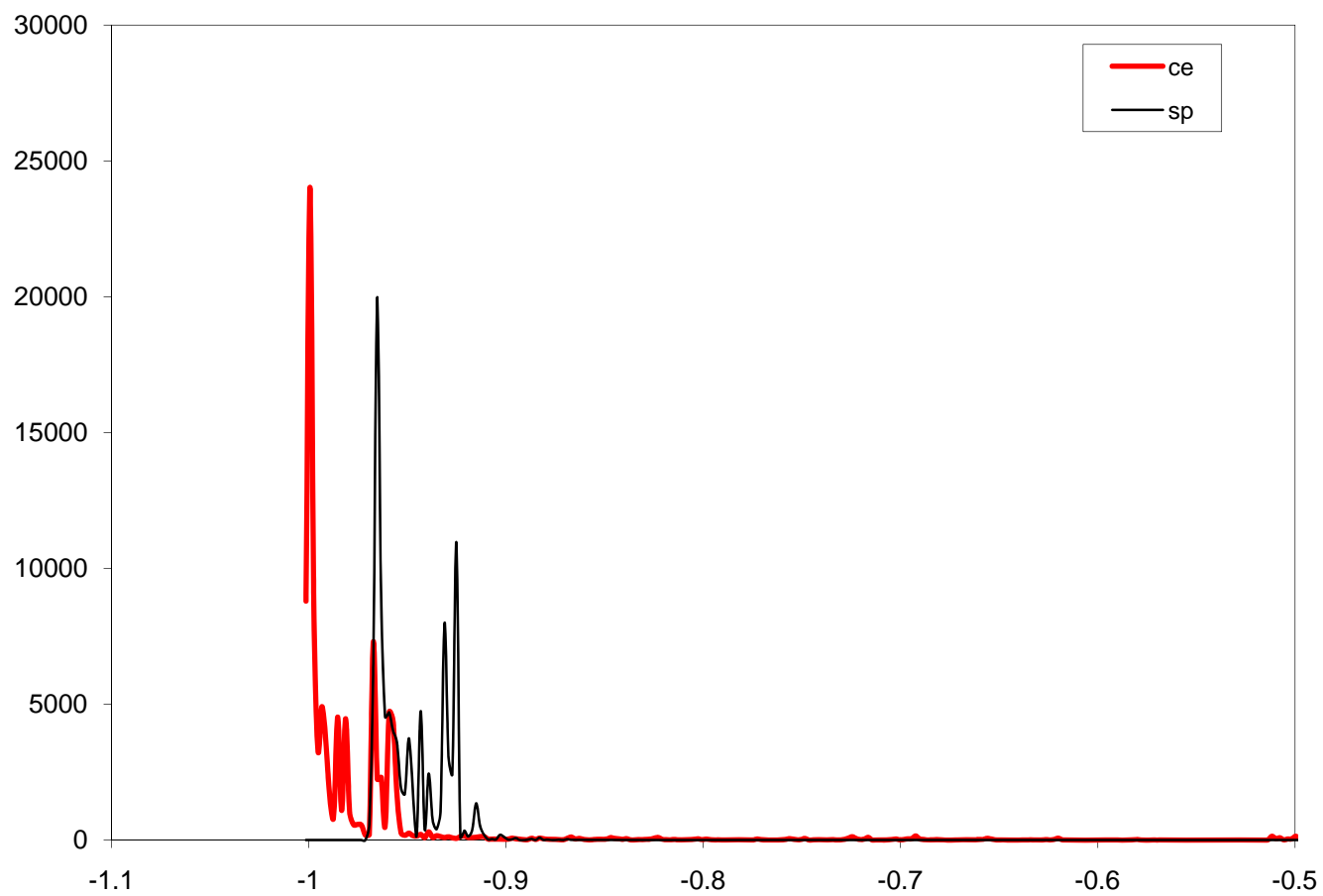
Figure 9: Endowment Economies under Alternative Calibrations (Competitive Equilibium and Social Planner)
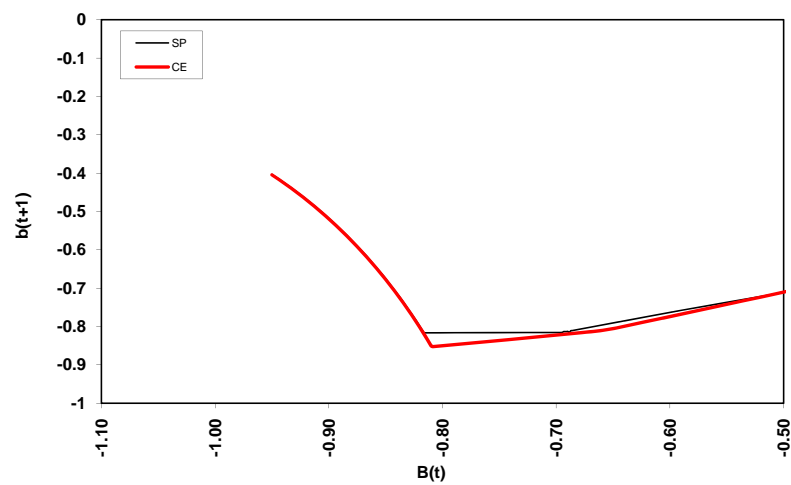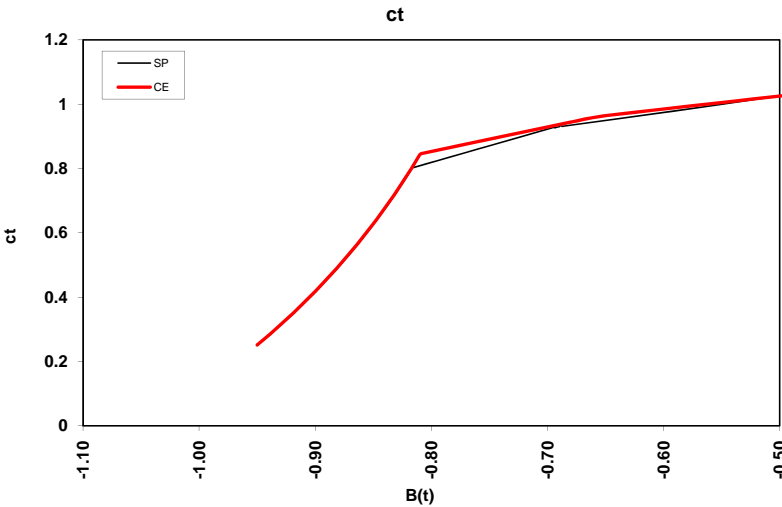Panel A. Baseline calibration
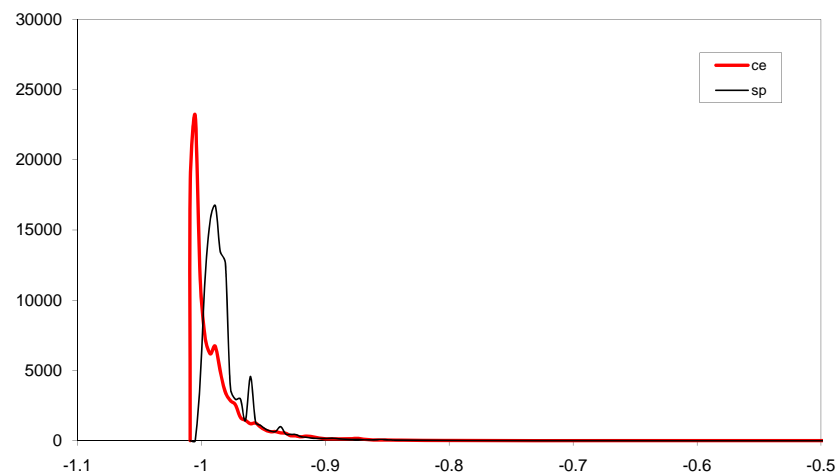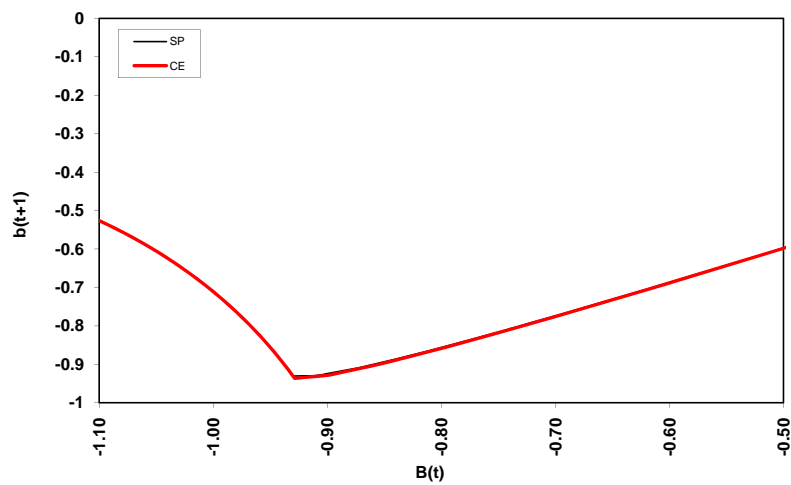
Panel B. Alternative calibration
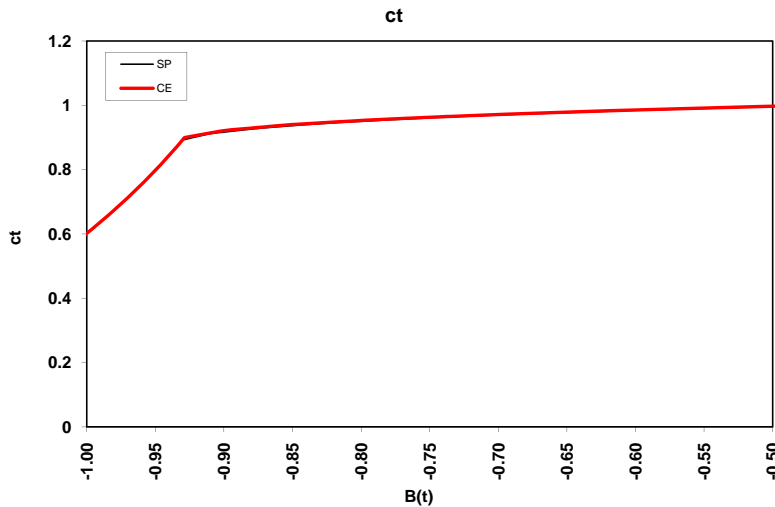
Figure 10: Endowment Economies under Alternative Calibrations (Competitive Equilibium and Social Planner)

Panel A. Baseline calibration

Panel B. Alternative calibration