Firm-Specific Productivity Risk over the Business Cycle: Facts and Aggregate Implications

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Abstract

Is time-varying firm-level uncertainty a major cause or amplifier of the business cycle? This paper investigates this question in the context of a heterogeneous-firm RBC model with persistent firm-level productivity shocks and lumpy capital adjustment, where cyclical changes in uncertainty correspond naturally to cyclical changes in the cross-sectional dispersion of firm-specific Solow residual innovations. We use a German firm-level data set to investigate the extent to which firm-level uncertainty varies over the cycle. This allows us to put empirical discipline on our numerical simulations. We find that, while firm-level uncertainty is indeed countercyclical, it does not fluctuate enough to significantly alter the dynamics of an RBC model with only first moment shocks. The mild changes we do find are mainly caused by a bad news effect: higher uncertainty today predicts lower aggregate Solow residuals tomorrow. This effect dominates the real option value effect of time-varying uncertainty, highlighted in the literature.

JEL Codes: E20, E22, E30, E32.

Keywords: Ss model, RBC model, lumpy investment, countercyclical risk, aggregate shocks, idiosyncratic shocks, heterogeneous firms, news shocks, uncertainty shocks.

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1 Introduction

> *"Crises feed uncertainty. And uncertainty affects behaviour, which feeds
> the crisis. [...] But there is more at work. If you think that another Depres-
> sion might be around the corner, better to be careful and save more."*
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> IMF Chief Economist, Olivier Blanchard, in: The Economist, 29 January, 2009

Is time-varying firm-level uncertainty a major cause or amplifier of the business cycle? This paper investigates this question in the context of a heterogeneous-firm RBC model with persistent firm-level productivity shocks and lumpy capital adjustment. We make three contributions to the existing literature: first, we discipline the calibration of time-varying uncertainty as a driving force through a detailed analysis of the behavior of the cross-section of firms over the cycle, rather than through stock market data; secondly, we analyze the implications of time-varying uncertainty in the context of a general equilibrium framework and using the detailed unconditional and conditional second moment analysis of model-simulated aggregate time series that has become a standard tool in the business cycle literature; and, thirdly, we show a new mechanism through which uncertainty shocks have (mild) aggregate effects: a bad news effect, rather than the time-varying option value effect (wait-and-see) that has been highlighted thus far (Dixit and Pindyck, 1994, Hassler, 1996 and 2001, and, more recently, Bloom, 2009). We find the wait-and-see-effect to be dominated by the bad news effect.

In the aforementioned model context, cyclical changes in uncertainty correspond naturally to cyclical changes in the cross-section of firms – more specifically the dispersions of change rates of firm-level Solow residuals, real value added and real sales. Thus, using the balance sheet data set of Deutsche Bundesbank (USTAN) – a private sector, annual, firm-level data set that allows us to investigate 26 years of data (1973-1998), in which the cross-sections of the panel have over 30,000 firms per year on average, and which has a broad ownership, size and sectoral coverage –, we first show that the cross-sectional standard deviations of the firm-level innovations in the Solow residual, real value added and sales are indeed robustly and significantly countercyclical. This confirms U.S.-related findings by Bloom (2009) and Bloom et al. (2009) for the implied volatility of the stock market, for Compustat data on publicly traded firms and for sectoral data. We interpret these changes as countercyclical fluctuations in firm-specific productivity uncertainty. Second, we show that for our broad German cross-section of firms the volatility in uncertainty over the cycle is substantially lower than has been considered in the literature. These two results are essentially robust to different choices for the cyclical indicator, to alternative calculations of the Solow residual, to various changes in the sample selection criteria and to explicitly considering cyclical sample selection and measurement error in the micro data. We also use IFO survey data on business expectations to confirm our findings. Figure 1 il-

lustrates the countercyclicality of firm-level risk. It displays for Germany the annual time series of the dispersion of firm-level Solow residual innovations, linearly detrended, and the cyclical component of GDP, as measured by log-HP(100) filtered aggregate output.[1]

Figure 1: Time Series of the Dispersion of Firm-Level Solow Residual Innovations and the Cyclical Component of GDP
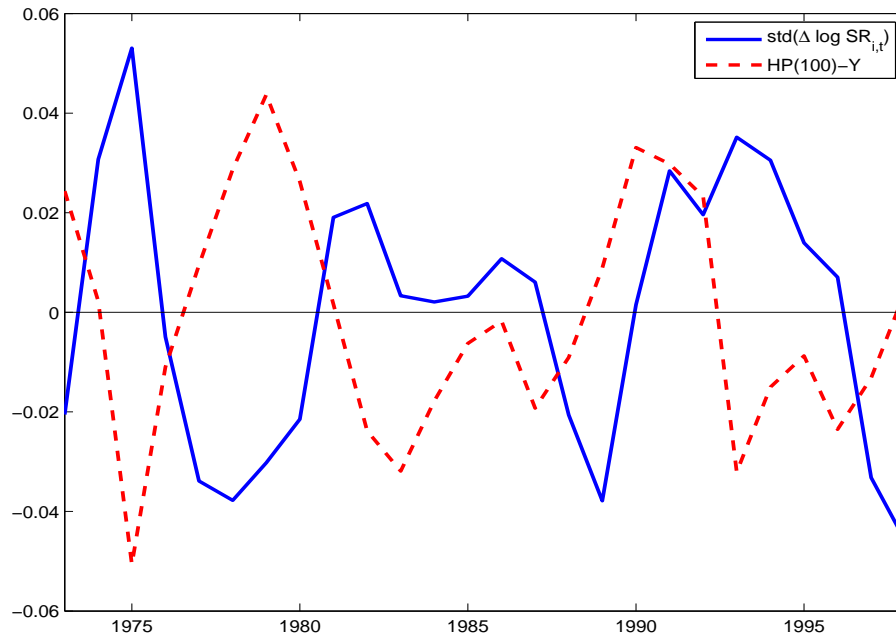
Notes: std: cross-sectional standard deviation, linearly detrended and normalized by the average dispersion. $HP(\lambda) - Y$: Cyclical component of log-GDP after HP-filtering using smoothing parameter λ.

We then explore the quantitative importance of these shocks to uncertainty in a heterogeneous-firm RBC model similar to the one in Khan and Thomas (2008) and Bachmann et al. (2008). We have developed the computational techniques to study time-varying uncertainty in general equilibrium in parallel with Bloom et al. (2009). We find that uncertainty shocks alone - if of a size in line with our empirical evidence - do not introduce quantitatively significant business cycle fluctuations. Moreover, purely uncertainty driven business cycles lead to counterfactual acyclicality of aggregate consumption.

Shocks to uncertainty nonetheless modestly alter the aggregate behavior of the model if they are introduced alongside standard first moment fluctuations in aggregate Solow residuals and compared to a model with only these first moment shocks (essentially the standard RBC

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[1]See Appendix B.1 for analogous figures for the time series of the dispersions of firm-level innovations to value added and sales, as well as scatter plots between dispersion measures and the cyclical component of GDP.

model). Yet, this change does not come through the real options effect often associated with time-varying uncertainty. Instead, we identify a new economic channel, by which fluctuations in uncertainty influence the business cycle that results only from their correlation with aggregate productivity. Since changes in uncertainty empirically correlate with future developments in the aggregate Solow residual, the former constitute news about the latter.

Specifically, we find that an increase in productivity risk essentially acts as a bad news shock. In general equilibrium, this means that households decrease their consumption and increase their labor supply, which we find to be consistent with the data. We also find a decrease in the real wage as a result of a shock to uncertainty not only in our model but again also in the impulse response function of the data. As a result of this decrease in wages, we find – again both in the data as well as in our model – an increase in economic activity at the moment of the uncertainty shock, followed by a recession, when the predicted downturn in productivity occurs. The effects of uncertainty shocks that we find in our model almost entirely operate through this channel.

Comparing a partial equilibrium version to our general equilibrium model in terms of the impulse response functions of investment with respect to shocks to uncertainty illustrates this mechanism further. While in partial equilibrium the bad news on impact causes aggregate investment activity to collapse, in general equilibrium, this effect is reversed by the decrease in the real interest rate, such that investment rates even expand on impact and only turn negative as lower productivity actually realizes. Indeed, the point estimate of the empirical impulse response function of uncertainty on national saving is positive on impact. This shows that general equilibrium analysis is paramount in understanding the aggregate effects of firm-level uncertainty shocks.

The remainder of this paper is organized as follows: Section 2 describes our data set, the USTAN data, discusses briefly the selection of the final sample and details our empirical results. Section 3 explains the model. Section 4 describes its calibration and Section 5 discusses the numerical results. Appendices provide more details on the data set, give further robustness checks for the empirical findings as well as for the simulation results.

Related Literature

Over the last two decades, the economic literature has used models with fixed costs of capital adjustment or irreversibility as a natural starting point to study the effects of uncertainty on economic activity (see e.g. Dixit and Pindyck, 1994). More recently, Bloom (2009) and Bloom et al. (2009) document that increases in stock market volatility are correlated with a reduction in aggregate economic activity. The former in partial equilibrium, the latter in general equilibrium then provide a formal model qualitatively similar to the one used here, where an increase in uncertainty leads to a higher real option value of investment, letting more firms 'wait and see'

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after such an increase, which in turn leads to a fall in aggregate investment and employment. In a similar vein and also in general equilibrium, but in a different model context, Sim (2008) puts forth a model that explains the cyclical patterns in firms' entry and exit with cyclical variations in uncertainty. Gilchrist et al. (2009) explore the implications of time-varying uncertainty for bond risk premia and via this channel for aggregate investment in a general equilibrium model. Alexopoulos and Cohen (2009) propose the 'number of New York Times articles on uncertainty and economic activity' as an alternative measure for uncertainty shocks and find in an empirical exercise that this measure of uncertainty is negatively correlated with aggregate activity. Finally, in a somewhat different context, Fernandez-Villaverde et. al (2009) find that increases in interest rate volatility suppress real activity in small open emerging market economies in Latin America.

None of the above draws its empirical evidence on broad-based cross-sectional firm data. The data that is typically used is stock market data or analyst data available only for a subset of large publicly traded companies or even sectoral data. Using stock market data to identify fluctuations in productivity, however, implies strong market efficiency assumptions (see for instance Shiller, 1981). We use firm-level balance sheet data instead.[2] This links the empirical part of this paper most closely to a series of papers by Higson and Holly et al. (2002, 2004), Doepke and Holly et al. (2005, 2008), Doepke and Weber (2006), as well as Holly and Santoro (2008). Higson and Holly et al. (2002), using Compustat data, study empirically the cyclicality of the standard deviation, skewness and kurtosis of the sales growth rate distribution and find them to be countercyclical, countercyclical and procyclical, respectively. Higson and Holly et al. (2004) repeat this analysis for UK data on quoted firms, and Doepke and Holly et al. (2005) for Germany, using the USTAN database, with similar findings. Doepke and Weber (2006) study, again using USTAN data, the cyclicality of transitions between sales growth regimes in firm-level data. In contrast to these papers, we focus on the cyclicality of cross-sectional second moments only, but include real value added and Solow residuals into the analysis. These are arguably the right firm-level counterpart for GDP and aggregate Solow residuals, respectively.

The quantitative part of this paper draws heavily on the recent literature on heterogenous-firm RBC models, developed in Khan and Thomas (2008) as well as Bachmann et al. (2008). Finally, this paper is related to the work by Beaudry and Portier (2006), Jaimovich and Rebelo (2008), Sims (2008) as well as Schmidt-Grohe and Uribe (2008) on the impact of news shocks on business cycle dynamics. In a companion paper (Bachmann and Bayer, 2009), we focus on the implications of countercyclical dispersion in the firm-level Solow residual innovations for cross-sectional dynamics as opposed to aggregate dynamics.

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[2]On the household side, using PSID data, Storesletten et al. (2004) have extensively demonstrated the counter-cyclicality of labor income risk. This paper fills the gap on the production side of the economy.

2 The Facts

In Section 2.1 we briefly describe the USTAN data set and the main sample selection criteria we use. Details are relegated to Appendix A.1. In Section 2.2 we present the baseline facts: the contemporaneous correlations of cyclical aggregate output and the cross-sectional standard deviations of log changes in firm-level Solow residuals, real value added and sales are negative, but the cyclical variations are small, at most 1.5 times the volatility of aggregate output. In Section 2.3 we perform extensive robustness checks and also show, how these facts depend on observable firm characteristics.

2.1 A Brief Data Description

2.1.1 USTAN Data

USTAN is a large annual firm-level balance sheet data base (*Unternehmensbilanzstatistik*) collected by *Deutsche Bundesbank*. It is unique in its combination of size and coverage as well as detail of available variables. It provides annual firm level data from 1971 to 1998 from the balance sheets and the profit and loss accounts of over 60,000 firms per year (see Stoess (2001), von Kalckreuth (2003) and Doepke et al. (2005) for further details). In the days when the discounting of commercial bills were one of the principal instruments of German monetary policy, Bundesbank law required the Bundesbank to assess the creditworthiness of all parties backing a commercial bill put up for discounting. The Bundesbank implemented this regulation by requiring balance sheet data of all parties involved. These balance sheet data were then archived and collected into a database.

Although the sampling design – one's commercial bill being put up for discounting – does not lead to a representative selection of firms in a statistical sense, the coverage of the sample is very broad. USTAN covers incorporated firms as well as privately-owned companies, which distinguishes it positively from Compustat data.[3] Its sectoral coverage – while still somewhat biased to manufacturing firms – includes the construction, the service as well as the primary sectors. This makes it different from, for instance, the Annual Survey of Manufacturing (ASM) in the U.S.[4] The following table 1 displays the sectoral coverage of our final baseline sample.

Moreover, while there remains a bias to somewhat larger and financially healthier firms, the size coverage is still fairly broad: 31% of all firms in our final baseline sample have less than

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[3]Davis et al. (2006) show that studying only publicly traded firms can lead to wrong conclusions, in particular when higher cross-sectional moments are concerned. See Appendix A.1 Table 27 for ownership coverage in our final sample.

[4]An additional advantage of these data is easy access: while access is on-site, it is practically free for researchers, so that results derived from this data base can be easily tested and checked.

Table 1: SECTORAL COVERAGE

1-digit Sector	Firm-year observations	Percentage
Agriculture	12,291	1.44
Mining & Energy	4,165	0.49
Manufacturing	405,787	47.5
Construction	54,569	6.39
Trade (Retail & Wholesale)	355,208	41.59
Transportation & Communication	22,085	2.59

20 employees and 57% have less than 50 employees (see Table 25 in Appendix A.1 for details). Finally, the Bundesbank itself frequently used the USTAN data for its macroeconomic analyses and for cross-checking national accounting data. We take this as an indication that the bank considers the data as sufficiently representative and of sufficiently high quality. This makes the USTAN data an exceptionally suitable data source for the study of cross-sectional business cycle dynamics.

2.1.2   Selection of the Baseline Sample

From the original USTAN data, we select only firms that report complete information on payroll, gross value added and capital stocks. Moreover, we drop observations from East German firms to avoid a break of the series in 1990. In addition, we remove observations that stem from irregular accounting statements, e.g. when filing for bankruptcy or when closing operations. We deflate all but the capital and investment data by the implicit deflator for gross value added from the German national accounts.

Capital is deflated with one-digit sector- and capital-good specific investment good price deflators within a perpetual inventory method. Even though USTAN data can be considered as particularly high quality data, we cannot directly use capital stocks as reported. Tax motivated depreciation and price developments of capital goods lead to a general understatement of the stock of capital a firm holds. Thus, capital stocks have to be recalculated using a perpetual inventory method (see Appendix A.2, for details). Similarly, we recover the amount of labor inputs from wage bills, as information on the number of employees (as opposed to payroll data) is only updated infrequently for some companies (see Appendix A.3, for details). Finally, the firm-level Solow residual is calculated from data on gross value added and factor inputs.

We remove outliers according to the following procedure: we calculate log changes in real gross value added, the Solow residual, real capital and employment, as well as the firm-level investment rate and drop all observations where a change falls outside a three standard deviations

interval around the year-specific mean.[5] We also drop those firms for which we do not have at least five observations in first differences. This leaves us with a sample of 854,105 firm-year observations, which corresponds to observations on 72,853 firms, i.e. the average observation length of a firm in the sample is 11.7 years. The average number of firms in the cross-section per year is 32,850. We perform numerous robustness checks with respect to each of the selection criteria and measurement choices: we use sectoral deflators for value added, an aggregate investment good price deflator, change the cut-off rule to 2.5 and 5 standard deviations and leave all firms in the sample with two and twenty observations in first differences, respectively. None of these choices change our baseline results (see Table 38 in Appendix B.1 for details).

2.1.3 Solow Residual and Productivity Innovations

We compute the firm-level Solow residual based on the following Cobb-Douglas production function in accordance with our model:

$$y_{i,t} = z_t \epsilon_{i,t} k_{i,t}^{\theta} n_{i,t}^{\nu},$$

where $\epsilon_{i,t}$ is firm-specific productivity, and z_t is aggregate productivity. We assume that labor input $n_{i,t}$ is immediately productive, whereas capital $k_{i,t}$ is pre-determined and inherited from last period. In our main specification, we estimate the output elasticities of the production factors, ν and θ, as median shares of factor expenditures over gross value added within each industry.[6] We use log-differences in the Solow residual to capture Solow residual innovations, as the persistence of firm-level Solow residuals exhibits behavior close to a unit root. In order to check, whether our results are driven by measurement error in a difficult-to-measure driving force, we also study log-differences in outcome variables: firm-level real value added and sales. We remove firm fixed and sectoral-year[7] effects from these first-difference variables to focus on idiosyncratic fluctuations that do not capture differences in sectoral responses to aggregate shocks or permanent ex-ante heterogeneity between firms.[8] We focus on the cross-sectional

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[5]This outlier removal is done after removing firm and sectoral fixed effects. Centering the outlier removal around the year mean is important to avoid artificial and countercyclical skewness of the respective distributions.

[6]To check the robustness of our results, we try alternative specifications with predefined elasticities common across sectors. We also change the timing assumption to include a predetermined employment stock, as well as immediate adjustment in both factors. All results are very robust to the alternative ways to generate the firm-specific Solow residual (see Table 31 Appendix B.1 for details).

[7]The sectoral fixed effects are essentially computed at the 2-digit level, see Table 24 in Appendix A.1 for details.

[8] Specifically, we decompose a variable observed of firm i in sector j at time t into firm a fixed effect α_i, a sectoral time effects γ_{jt} and an idiosyncratic unpredictable error term ε_{ijt}, which are orthogonal to each other

$$x_{ijt} = \alpha_i^x + \gamma_{jt}^x + \varepsilon_{ijt}^x.$$

We then focus on the error term ε_{ijt}.

dispersion of these three variables and measure the dispersion in terms of standard deviations.[9]
In a standard model with ex-ante homogenous firms, these standard deviations characterize
the uncertainty or risk of a firm with regard to its idiosyncratic productivity growth. In line with
this, the literature – e.g. Davis et al. (2006), Bloom et al. (2007), Sim (2008), Bloom et al. (2009),
Bloom (2009) and Gilchrist et al. (2009) – has modeled fluctuations in idiosyncratic uncertainty
as fluctuations in the standard deviation of firm-specific innovations in the Solow residual. We
follow this view and study the quantitative aggregate importance of fluctuations in the cross-
sectional dispersion of productivity innovations.

2.1.4 Macro data

When combining this micro data with aggregate data, we have to take a stance on the sectoral
aggregate that is the empirical counterpart to our model. We chose to include firms from the
following six sectors in our analysis: agriculture, mining and energy, manufacturing, construc-
tion, trade (both retail and wholesale) as well as the transportation and communication sector.
This aggregate can be roughly characterized as the non-financial private business sector (NF-
PBS) in Germany. Whenever we use the term aggregate in the following, we mean this sector.

German national accounting data per one-digit sector (see Appendix A.1, Footnote 49, for
a detailed description of the data sources used) allow us to compute real value added, invest-
ment, capital and employment data for this sectoral aggregate, and therefore also an aggregate
Solow residual. Our USTAN sample captures on average 70% of sectoral value added and 78%
of sectoral real gross output (sales). The NFPBS, in turn, comprises 59% and 69%, respectively,
of aggregate real value added and real gross output.

In addition to representing a large part of the non-financial private business sector in Ger-
many, USTAN also represents its cyclical behavior very well, as the following Table 2 shows:[10]

Table 2: CYCLICALITY OF CROSS-SECTIONAL AVERAGES

Cross-sectional Moment	$\rho(\cdot, HP(100) - Y)$	$\rho(\cdot, Log - diff - Y)$
$mean(\Delta \log \epsilon_{i,t})$	0.592	0.923
$mean(\Delta \log y_{i,t})$	0.663	0.920
$mean(\Delta \log sales_{i,t})$	0.410	0.899

Notes: mean: cross-sectional average, linearly detrended. ρ: time series correlation coefficient.
$HP(\lambda) - Y$: Cyclical component of aggregate log-GDP after HP-filtering using smoothing parameter λ. Y refers to
the output of the aggregate of the non-financial private business sector.

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[9]The baseline within-transformed cross-sectional data for these dispersions can be found in Table 30 in Ap-
pendix A.5.

[10]We further illustrate the good representation properties of USTAN in Appendix A.1.

2.2 Main Facts: Idiosyncratic Productivity Risk – Large and Countercyclical

Using the micro data from USTAN, we can compute the cross-sectional dispersions for innovations in firm-level Solow residuals, real value added and real sales. We analyze both their time-averaged long-run as well as their cyclical properties. Table 3 presents our main results:

Table 3: CYCLICALITY AND VOLATILITY OF CROSS-SECTIONAL DISPERSION

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100) - Y)$	$\mu(\cdot)$
$std(\Delta \log \epsilon_{i,t})$	2.67%	-0.481	0.120
$std(\Delta \log y_{i,t})$	3.73%	-0.451	0.142
$std(\Delta \log sales_{i,t})$	3.82%	-0.405	0.187

Notes: std: cross-sectional standard deviation, linearly detrended. *cv*: time series coefficient of variation. *μ*: time series average. See further notes to Table 2.

- The time-averaged cross-sectional standard deviation of the log-changes in firm-level Solow residuals is large: 12%. In contrast, the volatility of the linearly detrended aggregate Solow residual is 2.06%, i.e. an order of magnitude smaller; the one for the linearly detrended cross-sectional average Solow residual 2.37%.[11] This also means that firm-specific productivity risk is an order of magnitude larger than in Kahn and Thomas (2008) as well as Bloom et al. (2009) who, without the discipline of firm-level data, are using for the standard deviation of the innovations to firm-level Solow residuals in their models: 2.2% annually.

- The cross-sectional standard deviation of the innovations to firm-level Solow residuals is countercyclical with a correlation coefficient with HP-filtered aggregate GDP of -0.481. Table 4 shows that this countercyclicality is significant, as the 5% and 95% confidence bands from 10,000 parametric bootstrap simulations are both negative.[12] The last column of Table 4 displays the fraction of positive correlations in these bootstrap simulations.

- The size of annual fluctuations in risk is relatively small, with a time-series coefficient of variation of 2.67%.

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[11]This order-of-magnitude difference also suggests that the fluctuations-in-uncertainty-story has to be likely one about idiosyncratic productivity, rather than aggregate productivity. Firms' policy functions are mainly determined by the idiosyncratic state given this difference, and fluctuations in anyway small aggregate risk do not seem to be a plausible candidate for business cycle fluctuations. In addition, we find for the cyclical components of aggregate quarterly output and employment in Germany that moving-window volatility estimates as well as the squared residuals from AR(1) regressions are essentially uncorrelated with the cycle.

[12]We use a pairwise unrestricted VAR with one lag as the parametric model. The results from a nonparametric overlapping block bootstrap with a block size of four are similar to the parametric bootstrap.

For comparison, Bloom et al. (2009) find a correlation of their stock market-based uncertainty index with GDP of -0.606. Gourio (2008), using Compustat data, finds the correlation between GDP and the standard deviation of the innovation to a permanent productivity shock to be - 0.56.[13] Thus our finding aligns well in this point with previous work. For the size of the fluctuations, there is little previous evidence. In his numerical simulations, Bloom (2009) considers a productivity process for monthly data, which exhibits a time-series coefficient of variation of the dispersion in annual productivity growth of roughly 17%.

One could be concerned that the Solow residual does not measure productivity well. For this reason, we provide evidence also from log-differences in endogenous outcome variables, namely real value added and real sales. As balance sheet items these are arguably less subject to measurement error.[14] Using these *indirect* measures makes no difference: their cross-sectional dispersions are large on average, they are significantly countercyclical, but their business cycle fluctuations are small.

Table 4: CYCLICALITY OF CROSS-SECTIONAL DISPERSION - SIGNIFICANCE

Cross-sectional Moment	$\rho(\cdot, HP(100) - Y)$	5%	95%	Frac. w. opposite sign
$std(\Delta \log \epsilon_{i,t})$	-0.481	-0.678	-0.306	0.000
$std(\Delta \log y_{i,t})$	-0.451	-0.675	-0.196	0.005
$std(\Delta \log sales_{i,t})$	-0.405	-0.711	0.042	0.065

Notes: The columns 5% and 95% refer to the top and bottom 5-percentiles in a parametric bootstrap of the correlation coefficient. The last column displays the fraction of simulations with the opposite sign of the point estimate. See further notes to Tables 2 and 3.

2.3 Robustness

Two questions arise: first, are these results sensitive to the choice of the cyclical indicator and the dispersion measure used? Secondly, are they driven by sample composition or cyclical sample selection? We show they are not.

Cyclical Indicator and Dispersion Measure

The following Table 5 shows that countercyclical idiosyncratic uncertainty as measured by the cross-sectional dispersion of firm-level Solow residual innovations is robust to the choice of the cyclical indicator (for $std(\Delta \log y_{i,t})$ and $std(\Delta \log sales_{i,t})$ see Table 32 in Appendix B.1).

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[13]Interestingly, Comin and Mulani (2006), using also Compustat data, find procyclicality of a moving-standard deviation estimator of firm-level risk in real sales innovations.

[14]Bachmann and Bayer (2009) show that when looking at inputs, log-changes in firm-level employment are similarly countercyclical, firm-level investment rates are strongly and significantly procyclical.

Table 5: COUNTERCYCLICALITY OF IDIOSYNCRATIC UNCERTAINTY - ROBUSTNESS TO CYCLICAL INDICATOR

Cyclical Indicator	$\rho(std(\Delta\log\epsilon_{i,t}),\cdot)$
HP(6.25)-Y	-0.485
Log-diff-Y	-0.583
$mean(\Delta\log\epsilon_{i,t})$	-0.591
Log-diff-N	-0.542
Log-diff-Solow Residual	-0.465

Notes: See notes to Tables 2 and 3. *N* refers to aggregate employment.

The result stands irrespective of whether we choose as cyclical indicators output filtered using a smaller smoothing parameter for the HP filter, following Ravn and Uhlig (2002), apply a log-difference filter to output, or use the linearly detrended average cross-sectional investment rate, or the HP(100)-filtered aggregate employment, or aggregate Solow residuals.

Vice versa, as Table 6 shows, our three findings are also robust to the numerous choices we have made for the other part of the correlation, $std(\Delta\log\epsilon_{i,t})$, for instance (for $std(\Delta\log y_{i,t})$ and $std(\Delta\log sales_{i,t})$ see Table 33 in Appendix B.1). For instance, one can use the interquartile range (iqr) as the dispersion measure. Moreover, it is not the removal of firm-level and sectoral fixed effects that induces the countercyclicality, as row three of this table shows. Finally, the last two rows demonstrate that the results are neither driven by the German reunification, nor by the strong recession in 1975.

Table 6: MORE ROBUSTNESS

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100)-Y)$	$\mu(\cdot)$
$std(\Delta\log\epsilon_{i,t})_{\text{Baseline}}$	2.67%	-0.481	0.120
$iqr(\Delta\log\epsilon_{i,t})$	4.56%	-0.386	0.130
$std(\Delta\log\epsilon_{i,t})_{\text{raw}}$	2.90%	-0.449	0.126
$std(\Delta\log\epsilon_{i,t})_{1973-1990}$	2.51%	-0.680	0.121
$std(\Delta\log\epsilon_{i,t})_{1977-1998}$	2.55%	-0.375	0.119

Notes: See notes to Tables 2 and 3. *iqr* stands for interquartile range, which is linearly detrended.

Sample Composition

USTAN is no random sample. Consequently, we need to check whether our findings are specific to certain sectors or types of firms overrepresented in this sample. Our findings for $std(\Delta\log\epsilon_{i,t})$ in Tables 7 to 9[15] are twofold: on the one hand, we find that *qualitatively* our em-

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[15]For $std(\Delta\log y_{i,t})$ and $std(\Delta\log sales_{i,t})$ see Tables 34, 35 and 36 in Appendix B.1.

pirical results hold across various observable characteristics; on the other hand, we also show that, *quantitatively*, using data sets that overrepresent large or publicly traded firms or are limited to manufacturing, as is the case with most U.S. data sets, one may overstate the cyclical fluctuations in firm-level uncertainty.

Table 7: RESULTS BY SECTOR

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100) - Y)$	$\rho(\cdot, HP(100) - Y^{Sect.})$	$\mu(\cdot)$
$std(\Delta\log\epsilon_{i,t})_{\text{Aggregate}}$	2.67%	-0.481	-	0.120
$std(\Delta\log\epsilon_{i,t})_{\text{AGR}}$	4.44%	-0.045	-0.283	0.173
$std(\Delta\log\epsilon_{i,t})_{\text{MIN/ENE}}$	11.46%	-0.166	0.107	0.116
$std(\Delta\log\epsilon_{i,t})_{\text{MAN}}$	3.54%	-0.607	-0.397	0.115
$std(\Delta\log\epsilon_{i,t})_{\text{CON}}$	4.56%	-0.483	0.037	0.112
$std(\Delta\log\epsilon_{i,t})_{\text{TRD}}$	2.68%	-0.192	-0.387	0.124
$std(\Delta\log\epsilon_{i,t})_{\text{TRA/COM}}$	3.28%	-0.036	0.034	0.136

Notes: See notes to Tables 2 and 3. AGR: Agriculture; MIN/ENE: Mining & Energy; MAN: Manufacturing; CON: Construction ; TRD: Trade (Retail & Wholesale); TRA/COM: Transportation & Communication. Y refers to the output of the aggregate of the non-financial private business sector. $Y^{Sect.}$ refers to the output of the corresponding sector.

Table 7 displays the behavior of productivity risk for the six single-digit sectors in the USTAN sample. The relatively large idiosyncratic risk is obviously not specific to any sector, in fact the numbers are surprisingly uniform across sectors. When one disregards the small agricultural sector, average productivity uncertainty ranges from 11.2% in the construction sector to 13.6% in the transportation and communication sector. For the two sectors that comprise almost 90% of all firm-year observations in the data – manufacturing and trade – also the countercyclicality-of-uncertainty result is robust.[16] Finally, also the result that risk fluctuations are not large is robust across sectors (again with the exception of the small mining and energy sector). Manufacturing and construction exhibit somewhat larger risk fluctuations, but the difference to both the trade sector and the aggregate is small.

The non-random design of USTAN manifests itself not only in the sectoral but also the size composition of the sample. To understand the effect of this, we split the sample according to firm size, as measured by employment. Again this yields no different results. Table 8 shows that large firms exhibit a somewhat smaller average uncertainty that fluctuates more than the uncertainty of small firms, but overall the results are surprisingly uniform across size classes.[17]

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[16]Had we used Log-diff-Y as the cyclical measure for the trade sector, the countercyclicality indicator would be -0.474, and -0.455 in the case of Log-diff-Solow Residual. Moreover, the acyclicality of uncertainty in the construction sector, when the own sectoral output is used, goes away, when instead we use the average sectoral Solow residual innovation from USTAN as the cyclical indicator: -0.059, 0.291, -0.638, -0.596, -0.270 and -0.290, respec-

Table 8: RESULTS BY FIRM SIZE (EMPLOYMENT)

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100) - Y)$	$\mu(\cdot)$
$std(\Delta \log \epsilon_{i,t})_{\text{Aggregate}}$	2.67%	-0.481	0.120
$std(\Delta \log \epsilon_{i,t})_{\text{First Quartile}}$	1.75%	-0.334	0.142
$std(\Delta \log \epsilon_{i,t})_{\text{Second Quartile}}$	3.21%	-0.483	0.119
$std(\Delta \log \epsilon_{i,t})_{\text{Third Quartile}}$	3.13%	-0.414	0.112
$std(\Delta \log \epsilon_{i,t})_{\text{Fourth Quartile}}$	4.49%	-0.502	0.104
$std(\Delta \log \epsilon_{i,t})_{\text{Largest 5\%}}$	5.55%	-0.508	0.098

Notes: See notes to Tables 2 and 3. First Quartile refers to the smallest 25% firms in a given year, Second Quartile to the firms with size in the 25%-50% range, etc., if size is measured in employment.

Nevertheless, it shows that using data sets, which suffer from an even stronger overrepresentation of large firms, such as Compustat, may lead to an overstatement of the cyclical fluctuations in firm-level uncertainty. A similar point is made in Table 9, where we split the sample according to a broad definition of legal form. Again, the publicly traded firms, which are also typically large, are slightly different from the aggregate.

Table 9: RESULTS BY LEGAL FORM

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100) - Y)$	$\mu(\cdot)$
$\sigma(\Delta \log \epsilon_{i,t})_{\text{Aggregate}}$	2.67%	-0.481	0.120
$std(\Delta \log \epsilon_{i,t})_{\text{Publicly Traded}}$	4.01%	-0.248	0.106
$std(\Delta \log \epsilon_{i,t})_{\text{Limited Liability}}$	3.02%	-0.520	0.121
$std(\Delta \log \epsilon_{i,t})_{\text{Unlimited Liability}}$	2.66%	-0.395	0.121

Notes: See notes to Tables 2 and 3. Publicly Traded refers incorporated entities (AG and KGaA), Limited Liability refers to limited liability companies (GmbH and GmbH&Co.), Unlimited Liability refers to fully liable partnerships, unincorporated associations and municipal agencies (OHG, KG, e.V., KöR), where the latter two categories are only a small fraction.

Cyclical Sample Selection

So far we have focused on permanent misrepresentation of certain kinds of firms, like a bias towards manufacturing. As we have shown, this can be dealt with by splitting the sample along observable characteristics. The possibility of systematic cyclical variation in sample selection, for instance through firm entry and exit, poses a more severe problem to our analysis.

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tively, for the six sectors.

[17]We tried alternative splits according to the stock of capital and real value added. The results are very similar, details can be found in Table 35 in Appendix B.1. Numbers for the dispersions of innovations to firm-level value added and sales across size classes are available on request. We also studied a stratification according to firm-level productivity and found the same results. Again, details are available on request.

We address the issue of cyclical sample selection in two different ways: in a first pass, we constrain our sample to those firms which we observe at least 20 times in the sample; secondly, we estimate a simple selection model, where lagged firm-level Solow residuals determine selection and firm-level Solow residual innovations are modeled as a mean regression.

Table 10: RESULTS FOR FIRMS WITH AT LEAST 20 OBSERVATIONS

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100) - Y)$	$\mu(\cdot)$
$std(\Delta \log \epsilon_{i,t})_{\text{Aggregate}}$	2.67%	-0.481	0.120
$std(\Delta \log \epsilon_{i,t})_{\text{Longer in sample}}$	4.61%	-0.341	0.106
$std(\Delta \log y_{i,t})_{\text{Aggregate}}$	3.73%	-0.451	0.142
$std(\Delta \log y_{i,t})_{\text{Longer in sample}}$	7.00%	-0.383	0.120
$std(\Delta \log sales_{i,t})_{\text{Aggregate}}$	3.82%	-0.405	0.187
$std(\Delta \log sales_{i,t})_{\text{Longer in sample}}$	7.90%	-0.466	0.149

Notes: See notes to Tables 2 and 3. The number of total firm-year observations goes down from 854,105 to 212,607.

When we constrain our sample to those firms which we observe at least 20 times (see Table 10), we find the same countercyclical pattern in productivity risk as in the baseline sampling with a higher volatility of uncertainty. The advantage of this sample is that its selection is by construction less subject to cyclical fluctuations. Its disadvantage is that the non-cyclical selection bias becomes stronger and the sample covers more large, publicly traded firms. This partly drives our finding, as the second procedure below shows.

Alternatively, we control for sample selection in the following way: we estimate a simple selection model, where lagged firm-level Solow residuals determine selection. Productivity levels are a natural determinant of market entry and exit. Firm-level Solow residual innovations are modeled as a mean regression. Lagged firm specific Solow residuals are excluded from this mean regression as productivity is highly persistent and thus lagged levels do not predict changes. We use the maximum likelihood estimator by Heckman (1976) to infer the selection-corrected standard deviation of the residual in the reduced-form regression for firm-level Solow residual innovations. The correlation between the latter and the sample standard deviation of firm-level Solow residual innovations is 0.972, indicating that our results are not influenced by systematic sample selection. Table 11 shows that this indeed is the case.

Table 11: CROSS-SECTIONAL DISPERSION - SELECTION-CORRECTED

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100) - Y)$	$\mu(\cdot)$
$std(\Delta \log \epsilon_{i,t})_{\text{Aggregate}}$	2.67%	-0.481	0.120
$std(\Delta \log \epsilon_{i,t})_{\text{Selection-Corrected}}$	2.63%	-0.439	0.118

Notes: See notes to Tables 2 and 3.

On the basis of these two pieces of evidence – robustness of our results to a sample with surviving firms and a high correlation between selection-corrected and sample standard deviations of firm-level Solow residual innovations – we conclude that cyclical sample attrition is not central to our findings.[18]

Measurement Error

In any Solow residual calculation that is based on a simple Cobb-Douglas production function with only labor and capital, there is the potential problem of attributing optimal changes in utilization, hours per worker or effort to random productivity changes and therefore of overstating (average) firm-level risk. Therefore we focus in Table 12 on those firms who did not change the intensity of their material usage between two periods, assuming that observed changes in material usage are a good proxy for unobserved changes in utilization (see Basu, 1996, for this idea). These firms indeed exhibit a somewhat smaller productivity risk that fluctuates slightly more in relative terms than the productivity risk of the average firm in the sample. The measured countercyclicality of idiosyncratic uncertainty is smaller,[19] but overall our results carry over to this group of firms.

Table 12: RESULTS BY MATERIAL USAGE

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100) - Y)$	$\mu(\cdot)$
$std(\Delta \log \epsilon_{i,t})_{\text{Aggregate}}$	2.67%	-0.481	0.120
$std(\Delta \log \epsilon_{i,t})_{\text{Material Intensity const.}}$	4.19%	-0.247	0.103

Notes: See notes to Tables 2 and 3. Material Intensity const. refers to firms whose material expenditures over sales $\frac{m_{i,t}}{sales_{i,t}}$ have changed by less then one percentage point in absolute terms. Results for $std(\Delta \log y_{i,t})$ and $std(\Delta \log sales_{i,t})$ can be found in tables 37 in Appendix B.1.

More generally, we take measurement error at the micro level into account in the following way: Assuming additive classical measurement error in firm-level log-Solow residuals and log-value added and a time-invariant measurement error variance in both, we can use the time-average of the difference between the one-period innovation to observed firm-level log-Solow residuals/log-value added and half the variance of the two-period innovations to estimate the variances of measurement error for both quantities. Subtracting this number (twice) from the observed variance of one-period innovations yields an estimate of the true variance of the inno-

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[18]We also investigated whether the time-average and the cyclical behavior of firm-level fixed effects is large enough to indicate systematic cyclical sample selection as a problem. We did not find that it is. Details are available on request.

[19]The decline happens, irrespective of the cyclical measure used, but its magnitude is somewhat sensitive to the choice of the cyclical measure. For instance, had we used Log-diff-Y, the decline would have been from -0.583 to -0.460, and from -0.465 to -0.393 in the case of Log-diff-Solow Residual.

vations. In other words, we obtain the variance of the measurement error from a permanent-transitory decomposition of changes in productivity. This means, we attribute *all* temporary changes in firm-level log-Solow residuals/log-value added to measurement error. For this reason one should view the obtained numbers as upper bounds. Table 13 shows that the cyclicality measure for firm-level uncertainty is basically unaffected, but that its fluctuations might be somewhat underestimated when measurement error is not taking into account. Conversely, the last column of Table 13 shows that part of the observed cross-sectional dispersion in innovations to both firm-level Solow residuals and value added might be measurement error. Nevertheless, even when measurement error is taking into account, idiosyncratic uncertainty is significantly larger than aggregate volatility.

Table 13: RESULTS AND MEASUREMENT ERROR

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100) - Y)$	$\mu(\cdot)$
$std(\Delta \log \epsilon_{i,t})_{\text{Aggregate}}$	2.67%	-0.481	0.120
$std(\Delta \log \epsilon_{i,t})_{\text{Corrected}}$	4.72%	-0.472	0.091
$std(\Delta \log y_{i,t})_{\text{Aggregate}}$	3.73%	-0.451	0.142
$std(\Delta \log y_{i,t})_{\text{Corrected}}$	4.63%	-0.448	0.127

Notes: See notes to Tables 2 and 3. *Corrected* refers to the standard deviation corrected for measurement error, as explained in the main text.

2.4 External Validation

In this section we use an additional data set to confirm the countercyclicality of firm-level uncertainty. Specifically, we make use of survey evidence on firms's general six-month-ahead business outlook from the IFO institute in Germany to validate our findings from the USTAN data set. One of the items in this monthly business survey is: *"Expectations for the next six months: our business situation with respect to XY[20] will, in a cyclical view, improve, remain about the same, develop unfavorably.".* There is a widespread literature which argues that (qualitative) survey disagreement is a good proxy for true firm-level uncertainty, see, for instance and for an overview, Fuss and Vermeulen (2004). We make use of this idea and compute a firm-level uncertainty index as follows, where p_t^+ denotes the fraction of firms that see their business outlook improve in a given month, and p_t^- the fraction of firms that see their business outlook deteriorate:

$$Uncertainty_t^{IFO} = p_t^+ + p_t^- - (p_t^+ - p_t^-)^2.$$

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[20]The IFO survey is at the product category level, multiproduct firms can fill out more than one questionnaire.

If all responders think their business situation will improve, this index will be minimized at zero, if they split in half about their business outlook, this index will be maximized at unity.

Two issues arise, when comparing the IFO evidence to the USTAN data. First, the IFO institute conducts this survey in four major sectors: manufacturing, construction, retail and wholesale. Due to limited data availability in the German system of national accounting we combined the latter two into one trade sector for the analysis of the USTAN data. However, as Table 24 in Appendix A.1 shows, roughly 60% of the USTAN observations in the trade sector are from wholesale, which is why we use the IFO wholesale data in our comparison with the USTAN trade sector. Secondly, the IFO data are available on a monthly basis – looking six months ahead –, whereas the USTAN are annual. Thus, some time aggregation of the uncertainty index is necessary. We use the simple year average of $Uncertainty_t^{IFO}$ as our baseline case.[21] Table 14 displays our findings:[22]

Table 14: COMPARISON USTAN AND IFO

Sector	$\rho(std(\Delta \log \epsilon_{i,t}), Uncertainty^{IFO})$	$\rho(Uncertainty^{IFO}, HP(100) - Y)$
Manufacturing	0.642	-0.525
Construction	0.396	-0.606
Wholesale	0.536	-0.519
Retail	0.0767	-0.294

Notes: See notes to Tables 2 and 3.

It shows in the first column the correlation between the dispersion of USTAN firm-level Solow Residual innovations and the IFO uncertainty index, and the correlation between the latter and the cyclical component of aggregate output in the second column. The correlation between the two measures of firm-level uncertainty is remarkably high, in particular for the manufacturing sector, which the majority of the firms in the IFO survey belong to.[23] Given the smaller numbers of firms in the construction and the trade sector, which is further complicated by the imperfect match to the USTAN trade sector, a small decline in correlation there is not surprising. And the second column shows again the very consistent countercyclicality of this alternative firm-level uncertainty measure. These findings give us further confidence in the robustness of our findings in the USTAN data set.

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[21]We experiment with semi-annual and differently timed averages as well as the June uncertainty index of a given year. The results are robust

[22]Figure 14 in Appendix B.1 displays the appropriately normalized time series of the dispersion of firm-level Solow residual innovations from USTAN and the IFO uncertainty index for the manufacturing, construction and wholesale sectors. The comove remarkably well.

[23]According to IFO personnel there are, on average, roughly 3400 firms from manufacturing, 1900 from retail and wholesale and 1100 from construction participation in the survey every month.

In Table 38 in Appendix B.1 we provide further robustness checks for our results. In all these robustness checks we find a strongly countercyclical productivity risk with relatively small cyclical fluctuations in this risk. Next, we explore the quantitative implications of our three findings: large idiosyncratic uncertainty, countercyclical fluctuations in and relatively low volatility of this uncertainty, in a heterogeneous firm dynamic stochastic general equilibrium model with fixed cost of capital adjustment.

3   The Model

In this section we describe our model economy. We start with the firm's problem, followed by a brief description of the households and the definition of equilibrium. We conclude with a sketch of the equilibrium computation. Our model follows closely Khan and Thomas (2008) and Bachmann et al. (2008). Since there the model set up is discussed in detail, we will be rather brief here.

The main departure from either papers is the introduction of a second exogenous aggregate state. Following Bloom (2009) we assume that firms today observe the standard deviation of the distribution of idiosyncratic productivity shocks tomorrow, $\sigma(\epsilon')$. Following Khan and Thomas (2008), we approximate this now bivariate aggregate state process with a discrete Markov chain.

3.1   Firms

The economy consists of a unit mass of small firms. We do not model entry and exit decisions. There is one commodity in the economy that can be consumed or invested. Each firm produces this commodity, employing its pre-determined capital stock (k) and labor (n), according to the following Cobb-Douglas decreasing-returns-to-scale production function ($\theta > 0$, $\nu > 0$, $\theta + \nu < 1$):

$$y = z\epsilon k^{\theta} n^{\nu}, \tag{1}$$

where z and ϵ denote aggregate and firm-specific (idiosyncratic) technology, respectively.

The idiosyncratic technology process has autocorrelation ρ_I. It follows a Markov chain, whose transition matrix depends on the aggregate state of its time-varying standard deviation, $\sigma(\epsilon')$. In contrast, its support is independent of the aggregate state. To also capture observed excess kurtosis in the idiosyncratic productivity shocks, we use a mixture of two Gaussian distributions in the Tauchen-approximation algorithm instead of the usual normal distribution.[24]

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[24]Tauchen (1986). For details, see Section 4.

We denote the trend growth rate of aggregate productivity by $(1-\theta)(\gamma-1)$, so that aggregate y and k grow at rate $\gamma-1$ along the balanced growth path. From now on we work with k and y (and later C) in efficiency units. The linearly detrended logarithm of aggregate productivity levels as well as linearly detrended $\sigma(\epsilon)$ evolve according to a VAR(1) process, with normal innovations v that have zero mean and covariance Ω:

$$
\begin{pmatrix} \log z' \\ \sigma(\epsilon'') - \bar{\sigma}(\epsilon) \end{pmatrix} = \varrho_A \begin{pmatrix} \log z \\ \sigma(\epsilon') - \bar{\sigma}(\epsilon) \end{pmatrix} + v,
\tag{2}
$$

where $\bar{\sigma}(\epsilon)$ denotes the steady state standard deviation of idiosyncratic productivity innovations.[25]

Productivity innovations at different aggregation levels are independent. Also, idiosyncratic productivity shocks are independent across productive units. In contrast, we do not impose any restrictions on Ω or $\varrho_A \in \mathbb{R}^{2 \times 2}$.

Each period a firms draws from a time-invariant distribution, G, its current cost of capital adjustment, $\xi \geq 0$, which is denominated in units of labor. G is a uniform distribution on $[0, \bar{\xi}]$, common to all firms. Draws are independent across firms and over time, and employment is freely adjustable.

At the beginning of a period, a firm is characterized by its pre-determined capital stock, its idiosyncratic productivity, and its capital adjustment cost. Given the aggregate state, it decides its employment level, n, production and depreciation occurs, workers are paid, and investment decisions are made. Then the period ends.

Upon investment, i, the firm incurs a fixed cost of $\omega\xi$, where ω is the current real wage rate. Capital depreciates at a rate δ. We can then summarize the evolution of the firm's capital stock (in efficiency units) between two consecutive periods, from k to k', as follows:

	Fixed cost paid	$\gamma k'$
$i \neq 0$:	$\omega\xi$	$(1-\delta)k + i$
$i = 0$:	0	$(1-\delta)k$

Given the i.i.d. nature of the adjustment costs, it is sufficient to describe differences across firms and their evolution by the distribution of firms over (ϵ, k). We denote this distribution by μ. Thus, $\big(z, \sigma(\epsilon'), \mu\big)$ constitutes the current aggregate state and μ evolves according to the law of motion $\mu' = \Gamma(z, \sigma(\epsilon'), \mu)$, which firms take as given.

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[25] Specifying this process in terms of $\log\big(\sigma(\epsilon)\big)$, in order to avoid negativity of the standard deviation of idiosyncratic productivity shocks is – given its high steady state value and relatively low variability – an unnecessary precaution that does not change the results. Simulation results are available upon request.

Next we describe the dynamic programming problem of each firm. We will take two short-cuts (details can be found in Khan and Thomas, 2008). First, we state the problem in terms of utils of the representative household (rather than physical units), and denote by $p = p(z, \sigma(\epsilon'), \mu)$ the marginal utility of consumption. Second, given the i.i.d. nature of the adjustment costs, continuation values can be expressed without explicitly taking into account future adjustment costs.

Let $V^1(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu)$ denote the expected discounted value—in utils—of a firm that is in idiosyncratic state (ϵ, k, ξ), given the aggregate state $(z, \sigma(\epsilon'), \mu)$. Then the expected value prior to the realization of the adjustment cost draw is given by:

$$V^0(\epsilon, k; z, \sigma(\epsilon'), \mu) = \int_0^{\bar{\xi}} V^1(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu) G(d\xi). \tag{3}$$

With this notation the dynamic programming problem is given by:

$$V^1(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu) = \max_n \{CF + \max(V_{\text{no adj}}, \max_{k'}[-AC + V_{\text{adj}}])\}, \tag{4}$$

where CF denotes the firm's flow value, $V_{\text{no adj}}$ the firm's continuation value if it chooses inaction and does not adjust, and V_{adj} the continuation value, net of adjustment costs AC, if the firm adjusts its capital stock. That is:

$$CF = [z\epsilon k^\theta n^\nu - \omega(z, \sigma(\epsilon'), \mu) n] p(z, \sigma(\epsilon'), \mu), \tag{5a}$$

$$V_{\text{no adj}} = \beta E[V^0(\epsilon', (1-\delta)k/\gamma; z', \sigma(\epsilon''), \mu')], \tag{5b}$$

$$AC = \xi \omega(z, \sigma(\epsilon'), \mu) p(z, \sigma(\epsilon'), \mu), \tag{5c}$$

$$V_{\text{adj}} = -ip(z, \sigma(\epsilon'), \mu) + \beta E[V^0(\epsilon', k'; z', \sigma(\epsilon''), \mu')], \tag{5d}$$

where both expectation operators average over next period's realizations of the aggregate and idiosyncratic shocks, conditional on this period's values, and we recall that $i = \gamma k' - (1-\delta)k$. Also, β denotes the discount factor of the representative household.

Taking as given $\omega(z, \sigma(\epsilon'), \mu)$ and $p(z, \sigma(\epsilon'), \mu)$, and the law of motion $\mu' = \Gamma(z, \sigma(\epsilon'), \mu)$, the firm chooses optimally labor demand, whether to adjust its capital stock at the end of the period, and the optimal capital stock, conditional on adjustment. This leads to policy functions: $N = N(\epsilon, k; z, \sigma(\epsilon'), \mu)$ and $K = K(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu)$. Since capital is pre-determined, the optimal employment decision is independent of the current adjustment cost draw.

3.2 Households

We assume a continuum of identical households that have access to a complete set of state-contingent claims. Hence, there is no heterogeneity across households. Moreover, they own shares in the firms and are paid dividends. We do not need to model the household side in detail (see Khan and Thomas (2008) for the details), and concentrate instead on the first-order conditions to determine the equilibrium wage and the marginal utility of consumption.

Households have a standard felicity function in consumption and (indivisible) labor:

$$U(C, N^h) = \log C - A N^h, \tag{6}$$

where C denotes consumption and N^h the household's labor supply. Households maximize the expected present discounted value of the above felicity function. By definition we have:

$$p(z, \sigma(\epsilon'), \mu) \equiv U_C(C, N^h) = \frac{1}{C(z, \sigma(\epsilon'), \mu)}, \tag{7}$$

and from the intratemporal first-order condition:

$$\omega(z, \sigma(\epsilon'), \mu) = -\frac{U_N(C, N^h)}{p(z, \sigma(\epsilon'), \mu)} = \frac{A}{p(z, \sigma(\epsilon'), \mu)}. \tag{8}$$

3.3 Recursive Equilibrium

A *recursive competitive equilibrium* for this economy is a set of functions

$$\left(\omega, p, V^1, N, K, C, N^h, \Gamma \right),$$

that satisfy

1. *Firm optimality*: Taking ω, p and Γ as given, $V^1(\epsilon, k; z, \sigma(\epsilon'), \mu)$ solves (4) and the corresponding policy functions are $N(\epsilon, k; z, \sigma(\epsilon'), \mu)$ and $K(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu)$.

2. *Household optimality*: Taking ω and p as given, the household's consumption and labor supply satisfy (7) and (8).

3. *Commodity market clearing*:

$$C(z, \sigma(\epsilon'), \mu) = \int z \epsilon k^\theta N(\epsilon, k; z, \sigma(\epsilon'), \mu)^\nu d\mu - \int \int_0^{\bar{\xi}} [\gamma K(\epsilon, k, \xi; z, \sigma(\epsilon'), \mu) - (1 - \delta) k] dG d\mu.$$

4. *Labor market clearing*:

$$N^h(z,\sigma(\epsilon'),\mu) = \int N(\epsilon,k;z,\sigma(\epsilon'),\mu)d\mu + \int\int_0^{\bar{\xi}} \xi \mathcal{J}\Big(\gamma K(\epsilon,k,\xi;z,\sigma(\epsilon'),\mu) - (1-\delta)k\Big)dGd\mu,$$

where $\mathcal{J}(x) = 0$, if $x = 0$ and 1, otherwise.

5. *Model consistent dynamics*: The evolution of the cross-section that characterizes the economy, $\mu' = \Gamma(z,\sigma(\epsilon'),\mu)$, is induced by $K(\epsilon,k,\xi;z,\sigma(\epsilon'),\mu)$ and the exogenous processes for z, $\sigma(\epsilon')$ as well as ϵ.

Conditions 1, 2, 3 and 4 define an equilibrium given Γ, while step 5 specifies the equilibrium condition for Γ.

3.4 Solution

As is well-known, (4) is not computable, since μ is infinite dimensional. Hence, we follow Krusell and Smith (1997, 1998) and approximate the distribution μ by its first moment over capital, and its evolution, Γ, by a simple log-linear rule. In the same vein, we approximate the equilibrium pricing function by a log-linear rule discrete – aggregate state by discrete aggregate state:

$$\log\bar{k}' = a_k\big(z,\sigma(\epsilon')\big) + b_k\big(z,\sigma(\epsilon')\big)\log\bar{k}, \tag{9a}$$

$$\log p = a_p\big(z,\sigma(\epsilon')\big) + b_p\big(z,\sigma(\epsilon')\big)\log\bar{k}, \tag{9b}$$

where $\bar{k}$ denotes aggregate capital holdings. Given (8), we do not have to specify an equilibrium rule for the real wage. As usual with this procedure, we posit this form and check that in equilibrium it yields a good fit to the actual law of motion. We use this simple forecasting rule because it is much less computationally involved than a rule that includes higher moments of the capital distribution. In models without second moment shocks, it has been extensively shown that the first moment suffices. Unfortunately, we show here that the pure R^2 goodness-of-fit metric does not favor the simple rule (9a): R^2 below 0.9 are possible, as we shall see in Section 5.2. However, we show for our baseline calibration that the aggregate dynamics of the economy are hardly affected, when higher moments of the capital distribution are included and the R^2 are pushed closer to unity (see Bachmann et al. (2008) for a similar observation). Therefore, we prefer the increase in computational speed and report our results, unless otherwise noted, with the first moment only as a state variable.

Combining these assumptions and substituting $\bar{k}$ for μ into (4) and using (9a)–(9b), we have that (4) becomes a computable dynamic programming problem with policy functions $N = N(\epsilon, k; z, \sigma(\epsilon'), \bar{k})$ and $K = K(\epsilon, k, \xi; z, \sigma(\epsilon'), \bar{k})$. We solve this problem via value function iteration on V^0.

With these policy functions, we can then simulate a model economy *without* imposing the equilibrium pricing rule (9b), but rather solve for it along the way. We simulate the model economy for 1,600 time periods and discard the first 100 observations, when computing any statistics. This procedure generates a time series of $\{p_t\}$ and $\{\bar{k}_t\}$ endogenously, with which assumed rules (9a)–(9b) can be updated via a simple OLS regression. The procedure stops when the updated coefficients $a_k(z, \sigma(\epsilon'))$ and $b_k(z, \sigma(\epsilon'))$, as well as $a_p(z, \sigma(\epsilon'))$ and $b_p(z, \sigma(\epsilon'))$ are sufficiently close to the previous ones. We skip the details of this procedure, as this has been outlined elsewhere – see Khan and Thomas (2008) and Bachmann et al. (2008).

4 Calibration

4.1 Baseline

The model period is a year – in congruence with the data frequency in USTAN. The following parameters have standard values: $\beta = 0.98$ and $\delta = 0.094$, which we compute from German national accounting data for the sectoral aggregate that the USTAN sample corresponds to: the non-financial private business sector. Given this depreciation rate, we pick $\gamma = 1.014$, in order to match the time-average aggregate investment rate of 0.108. This number is also consistent with German long-run growth rates. The log-felicity function features an elasticity of intertemporal substitution (EIS) of one. The disutility of work parameter, A, is chosen to generate an average time spent at work of 0.33: $A = 2$ for the baseline calibration.

We set the output elasticities of labor and capital to $\nu = 0.5565$ and $\theta = 0.2075$, respectively, which correspond to the measured median labor and capital shares in manufacturing in the USTAN data base (see Appendix A.4). While our data also include a considerable amount of firms from other sectors, any weighted average or median of these shares would still be close to the manufacturing values, which is why we decided to use them in our baseline calibration.[26]

Next, we have to choose the parameters of the two-state aggregate shock process. Here we simply estimate a bivariate, unrestricted VAR with the linearly detrended natural logarithm of

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[26]If one views the DRTS assumption as a mere stand-in for a CRTS production function with monopolistic competition, than these choices would correspond to an employment elasticity of the underlying production function of 0.7284 and a markup of $\frac{1}{\theta+\nu} = 1.31$. Given the regulated product markets in Germany, this is a reasonable value. The implied capital elasticity of the revenue function, $\frac{\theta}{1-\nu}$ is 0.47. We have experimented with both elasticities quite a bit, but have not found any of our main results depending on them. Details are available on request.

the aggregate Solow residual[27] and the linearly detrended $\sigma(\epsilon)$-process from the USTAN data.[28] The parameters of this VAR are as follows:[29]

$$\varrho_A = \begin{pmatrix} 0.4474 & -3.7808 \\ 0.0574 & 0.7794 \end{pmatrix} \quad \Omega = \begin{pmatrix} 0.0146 & 0.1617 \\ 0.1617 & 0.0023 \end{pmatrix} \tag{10}$$

When measurement error in firm-level Solow residuals is taken into account, as specified in Section 2.3, the driving force looks as follows:

$$\varrho_A = \begin{pmatrix} 0.4510 & -2.8283 \\ 0.0765 & 0.7034 \end{pmatrix} \quad \Omega = \begin{pmatrix} 0.0147 & 0.1688 \\ 0.1688 & 0.0031 \end{pmatrix} \tag{11}$$

This process is discretized on a $[5 \times 5]$−grid, using a bivariate analog of Tauchen's procedure. We measure the steady state standard deviation of idiosyncratic technology innovations as $\bar{\sigma}(\epsilon) = 0.1201$. Since these innovations also exhibit mild excess kurtosis – 4.4480 on average over our time horizon –,[30] and since the adjustment cost parameter $\bar{\xi}$ will be identified by the kurtosis of the firm-level investment rate (next to its skewness), we want to avoid attributing excess kurtosis in the firm-level investment rate to nonlinearities in the adjustment technology, when the driving force itself has kurtosis. Hence, we incorporate the measured excess kurtosis into the discretization process for the idiosyncratic technology state.[31] Finally, we set $\rho_I = 0.95$, in accordance with the high persistence of Solow residual innovations in the data. This process is discretized on a 19−state-grid, using Tauchen's procedure with mixed Gaussian normals.

Given the aforementioned set of parameters $\left(\beta, \delta, \gamma, A, \nu, \theta, \varrho_A, \Omega, \bar{\sigma}(\epsilon), \rho_I\right)$, we then calibrate the adjustment costs parameter $\bar{\xi}$ to minimize a quadratic form in the normalized differences between the time-average firm-level investment rate skewness produced by the model and the data, as well as the time-average firm-level investment rate kurtosis:[32]

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[27]We use again $\nu = 0.5565$ and $\theta = 0.2075$ in these calculations.

[28]After firm-level and sectoral fixed effects have been removed.

[29]With a slight abuse of notation, but for the sake of readability, Ω displays standard deviations on the main diagonal and correlations on the off diagonal. We also estimate a seemingly unrelated regressions (SUR) specification of this bivariate VAR and use it for simulation. This specification mitigates somewhat the obvious problems with estimating a bivariate VAR with 7 independent parameters from 2×26 data points. We show that both the actual estimates of the remaining VAR parameters as well as the implications for our simulation results are remarkably robust. Results can be seen in Table 40 in Appendix B.2.

[30]We find no skewness.

[31]We achieve this by using a mixture of two Gaussian distributions: $N(0, 0.0777)$ and $N(0, 0.1625)$ – the standard deviations are 0.1201 ± 0.0424 – with a weight of 0.4118 on the first distribution.

[32]The normalization constants in (12) are, respectively, the time series standard deviation of the investment rate skewness and the time series standard deviation of the investment rate kurtosis.

$$\min_{\bar{\xi}} \Psi(\bar{\xi}) \equiv 0.5 \cdot \left[\left(\left(\frac{1}{T} \sum_t skewness(\frac{i_{i,t}}{0.5 * (k_{i,t} + k_{i,t+1})})(\bar{\xi}) - 2.1920 \right) / 0.6956 \right)^2 + \right.$$
$$\left. \left(\left(\frac{1}{T} \sum_t kurtosis(\frac{i_{i,t}}{0.5 * (k_{i,t} + k_{i,t+1})})(\bar{\xi}) - 20.0355 \Big/ 5.5064 \right)^2 \right]. \quad (12)$$

As can be seen from (12), the distribution of firm-level investment rates exhibits both substantial positive skewness – 2.1920 – as well as kurtosis – 20.0355. Caballero et al. (1995) document a similar fact for U.S. manufacturing plants. They also argue that non-convex capital adjustment costs are an important ingredient to explain such a strongly non-Gaussian distribution, given a close-to-Gaussian shock process. We therefore use the deviation from Gaussianity in firm-level investment rates to identify $\bar{\xi}$.

The following Table 15 demonstrates identification of $\bar{\xi}$, as cross-sectional skewness and kurtosis of the firm-level investment rates are both monotonically increasing in $\bar{\xi}$. The minimum of the distance measure Ψ is achieved for $\bar{\xi} = 0.3$, our baseline case.[33] This implies costs conditional on adjustment equivalent to 15.4% of annual firm-level output on average, which is well in line with estimates from the U.S. (see Bloom, 2009). With measurement error in $\epsilon_{i,t}$, $\bar{\xi} = 0.25$.

Table 15: CALIBRATION OF ADJUSTMENT COSTS - $\bar{\xi}$

$\bar{\xi}$	Skewness	Kurtosis	$\Psi(\bar{\xi})$	Adj. costs/ Unit of Output
0.01	0.7852	5.0389	11.5082	1.5%
0.05	1.5168	7.6444	6.0062	4.2%
0.1	1.9340	9.3327	3.9157	6.8%
0.2	2.4011	11.4056	2.547	11.3%
0.3 (BL)	2.6915	12.8042	2.2402	15.4%
0.5	3.0686	14.7669	2.5035	23.3%
1	3.5926	17.8112	4.2169	43.3%

4.2 Variants of the Driving Processes

The empirical analysis has shown a negative comovement between the dispersion of firm-level Solow residual innovations and aggregate Solow residuals. The dynamics of this comovement – as in (2) and (10) – imply that one variable conveys information about the future development of

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[33]We searched over a finer grid of $\bar{\xi}$ than displayed in the table, in order to find the optimal $\bar{\xi}$.

the other. In other words, a shock to idiosyncratic uncertainty affects aggregate activity not only through the option value channels highlighted by Bloom (2009) and Bloom et al. (2009), but also as a news shock in the spirit of Beaudry and Portier (2006), Jaimovich and Rebelo (2008), Sims (2008) as well as Schmidt-Grohe and Uribe (2008). Upon observing an increase in idiosyncratic uncertainty today, households and firms rationally expect a future decrease in productivity ceteris paribus.

To disentangle the effects from the news about productivity contained in movements in idiosyncratic uncertainty, we solve a variant of our model where the actual risk that firms face remains constant over time at $\bar{\sigma}(\epsilon) = 0.1201$, but firms still observe a time-varying state, s, as a variable without further economic content that nonetheless contains information about future productivity as in (2). The parameters that describe the law of motion of s are the same as in (2) and (10), i.e. as in the model with time-varying uncertainty. More generally, we can reparameterize the model for the standard deviation of idiosyncratic productivity shocks, $\sigma(\epsilon)$, as follows:

$$\sigma(\epsilon) = \bar{\sigma}(\epsilon) + \vartheta\big(s - \bar{\sigma}(\epsilon)\big).$$

For $\vartheta = 0$, idiosyncratic uncertainty remains constant and in the following we use the term 'News Model' to describe this parametrization. The second state no longer influences actual risk, but only the conditional expectation of future aggregate Solow residuals, z'. The baseline specification in which idiosyncratic uncertainty fluctuates as well is termed 'Full Model', $\vartheta = 1$. Using an $\vartheta > 1$ we can scale up the fluctuations in idiosyncratic uncertainty without changing the underlying dynamics of the exogenous state variables, i.e. keeping the news content of s constant. Moreover, for comparison we consider a model with a standard univariate AR(1) process for the aggregate Solow residual, 'RBC Model', and a specification, where the only driving force of business cycles are shocks to idiosyncratic uncertainty, 'Risk Model'.[34]

5 Results

5.1 Baseline Results

Our main set of quantitative results can be summarized as follows: 1) Fluctuations in idiosyncratic uncertainty alone do not produce business cycle dynamics as observed in the data. 2) Adding orthogonal shocks to idiosyncratic uncertainty into a model with first-moment shocks to aggregate Solow residuals hardly changes the business cycle dynamics of the model. 3) Adding shocks to idiosyncratic uncertainty that comove with aggregate Solow residuals in a

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[34] The estimated autocorrelation of the aggregate Solow residual is 0.5259 and the standard deviation of its innovations 0.0182. The corresponding moments for the univariate risk process are: 0.5685 and 0.0028.

way observed in the data produces mild changes in the business cycle dynamics of the model, which we attribute to the bad news effect that uncertainty shocks have for aggregate productivity in the future. This bad news effect is embodied in the negative VAR coefficient (see 10) of tomorrow's (but known today) firm-level uncertainty on tomorrow's aggregate Solow residual.

Risk Model

Table 16: AGGREGATE BUSINESS CYCLE STATISTICS FOR THE PURE 'RISK MODEL'

Moment/Aggregate Quantity	Y	C	I	N
Baseline				
Volatility	0.30% (2.30%)	0.23% (1.79%)	2.01% (4.37%)	0.41% (1.80%)
Volatility relative to Y	1	0.77 (0.78)	6.76 (1.90)	1.38 (0.78)
Persistence	0.55 (0.48)	0.51 (0.67)	0.29 (0.42)	0.26 (0.61)
Correlation with Y	1	0.02 (0.66)	0.84 (0.83)	0.79 (0.68)
Correlation with C	0.02 (0.66)	1	-0.52 (0.60)	-0.60 (0.36)
Measurement error in $\epsilon_{i,t}$				
Volatility	0.35% (2.30%)	0.27% (1.79%)	2.51% (4.37%)	0.51% (1.80%)
Volatility relative to Y	1	0.77 (0.78)	7.23 (1.90)	1.47 (0.78)
Persistence	0.49 (0.48)	0.44 (0.67)	0.24 (0.42)	0.22 (0.61)
Correlation with Y	1	-0.124 (0.66)	0.86 (0.83)	0.82 (0.68)
Correlation with C	-0.124 (0.66)	1	-0.61 (0.60)	-0.67 (0.36)

Notes:

Business cycle statistics of aggregate output, Y, consumption C, investment I and employment N. N in the model includes the amount of labor used to adjust the firms' capital stocks. All variables are logged and then HP-filtered with a smoothing parameter of 100. The first numbers in a column refer to a simulation of the model over $T = 1500$ periods. Numbers in brackets refer to German aggregate NFPBS data (see Appendix A.1, Footnote 49, for a detailed description of sources). Volatility is percentage standard deviation. Persistence refers to the first order autocorrelation.

Table 16 shows that the pure 'Risk Model', in which the only driving force of business cycles is fluctuations in uncertainty, yields little fluctuations. Output fluctuations are almost eight times smaller than in the data, investment fluctuations just below 50% of the volatility in the data. The uncertainty fluctuations in the data are simply too small – with a time series coefficient of variation of 2.67% – to generate realistic business fluctuations. This finding is only mildly changed, when measurement error in firm-level Solow residuals is taken into account as described in Section 2.3. Even though the coefficient of variation of uncertainty fluctuations now increases by roughly three quarters to 4.72%, aggregate fluctuations become only somewhat stronger. We also note that general equilibrium price movements are partly responsible for this low volatility, as the volatility of output, investment and employment increases, respec-

tively, to 0.63%, 4.28% and 0.63%, i.e. more than doubles in a simulation, where we fix real wages and real interest rates to their average value from the general equilibrium simulation.[35] This shows that general equilibrium is important for understanding the aggregate implications of idiosyncratic uncertainty innovations and that previous work in partial equilibrium might have neglected an important channel. Finally, in terms of contemporaneous correlations with output, pure uncertainty shocks lead to a counterfactual de-coupling of consumption and output, which is accompanied by negative correlations between consumption on the one hand and investment and employment on the other. We conclude that uncertainty shocks alone cannot produce business cycles in an flexible price and market clearing model, augmented with persistent idiosyncratic productivity shocks and fixed capital adjustment costs.

We also experimented with mechanically and counterfactually doubling and quadrupling the uncertainty fluctuations, ($\vartheta = 2$ and $\vartheta = 4$), and found that this leads to an almost linear increase in all volatilities of the pure 'Risk Model', whereas persistence and correlations with output and consumption are practically unchanged.[36] This is a perhaps surprising finding: just as with first moment shocks, increasing the volatility of idiosyncratic shocks essentially amounts to re-scaling the model generated fluctuations. Consequently, we would need to observe uncertainty fluctuations that are roughly eight times as large as the fluctuations we observe in the data, in order for fluctuations in idiosyncratic uncertainty alone to produce realistic output volatility. This number would then be close to the volatility of uncertainty that based on stock market data has been suggested by Bloom (2009) for the U.S.: roughly 17%. However, it is unlikely that we underestimate the risk fluctuations by such an order of magnitude. In fact, as we have shown, the volatility of firm-level uncertainty only increases by a factor of 1.75 when measurement error is accounted for. Moreover, none of our different sample splits or robustness checks find an increase in the volatility of uncertainty by more than a factor of two relative to the baseline specification. To the contrary, as Table 7 in Section 2.3 shows, the service sector – still underrepresented in the USTAN data – exhibits less volatile uncertainty fluctuations than the goods-producing industries. Thus we should expect to have over- rather than underestimated the volatility of uncertainty fluctuations for the overall economy, especially if we extrapolate to other countries like the U.S., where it is well-known that the tertiary sectors comprise a much larger share of aggregate output and employment in the U.S., compared to Germany.

Risk Model and RBC Model

Although we do not find realistic business cycles caused by uncertainty fluctuations alone, they might well alter more standard first-moment-driven cycles. Therefore, as a next step, we briefly compare the 'Risk Model' with the simple 'RBC Model',[37] and include a specification,

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[35]These numbers are not reported in a table. Details for the partial equilibrium case are available on request.

[36]This holds for both specifications, i.e. with and without measurement error. Details are available on request.

[37]We keep the calibrated adjustment costs fixed across the variants for the driving process. This is justified, since

Table 17: AGGREGATE BUSINESS CYCLE STATISTICS FOR THE PURE 'RISK MODEL' AND THE 'RBC MODEL'

	RBC-Risk Combination	RBC Model	Risk Model	Data
Volatility of Output	3.18%	3.17%	0.30%	2.30%
Volatility of aggregate variables relative to output volatility				
Consumption	0.45	0.38	0.77	0.78
Investment	4.27	4.36	6.76	1.90
Employment	0.69	0.70	1.38	0.78
Persistence				
Output	0.32	0.31	0.55	0.48
Consumption	0.64	0.54	0.51	0.67
Investment	0.23	0.26	0.29	0.42
Employment	0.22	0.25	0.26	0.61
Contemporaneous Correlation with Aggregate Output				
Consumption	0.81	0.86	0.02	0.66
Investment	0.95	0.97	0.84	0.83
Employment	0.92	0.96	0.79	0.68
Contemporaneous Correlation with Aggregate Comsumption				
Investment	0.60	0.73	-0.52	0.60
Employment	0.53	0.69	-0.60	0.36

Notes: See notes to Table 16. 'RBC-Risk Combination' refers to a simulation, where we simply combine the two univariate processes for aggregate Solow residuals and idiosyncratic uncertainty, as specified in Footnote 34.

where we mechanically combine the two univariate processes for aggregate Solow residuals and idiosyncratic uncertainty, as specified in Footnote 34. This variant is referred to as 'RBC-Risk Combination'. It gives a first indication, whether second-moment shocks as found in firm-level data can alter the aggregate dynamics of a standard 'RBC Model'. Table 17 shows that this is – with one small exception – not the case: the 'RBC Model' displays second-moment statistics very similar to the combined one: 'RBC-Risk Combination'. Not even the notoriously too high contemporaneous output correlations are significantly mitigated by this second, independent shock. Put differently, the characteristic features of the 'Risk Model' vanish as soon as uncer-

they were calibrated to the steady state investment rate distribution, which is at least numerically invariant to the specification of the driving process. That means, by the 'RBC Model' we really mean a model with fixed capital adjustment costs and shocks to aggregate Solow residuals only. De facto, however, the calibrated adjustment cost level is so low that our baseline model is in the neutrality range explored by Khan and Thomas (2008).

tainty shocks are combined with standard aggregate Solow residual innovations. Only the contemporaneous correlations between aggregate consumption on the one hand and aggregate investment and employment on the other are pushed downward, closer to the data.

Perhaps surprisingly, the overall volatility of the 'RBC Model' is too high, relative to the data. In Table 20 in Section 5.2 we show that fixing the excess volatility of the model by re-scaling the volatility of the first-moment shock – to give the second moment shock maximum chance – so as to match the observed volatility of aggregate output does not change our main results.

Otherwise, the 'RBC Model' displays the well-known and typical failures: low persistence, too high contemporaneous correlations with output and the wrong mix between relative consumption and investment volatility, the former being too low, the latter too high. What matters here that neither the 'Risk Model' alone nor, as we shall see, in combination with first-moment shocks fixes these failures in a satisfying manner. Of course, there are well-known additions to the 'RBC Model', such as quadratic adjustment costs for capital on top of the fixed ones, or simply higher fixed adjustment costs, that would help to ameliorate its performance, but we aim here for cleanliness of exposition and view these issues as mainly orthogonal to the question, whether shocks to idiosyncratic uncertainty matter for aggregate dynamics.

Full Model and News Model

Integrating our empirical findings a bit further into the model simulations, we extend the driving process (2) such that it incorporates the correlation structure between firm-level uncertainty and aggregate Solow residuals found in the data. We proceed in three steps. First, we introduce in the 'RBC Model' the aggregate driving process estimated in the data, (10), but set $\vartheta = 0$, i.e. s has no actual implications for firm-level uncertainty. Also, we allow firms' policy functions to depend only on aggregate productivity z. We call this specification 'Naive Model', because firms could actually use s to get better forecasts about aggregate productivity. Second, we lift the latter assumption and allow firms to condition their optimal policy on s as well as z. The state s still has no implications for firm-level uncertainty, it merely helps to forecast aggregate productivity tomorrow. This variant – the aforementioned 'News Model' – adds the news character of uncertainty fluctuations to the model. Third, we explore the effect of uncertainty fluctuations, setting $\vartheta = 1$. This specification is called 'Full Model'.

Comparing 'Full Model' and 'RBC Model', we find that the standard business cycle statistics, volatilities, autocorrelations and correlations, change only modestly, see Table 18. As for autocorrelations, most of the change is mechanically due to the different driving process, as the persistence improvements happen, even when agents do not use the news content of uncertainty shocks ('Naive Model'). The same holds partially true for the increase in the volatility of aggregate output.[38] In other words, the excess volatility in output, featured in the 'RBC Model',

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[38] Of course, in general equilibrium this change is not purely mechanical, because the change in aggregate dynamics alters the Krusell-Smith rules, and therefore there is an indirect effect of news on agents' decisions.

Table 18: AGGREGATE BUSINESS CYCLE STATISTICS FOR THE 'FULL MODEL' AND 'NEWS MODEL'

|	Full Model	News Model	Naive Model	RBC Model	Data	ME-Full Model	ME-News Model
Volatility of Output	4.04%	4.24%	3.72%	3.17%	2.30%	3.95%	4.19%
Volatility of aggregate variables relative to output volatility							
Consumption	0.32	0.30	0.40	0.38	0.78	0.35	0.32
Investment	4.71	4.77	4.32	4.36	1.90	4.50	4.62
Employment	0.74	0.76	0.69	0.70	0.78	0.72	0.75
Persistence							
Output	0.42	0.46	0.47	0.31	0.48	0.43	0.47
Consumption	0.66	0.65	0.63	0.54	0.67	0.67	0.66
Investment	0.34	0.41	0.42	0.26	0.42	0.35	0.42
Employment	0.33	0.40	0.42	0.25	0.61	0.33	0.42
Contemporaneous Correlation with Aggregate Output							
Consumption	0.84	0.85	0.86	0.86	0.66	0.87	0.87
Investment	0.98	0.98	0.97	0.97	0.83	0.98	0.98
Employment	0.97	0.98	0.95	0.96	0.68	0.97	0.98
Contemporaneous Correlation with Aggregate Consumption							
Investment	0.73	0.75	0.71	0.73	0.60	0.76	0.78
Employment	0.69	0.72	0.66	0.69	0.36	0.72	0.75

Notes: See notes to Table 16. The 'Naive Model' is a simulation, where the realized aggregate shock process is the same as in the 'News Model', but we allow agents only to condition their policy functions on aggregate Solow residuals. ME designates the measurement error specification from equation (11).

is actually exacerbated when introducing the further state *s*. However, not all differences are mechanic to the driving process. We can see that agents using the information content of the second underlying state leads to altered relative volatilities, which is marked by the change from the 'Naive Model' to the 'News Model'. Yet only in the case of the relative volatility of employment, the 'Full Model' actually brings improvement over the 'RBC Model'. However, the overall change from the 'RBC Model' to the 'Full Model' is *not* due to the specific nature of the second state being time-varying uncertainty and thus generating time-varying option values, but rather due to its news character for aggregate productivity: changes in uncertainty alter the conditional expectations of aggregate productivity. Hence, an increase in risk acts as a (bad) news shock. This bad news is embodied in the negative VAR coefficient of tomorrow's (but known to-

day) firm-level uncertainty on tomorrow's aggregate Solow residual, see (10).[39] Finally, the last panel of Table 18 shows almost no changes in the contemporaneous correlations with output and consumption. In particular, there is no decrease in the notoriously too high 'RBC Model' correlation coefficients, something that one might have hoped for with the introduction of a second aggregate shock. In summary, there is little difference between the 'Full Model' and the 'News Model'. This finding is robust to measurement error on the micro side, as the last two columns of Table 18 show. Figure 2 illustrates this further by simply plotting a time series trajectory of aggregate output simulations from both models for an identical bivariate shock series. Both time series look very similar.

Figure 2: Comparison of the 'Full Model' and the 'News Model'
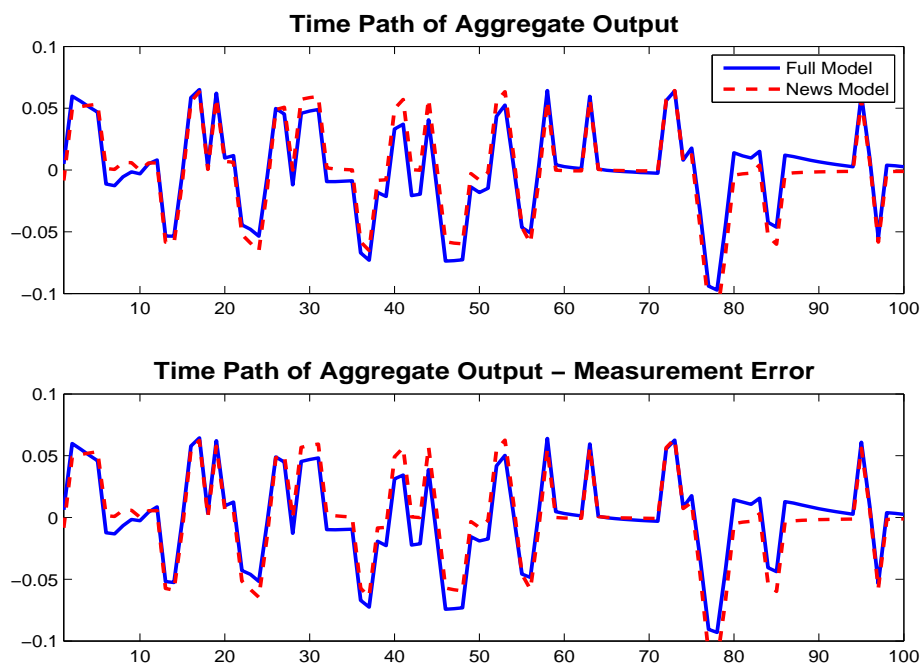
*Notes:* Raw aggregate output from simulations of the 'Full Model' and the 'News Model'.

Uncertainty Shocks as News Shocks

To further understand the mechanisms behind the effect of shocks to idiosyncratic uncertainty, we run a series of trivariate SVARs with one lag, where we augment the aggregate Solow residual (ordered first) and the idiosyncratic uncertainty (ordered second) by a third variable: aggregate output, the aggregate investment rate, aggregate consumption, aggregate saving, ag-

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[39]We also estimate a SUR specification of this bivariate VAR and use it for simulation. We show that most of the changes between the 'RBC Model' and the 'Full/News Model' are mainly due to the interaction effect between aggregate Solow residuals and firm-level uncertainty. Results can be seen in Table 40 in Appendix B.2

gregate employment and hours as well as the real wage.[40] We estimate these SVARs both from the actual data for 1973 - 1998 and for 60 independent $T = 26$-samples from simulations of both 'Full Model' and 'News Model'. With a series of impulse response functions, we show first that 'Full Model' and 'News Model' are practically indistinguishable in their conditional responses to shocks to idiosyncratic uncertainty.[41] Secondly, we also show that – for the most part – model and data impulse responses look very much alike, i.e. our model replicates the empirical aggregate dynamics resulting from a shock to uncertainty. Finally, these impulse responses are exactly like the impulse responses to a bad news shock in a standard RBC model with King-Rebelo-Plosser preferences (see Beaudry and Portier, 2006, Jaimovich and Rebelo, 2008 and Sims, 2008): output and investment increase, consumption decreases. The intertemporal wealth effect increases labor supply and this depresses the real wage. As the figures below show, all these findings are robust to measurement error in the firm-level Solow residual.

Figure 3: Impulse Response of Aggregate Output to a Shock in Idiosyncratic Uncertainty
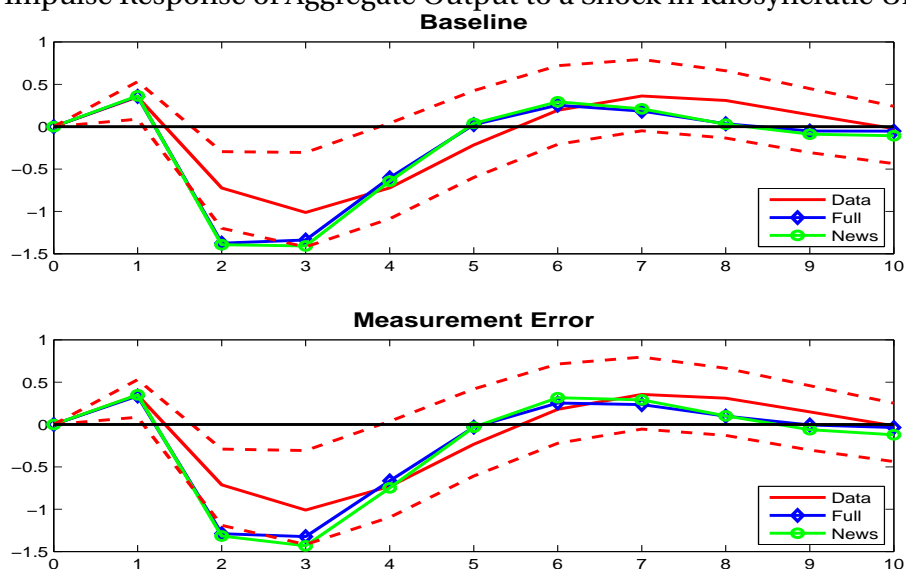

Notes: Impulse response functions from SVAR estimations of the linearly detrended aggregate Solow residual (ordered first), the linearly detrended idiosyncratic uncertainty (ordered second) and HP(100)-filtered aggregate output (ordered third). The dotted lines reflect 95% confidence bounds for the estimates from the data from 10,000 bootstrap replications. Estimates from data are in red, estimates from simulated data in blue ('Full Model') and green ('News Model'), respectively. Estimates from simulated data are the average of 60 impulse response functions estimated on 60 independent time series of $T = 26$, the length of our sample.

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[40]We use a simple Cholesky decomposition, assuming the aggregate Solow residual has an instantaneous effect on idiosyncratic uncertainty but not vice versa. This is to identify a pure innovation in contemporaneous uncertainty. Notice that studying SVARs in this context is really studying and comparing an interesting summary statistic between data and models. There is nor claim on causal empirical identification.

[41]The impulse responses with respect to an innovation to the aggregate Solow residual are available on request. Except for the general excess volatility of our baseline calibration, data and model impulse responses are very similar and theoretically reasonable. We take this as a good sign that our impulse responses with respect to a second-moment shock are estimated reasonably well despite the short sample problems.

Figure 3 displays the impulse response of aggregate output to a shock to idiosyncratic uncertainty. On impact, output goes up after an increase in uncertainty (larger dispersion of firm-level productivity innovations in the following period), both in the model and in the data. So, rather than causing an immediate recession, as has been argued in the literature on uncertainty shocks thus far, a shock in idiosyncratic uncertainty causes a boom in output on impact. Then the bad news is realized and the economy goes into a recession.

Figure 4: Impulse Response of the Aggregate Investment Rate and Aggregate Consumption to a Shock in Idiosyncratic Uncertainty
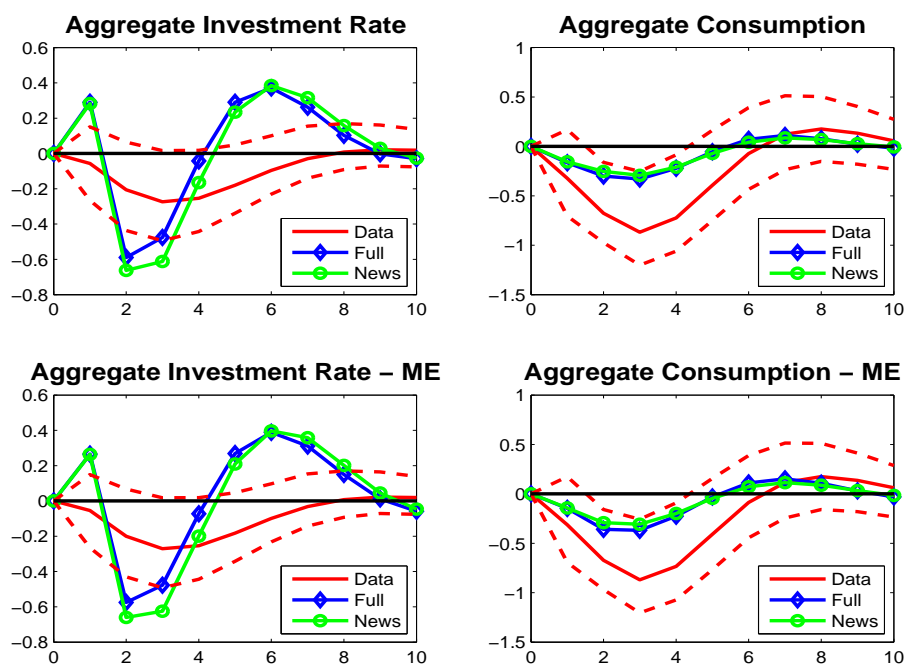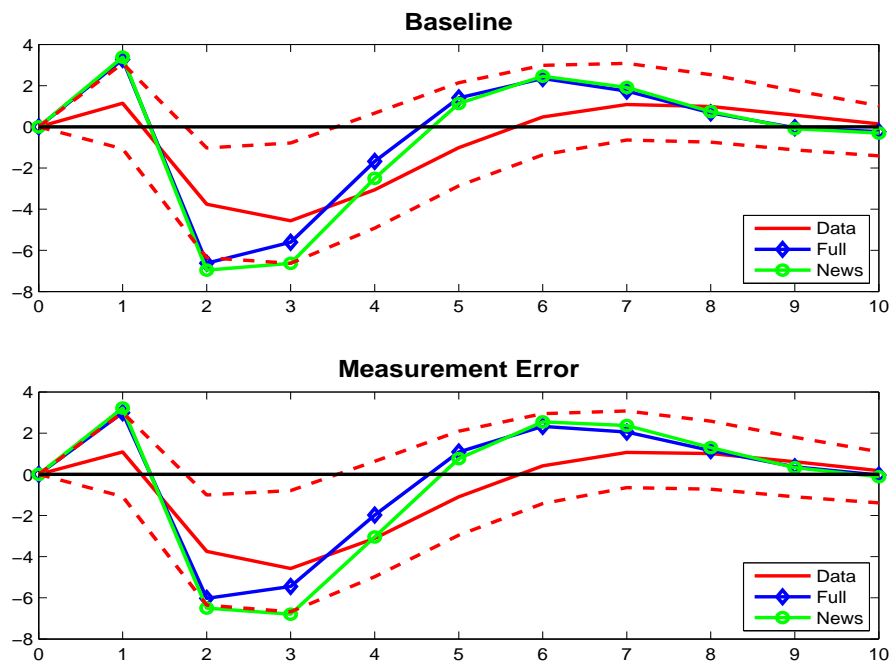

Notes: See notes to Figure 3. In this figure, the linearly detrended aggregate investment rate and HP(100)-filtered consumption, respectively, are ordered third. The aggregate investment rate is the sum of NFPBS investment divided by the average of the beginning-of-period and end-of-period aggregate capital stocks in NFPBS.

The left-hand panel of Figure 4 shows a similar expansion in the model aggregate investment rate as a response to an uncertainty shock, which is at least qualitatively consistent with the data, given the wide confidence bands around the downward sloping point estimate.[42] Of course, the impulse response of aggregate investment features the excess volatility we have discussed above. In contrast, the right-hand panel of Figure 4 illustrates that the opposite is true for the undershooting impulse response of aggregate consumption, which declines after a shock

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[42]An SVAR with aggregate investment instead of the aggregate investment rate shows a positive point estimate with similarly wide confidence bands.

to idiosyncratic uncertainty, because agents anticipate through it lower aggregate productivity and therefore wealth in the future. One can (re-)interpret the quote by Blanchard from the introduction in this context: higher uncertainty leads to a fall in consumption through a wealth effect, which in turn shifts the supply of funds curve outward, lowering the real interest rate and thus causing an investment increase on impact (of course in the current situation the economy has arguably been hit also by contemporaneous negative first moment shocks, which is why we would not necessarily observe this investment increase). Notice, however, that this is a very different mechanism from the wait-and-see interpretation of uncertainty increases that has been proposed in the literature and that operates through time-varying real option values.

Figure 5: Impulse Response of National Saving to a Shock in Idiosyncratic Uncertainty

Notes: See notes to Figure 3. In this figure, HP(100)-filtered saving is ordered third.

Figure 5 shows this increase of saving in the model as well as the data response. Of course, Germany is an open economy, and, thus, the impulse response of saving is really the correct data counterpart of the impulse response of saving/investment in the model. An open economy means that aggregate investment can be financed from abroad and that the transmission of the wealth effect onto investment is mitigated. The open economy response of aggregate investment is therefore closer to the negative partial equilibrium response that we show below in Figure 8.

Figure 6: Impulse Response of the Aggregate Employment and Aggregate Hours to a Shock in Idiosyncratic Uncertainty

Notes: See notes to Figure 3. In this figure, HP(100)-filtered aggregate employment and HP(100)-filtered aggregate total hours, respectively, are ordered third. Employment in the model includes the amount of labor used to adjust the firms' capital stocks.
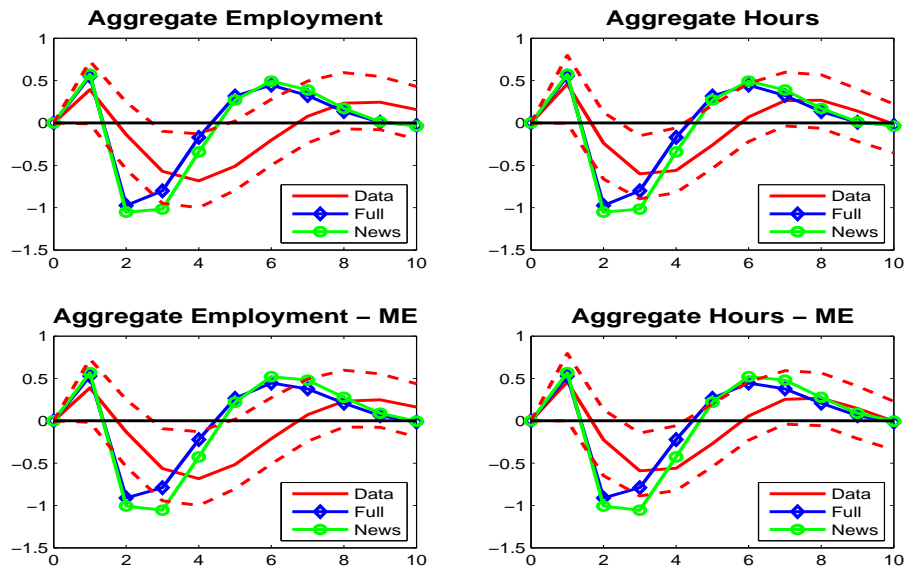
Figure 7: Impulse Response of the Real Wage to a Shock in Idiosyncratic Uncertainty

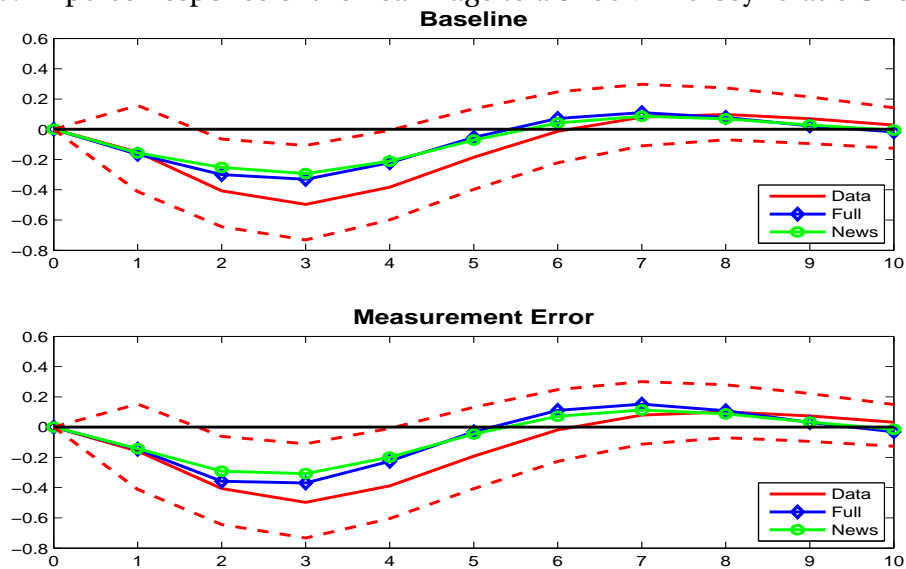*Notes:* See notes to Figure 3. In this figure, the HP(100)-filtered real wage is ordered third. The real wage is aggregate real payroll divided by the number of workers.

Consistent with the wealth effect both the data and the model impulse responses display an increase in aggregate employment, whether it is measured in the data by number of workers or aggregate hours (since the model features indivisible labor, this distinction is meaningless for

the model). This increase in aggregate labor inputs – and the output boom that is entailed – is made possible by a decline in the real wage. The labor market response, which is typical for a bad news shocks under standard preferences, is shown in Figures 6 and 7, respectively.

The final piece of the chain of arguments is displayed in Figure 8, which demonstrates that the expansion in the aggregate investment rate as a response to a shock to idiosyncratic uncertainty is a general equilibrium effect. The increase in investment is absent in partial equilibrium, where we hold constant both wages and interest rates at their average level from the general equilibrium simulation. With no increase in aggregate saving and thus a decrease in the real interest rate, the expectation of lower future productivity drives down the expected return on capital and hence investment.

Figure 8: Impulse Response of the Aggregate Investment Rate to a Shock in Idiosyncratic Uncertainty - GE vs. PE
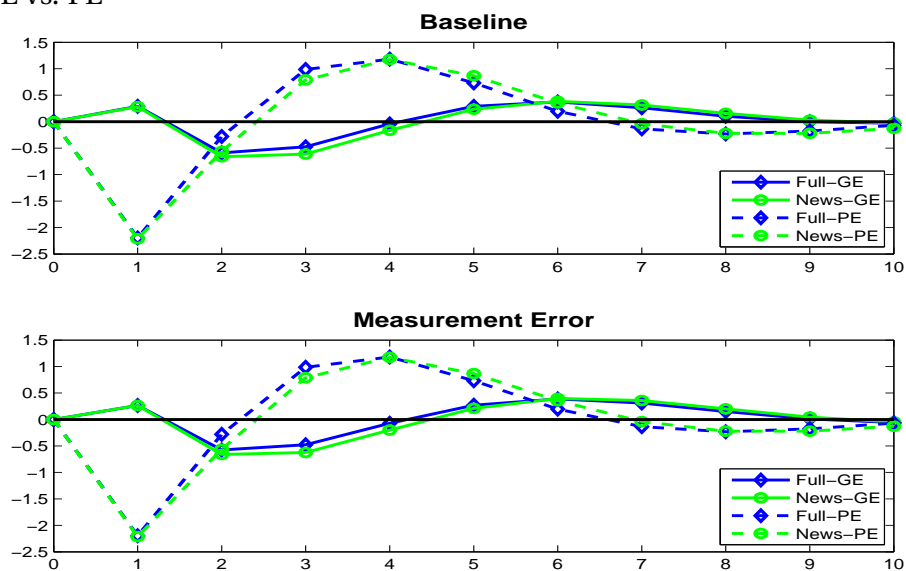
Notes: See notes to Figures 3 and 4. 'GE' stands for general equilibrium and means a model simulation with market clearing wages and interest rates. 'PE' stands for partial equilibrium and means a model simulation, where wages and interest rates are held constant at the average level in the 'GE'-simulation.

To summarize: quantitatively realistic uncertainty shocks have bad news effects in general equilibrium, and it is this news effect that dominates the time-varying real option value effect. This shows again that general equilibrium is paramount to understanding the aggregate implications of shocks to idiosyncratic uncertainty.

5.2 Robustness Checks

Table 19: AGGREGATE BUSINESS CYCLE STATISTICS FOR THE 'FULL MODEL' AND 'NEWS MODEL' - DOUBLE VOLATILITY OF UNCERTAINTY ($\vartheta = 2$)

	Full Model	ME-Full Model	News Model	ME-News Model	Data
Volatility of Output	3.84%	3.72%	4.24%	4.19%	2.30%
Volatility of aggregate variables relative to output volatility					
Consumption	0.36	0.41	0.30	0.32	0.78
Investment	4.67	4.40	4.77	4.62	1.90
Employment	0.74	0.71	0.76	0.75	0.78
Persistence					
Output	0.36	0.37	0.46	0.47	0.48
Consumption	0.68	0.69	0.65	0.66	0.67
Investment	0.25	0.23	0.41	0.42	0.42
Employment	0.23	0.21	0.40	0.42	0.61
Contemporaneous Correlation with Aggregate Output					
Consumption	0.78	0.81	0.85	0.87	0.66
Investment	0.97	0.96	0.98	0.98	0.83
Employment	0.95	0.93	0.98	0.98	0.68
Contemporaneous Correlation with Aggregate Consumption					
Investment	0.61	0.62	0.75	0.78	0.60
Employment	0.55	0.55	0.72	0.75	0.36

Notes: See notes to Table 16.

In this subsection we provide robustness checks for our simulation results along two dimensions. First, we double the volatility of firm-level uncertainty, while keeping its steady state value fixed at 0.1201. This amounts to setting $\vartheta = 2$. Bloom (2009) uses a roughly seven times more volatile firm-level uncertainty measure than we. While our empirical results are not consistent with $\vartheta = 7$, they are consistent with a doubling of the volatility of uncertainty. We explore this here. Second, we re-scale the volatility of the aggregate Solow residual so that the cyclical component of aggregate output in the model matches roughly the volatility of aggregate output in the data, in order to make sure that our results are not driven by the general excess volatility generated by the baseline specification. In particular, it could be that we falsely place too much weight on first-moment shocks by overestimating their volatility. None of these changes

overturn our results, whether we control for measurement error or not. We display a further robustness check in Table 39 in Appendix B.2, where we more than triple the adjustment costs parameter, $\bar{\xi}$, and set it to unity. This is to check whether higher nonconvex adjustment costs could give the wait-and-see effect are larger role. We find that this is not the case.[43]

If we increase the fluctuations in uncertainty, setting $\vartheta = 2$, the difference between 'Full Model' and the 'News Model' becomes somewhat larger, see Table 19. Interestingly, the overall volatility of the model decreases, the one of output from 4.04% to 3.84%, while the relative standard deviations are fairly robust to ϑ. This is a result of the mechanism highlighted in Bloom et al. (2007): when uncertainty increases firms react less to first moment shocks. We show here that this effect survives general equilibrium price movements. The performance of the 'Full Model' in terms of persistence is actually slightly worsened, while the contemporaneous correlation of consumption and output, investment as well as employment drops to levels closer to the data.[44]

The next robustness check concerns re-scaling the volatility of the aggregate Solow residual by a factor of 0.6 in the 'Full Model', in order to approximately match the observed volatility of cyclical aggregate output in the data. We keep this re-scaling factor the same for the 'News Model' and the 'RBC Model'. Despite the reduced importance of the first-moment shock both the absolute and the relative volatilities of the 'Full Model' and the 'News Model' remain very similar. As before in the case of an increased volatility of the firm-level uncertainty – which is essentially a weakening of the first-moment shock from the other side –, we see a deterioration (with the exception of consumption) in the persistence numbers, but an improvement in the contemporaneous correlation of consumption and output in the 'Full Model' compared to the data.

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[43]We performed other robustness checks with respect to choices for the risk aversion parameter, ρ_I, v and θ. Details are available on request.

[44]We also experimented with $\vartheta = 4$. The changes we find for $\vartheta = 2$ are enforced. However, persistence of aggregate investment and employment as well as the contemporaneous correlations between consumption on the one hand and investment and employment on the other drop to counterfactually low levels. Moreover, Bachmann and Bayer (2009) show that $\vartheta = 4$ is clearly incompatible with the procyclicality of investment dispersion found in the micro data.

Table 20: AGGREGATE BUSINESS CYCLE STATISTICS - RE-SCALED VOLATILITY OF THE AGGREGATE SOLOW RESIDUAL

	Full Model	ME-Full Model	News Model	ME-News Model	RBC Model	Data
Volatility of Output	2.34%	2.24%	2.54%	2.49%	1.90%	2.30%
Volatility of aggregate variables relative to output volatility						
Consumption	0.35	0.40	0.30	0.32	0.38	0.78
Investment	4.58	4.35	4.65	4.52	4.30	1.90
Employment	0.74	0.71	0.76	0.75	0.69	0.78
Persistence						
Output	0.38	0.39	0.46	0.47	0.31	0.48
Consumption	0.67	0.69	0.65	0.66	0.54	0.67
Investment	0.28	0.27	0.41	0.43	0.26	0.42
Employment	0.26	0.24	0.40	0.42	0.25	0.61
Contemporaneous Correlation with Aggregate Output						
Consumption	0.80	0.83	0.85	0.87	0.87	0.66
Investment	0.98	0.97	0.99	0.98	0.98	0.83
Employment	0.96	0.95	0.98	0.98	0.96	0.68
Contemporaneous Correlation with Aggregate Consumption						
Investment	0.66	0.67	0.75	0.78	0.74	0.60
Employment	0.61	0.60	0.72	0.75	0.69	0.36

Notes: See notes to Table 16. The re-scaling factor is 0.6.

Higher Moments in the Krusell and Smith Rules

It remains to be shown that our results are not driven by the choice of only the average capital stock in the Krusell and Smith rules (9a) and (9b). While it is the case that in the presence of countercyclical second-moment shocks the conventional R^2−measure is fairly low for the 'Full Model' – at least in some combinations of the discrete aggregate states, the minimum is 0.8701 –, and while it is also true that including the skewness of the capital distribution in the Krusell and Smith rules of the 'Full Model'[45] leads to an average increase of the R^2 for the capital regressions from 0.9378 to 0.9870 and for the marginal utility of consumption regressions from 0.9962 to 0.9986, the aggregate behavior of the 'Full Model' is essentially unchanged. Better forecasts

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[45]The 'News Model' features high R^2 over 0.9997 on any aggregate state with the average capital stock only. Including the standard deviation of capital does not yield any significant improvements in R^2. The average R^2 over all discrete states for the skewness regression that is analogous to (9a) is 0.9261.

do not necessarily induce the agents to behave differently (see Bachmann et al. (2008) for a similar finding).[46]

Table 21: AGGREGATE BUSINESS CYCLE STATISTICS - HIGHER MOMENTS

	Full Model MEAN	Full Model SKEWNESS	News Model MEAN	News Model SKEWNESS
Volatility of Output	4.04%	4.05%	4.24%	4.21%
Volatility of aggregate variables relative to output volatility				
Consumption	0.32	0.32	0.30	0.31
Investment	4.71	4.66	4.77	4.70
Employment	0.74	0.74	0.76	0.75
Persistence				
Output	0.42	0.44	0.46	0.46
Consumption	0.66	0.65	0.65	0.65
Investment	0.34	0.39	0.41	0.40
Employment	0.33	0.39	0.40	0.40
Contemporaneous Correlation with Aggregate Output				
Consumption	0.84	0.84	0.85	0.86
Investment	0.98	0.98	0.98	0.98
Employment	0.97	0.97	0.98	0.98
Contemporaneous Correlation with Aggregate Consumption				
Investment	0.73	0.73	0.75	0.76
Employment	0.69	0.70	0.72	0.73

Notes: See notes to Table 16. MEAN refers to a simulation, where only the aggregate capital stock is used in the Krusell-Smith rules (9a)–(9b). SKEWNESS refers to a simulation, where also the skewness of the firm-level capital stock distribution is used in the Krusell-Smith rules (9a)–(9b), plus an analogous rule to (9a) for skewness itself.

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[46]We find even somewhat better improvements in the R^2 and again unaltered aggregate dynamics, when instead of capital skewness we include the standard deviation of log firm-level Solow residuals as an additional moment in the Krusell and Smith rules. All findings are robust to including measurement error in the firm-level Solow residual. Numbers are available on request.

6   Discussion and Conclusion

This paper studies the cyclical behavior of the second moments of the cross-sections of firm-level innovations to Solow residuals, real value added and real sales. We show that firm-specific Solow residual innovations (likewise innovations in value added and sales) are more disperse in recessions than in booms. Thus, firm-level uncertainty is significantly and robustly counter-cyclical in the way Bloom (2009) finds this for U.S. stock market and Compustat data. We also show that the volatility of uncertainty is lower than has been previously found in this U.S. data.

We then explore the quantitative importance of these uncertainty fluctuations in an otherwise standard heterogeneous-firm dynamic stochastic general equilibrium model in the spirit of Khan and Thomas (2008) and Bachmann et al. (2008). We find that empirically realistic fluctuations in uncertainty about productivity growth on their own are not sufficient to generate realistic business cycles. Adding the second moment shocks to a model that has standard first moment shocks, we find that some aspects of the cyclical behavior of the model are altered. For the size of uncertainty fluctuations that we find in German firm-level data, we attribute these changes largely to the (bad) news uncertainty shocks carry about future aggregate productivity, rather than to the wait-and-see-effect of such fluctuations. In this context, we also find that general equilibrium price dynamics are paramount in understanding the aggregate effects of firm-level uncertainty shocks.

To sum up our findings: uncertainty shocks do not appear to be major drivers of year-to-year business cycle fluctuations. This, however, leaves open the possibility that uncertainty shocks are responsible for some particular episodes of the business cycle, recessions specifically. For instance, one may wonder whether the recession in 2007 was driven by an uncertainty shock and a wait-and-see amplification? The following Table 22 suggests that at least for the U.S. this does not seem to be the case. It displays for various investment and consumption categories the percentage peak-trough decline of the 2007 recession relative to the average percentage peak-trough decline of the nine other post-war U.S. recessions. It is obvious that the 2007 recession is amongst the most severe in U.S. post-war history; all macroeconomic aggregates fell by an above-average amount. However, investment and durable consumption decreased by relatively less than nondurable consumption categories. Whereas the former declined between twice and thrice as much as in the average recession, the latter declined fivefold or twelvefold, depending on the specific measure used. Also, whereas all durable categories saw some decline in previous recessions (in seven out of ten durable consumption at the NBER trough was lower than durable consumption at the NBER peak), the nondurable categories exhibited only in roughly half of the recessions any decline at all. This obviously includes the 2007 recession. All these statistics together show that the 2007 recession was especially severe for nondurable

consumption categories, whereas an uncertainty shock combined with a wait-and-see amplifi-
cation would at least suggest a particular severity for the durable expenditure elements. In fact,
the massive decline in nondurable consumption expenditures is rather consistent with a bad
news shock. Of course, this leaves open the possibility that other recessions in the past were
driven by the mechanism advocated in Bloom et al. (2009).

Table 22: THE RECESSION OF 2007 IN THE U.S.

	Relative Decline in this Crisis	# Declines Peak-to-Trough	# Declines
Total Fixed Investment	2.25	10	10
Nonresidential Fixed Investment	2.51	10	10
Durable Consumption	2.84	7	10
Nondurable Consumption	5.34	4	6
Nondurable Consumption + Services	11.89	2	5

Notes: Total Fixed Investment is GPDIC96 from the St. Louis FRED database; Nonresidential Fixed Investment PN-
FIC96; Durable Consumption PCDGCC96; Nondurable Consumption PCNDGC96; Services PCESVC96. All vari-
ables are real and in quarterly frequency. The first column reports the percentage peak-trough decline of the 2007
recession relative to the average percentage peak-trough decline of the nine other post-war U.S. recessions. Peak
and trough are defined by the official NBER recession dates. For the 2007 recession, we assume the trough to be in
the second quarter of 2009 owing to the lack of further data. The second column reports the number of recessions,
where the particular macroeconomic aggregate at the NBER trough was below its value from the NBER peak. The
third column reports the number of declines in the particular macroeconomic aggregate in a recession. There are
10 post-war recessions in the U.S.

In a companion paper (Bachmann and Bayer, 2009), using *cross-sectional* firm dynamics,
we provide further indirect evidence that the magnitude of uncertainty shocks over the busi-
ness cycle cannot be too large. We document there that the firm-level investment rates display
significantly and robustly procyclical dispersion. We explain this fact through a procyclical ex-
tensive margin effect caused by lumpy capital adjustment. However, we also demonstrate there
that increasing the volatility of firm-level uncertainty beyond the levels we find in the data miti-
gates this procyclical extensive margin effect. This allows us to bound the volatility of firm-level
uncertainty at at most double the value we measure in our baseline empirical specification.

Since we base our results on German data, we leave open the possibility that shocks to un-
certainty are a major driving force for the business cycle in the U.S. Bayer and Juessen (2009)
find for the cyclicality of wage risk striking differences between the U.S. and Germany, with the
former displaying countercyclicality (see also Storesletten et al., 2004) and the latter procyclical-
ity. Thus, there is the possibility of a fundamental difference between the German and the U.S.
business cycle, and we encourage future research in this cross-country dimension. However,

Table 22 and the finding that the service sector displays less strong countercyclicality of firm-level uncertainty together with the fact that this sector is larger in the U.S. than in Germany suggest otherwise. This is confirmed by Bachmann and Sims (2009), which uses monthly sectoral survey data, i.e. higher frequency data than balance sheet information, on business expectations in Germany and the U.S. to show that disagreement in survey responses – a well-known proxy for uncertainty – acts like a bad news shock in both countries without any evidence for wait-and-see. We finally leave open the possibility that the dispersion shocks we measure in the data can be a source of aggregate fluctuations in a significantly altered model environment; for instance one, where the asset markets and their volatility are more explicitly modeled (see Gilchrist et al., 2009, or Christiano et al., 2007). Also, Geanakoplos (2009) has proposed an interesting mechanism – the leverage cycle – where uncertainty shocks have to accompany first-moment shocks to transform 'bad news' into 'scary news'. We thus view future research into the precise interaction between uncertainty and aggregate dynamics as desirable.

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A   Appendix A - Data Appendix

A.1   Description of the Sample

The Bundesbank's corporate balance sheet database (*Unternehmensbilanzstatistik*, USTAN henceforth) has been originally created as a by-product of the bank's rediscounting activities, an important instrument of monetary policy before the introduction of the Euro. When a commercial bank wished to pledge a commercial bill of exchange to the Bundesbank, the commercial bank had to prove the creditworthiness of the bill. For that purpose the bank had to provide the Bundesbank with balance sheet information of all parties who backed the bill of exchange. By law, the Bundesbank could only accept bills backed by at least three parties known to be creditworthy. This procedure allowed the Bundesbank to collect a data set with information stemming from the balance sheets and the profit and loss accounts of firms (see Stoess (2001), von Kalckreuth (2003) and Doepke et al. (2005) for further details).

Quality standards of the data are particularly high. All mandatory data collected for USTAN have been double-checked by Bundesbank staff. Hence, the data should contain unusually few errors for a micro-data set. One drawback of USTAN is that with the introduction of the EURO, the Bundesbank stopped buying commercial bills and collected firm balance sheet data only irregularly and from publicly available sources. For this reason, the data set stops being useful in 1999. Therefore, we only use data from 1971 to 1998, which leaves us with essentially 26 year observations from 1973 to 1998 because of lagging and first-differencing.

The coverage of the sample is broad, although it is technically not a representative sample due to the non-random sample design. It was also more common to use bills of exchange in manufacturing and for incorporated companies, which biases our data somewhat towards these kinds of firms. And, of course, the Bundesbank would only rediscount bills with a good rating, so that the set of firms in USTAN is also somewhat biased to financially healthy and larger firms.

Nevertheless, USTAN covers a wide range of firms, since short-term financing through commercial bills of exchange was common practice for many German companies across all business sectors (see Table 24 below for the detailed sectoral composition of our final sample). USTAN also has a broad ownership coverage ranging from incorporated firms as well as privately owned companies, which distinguishes it from the Compustat data. Within the former group USTAN covers both untraded corporations (e.g. limited liability firms, *GmbH*) as well as publicly held companies (*AG*) – see Table 27 below. Finally, USTAN features also a relatively broad size coverage, as we will show in Table 25 below for our final sample, the creation of which we describe in some detail now.

We start out with the universe of observations in the USTAN data, merging the files for 1971-1986 and 1987-1998. In a first pass, we then drop all balance sheets that are irregular,

e.g. bankruptcy or closing balance sheets, or stem from a holding (*Konzernbilanz*). This leaves us with only regular balance sheets (*Handelsbilanz* or *Steuerbilanz*). We also drop all firms with missing payroll data and missing or negative sales data, which are basically non-operating firms. A small amount of duplicate balance sheets is removed as well. And finally, we drop the following sectors: hospitality (hotels and restaurants), which has only a small amount of firms in the database, financial and insurance institutions, the mostly public health and education sectors, as well as other public companies like museums, etc. and some other small service industries, such as hair cutters, dry cleaners and funeral homes;[47] or when sectoral information was missing. The sectoral aggregate we are studying can be roughly characterized as the non-financial private business sector in Germany. This leaves us with an initial data set of 1,764,846 firm-year observations and 259,614 firms. The average number of firms per year is 63,030.

From this initial data set we remove step-by-step more observations, in order to get an economically reasonable data set. We first drop observations from likely East German firms to avoid a break of the series in 1990. We identify a West German firm as a firm that has a West German address or has no address information but enters the sample before 1990. Then we recompute capital stocks with a modified perpetual inventory method (PIM) and employment levels. In the modified PIM we drop a small amount of observations from the top and bottom of the distribution of correction factors for the initial capital stock, see Appendix A.2. Extreme correction factors indicate that constant depreciation is not a good approximation for this particular firm. Such a firm will have had an episode of extraordinary depreciation (e.g. fire, a natural disaster, etc.) and the capital stocks by PIM will be a bad measure of the actual capital after the disaster. We remove observations that do not have a log value added and a log capital stock after PIM. Another large part is removed due to not featuring changes in log firm-level employment, capital and real value added, which obviously requires us to observe firms for two consecutive years. Then we remove outliers in factor changes and real value added changes. Specifically, we identify as outliers in our sample a firm-year in which the firm level investment rate or log changes in firm-level real value added, employment and capital stock fall outside a three standard deviations band around the firm and sectoral-year mean. Then we compute firm-level Solow residuals (see Appendix A.4 for details) and similarly remove observations with missing log changes in Solow residuals as well as outliers therein. We finally remove – before and after each step of the outlier removal – firms that have less than five observations in firm-level Solow residual changes. We conduct extensive robustness checks of our results to the choices for the outlier and observation thresholds (see Appendix B.1). Table 23 summarizes, how much observations are dropped in each step.

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[47]The number of firms from the public sector and these small industries is tiny to begin with, as they did not use commercial bills as a financing instrument. We left out financial and insurance institutions, as they arguably have a very different production function and investment behavior.

Table 23: SAMPLE CREATION

Criterion	Drops of Firm-Year Observations
East Germany	104,299
Outliers in PIM	7,539
Missing log value added	1,349
Missing log capital	31,819
Missing log-changes in N, K, VA	161,668
Outliers in factor and VA log-changes	41,453
Missing log-changes in Solow residual	126,086
Outliers in Solow residual log-changes	18,978
Not enough observations	417,550
Total	910,741

The final sample then consists of 854,105 firm-year observations, which amounts to observations on 72,853 firms and the average observation length of a firm in the sample is 11.7 years. The average number of firms per year is 32,850. The following Tables 24 and 25 as well as 26 and 27 show the average sectoral [48] and the size distributions in our sample, as well as the distributions over the number of observations and legal forms, respectively.

Table 24: SECTORAL DISTRIBUTION

ID	Sector	Observations	Fraction of Observations	WZ 2003
10	Agriculture	12,291	1.44%	A, B
20	Energy & Mining	4,165	0.49%	C, E
31	Chemical Industry, Oil	14,721	1.72%	DF, DG
32	Plastics, Rubber	23,892	2.80%	DH
33	Glass, Ceramics	28,623	3.35%	DI
34	Metals	30,591	3.58%	DJ
35	Machinery	162,407	19.01%	DK, DL, DM, DN
36	Wood, Paper, Printing	61,672	7.22%	DD, DE
37	Textiles, Leather	46,173	5.41%	DB, DC
38	Food, Tobacco	37,708	4.41%	DA
40	Construction	54,569	6.39%	F
61	Wholesale Trade	213,071	24.95%	G51
62	Retail Trade & Cars	142,137	16.64%	G50, G51
70	Transportation & Communication	22,085	2.59%	I
	Total	854,105		

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[48]WZ 2003 is the industry classification from 2003 that the German national accounting system (*Volkswirtschaftliche Gesamtrechnung, VGR*) uses.

Table 25: SIZE DISTRIBUTIONS OF FIRMS

Number of Employees	1-4	5-9	10-14	15-19	20-49	50-99	100-249	250-499	500+
Fraction	6.14%	9.46%	8.24%	7.30%	26.28%	17.04%	14.37%	5.68%	5.49%
Capital Stock (in 1000 1991-Euro)	0-299	300-599	600-999	1,000-1,499	1,500-2,499	2,500-4,999	5,000-9,999	10,000-24,999	25,000+
Fraction	8.23%	9.01%	9.67%	9.36%	13.08%	17.71%	13.87%	11.08%	7.99%
Real Value Added (in 1000 1991-Euro)	0-299	300-499	500-999	1,000-1,499	1,500-2,499	2,500-4,999	5,000-9,999	10,000-24,999	25,000+
Fraction	8.17%	7.93%	16.38%	11.56%	14.45%	16.28%	11.20%	8.25%	5.79%

Table 26: OBSERVATION DISTRIBUTION

Obs. per Firm	Firms	Percent	Cum.	Obs. per Firm	Firms	Percent	Cum.
5	8,973	12.32	12.32	16	2,487	3.41	78.10
6	7,592	10.42	22.74	17	2,225	3.05	81.16
7	6,609	9.07	31.81	18	2,024	2.78	83.93
8	5,724	7.86	39.67	19	1,849	2.54	86.47
9	4,901	6.73	46.39	20	1,619	2.22	88.69
10	4,338	5.95	52.35	21	1,479	2.03	90.72
11	3,960	5.44	57.78	22	1,351	1.85	92.58
12	3,528	4.84	62.63	23	1,446	1.98	94.56
13	3,134	4.30	66.93	24	988	1.36	95.92
14	3,006	4.13	71.05	25	892	1.22	97.14
15	2,647	3.63	74.69	26	2081	2.86	100
				Total	72,853		

Table 27: LEGAL FORM DISTRIBUTION

Legal Form	Observations	Frequency
Publicly Traded (AG, KGaA, etc.)	18,582	2.18%
Limited Liability Companies (GmbH, GmbH&Co., etc.)	506,184	59.26%
Fully Liable Partnerships (OHG, KG, etc.)	327,526	38.35%
Other: unincorporated associations (e.V.) municipal agencies (Körperschaften öR) etc.	1,813	0.21%
Total	854,105	100%

How well does the USTAN aggregate represent the non-financial private business sector (NFPBS) in Germany? USTAN represents on average 70% of the value added of the NFPBS and 78% of its gross output.[49] The NFPBS, in turn, comprises 59% and 69%, respectively, of aggregate real value added and real gross output.

Figure 9 shows that except for a certain overrepresentation of manufacturing (at least, when measured in value added) and a certain underrepresentation of the transportation and communication sector, USTAN represents the sectoral composition in NFPBS rather well.

Figure 9: Sectoral Composition in USTAN and NFPBS
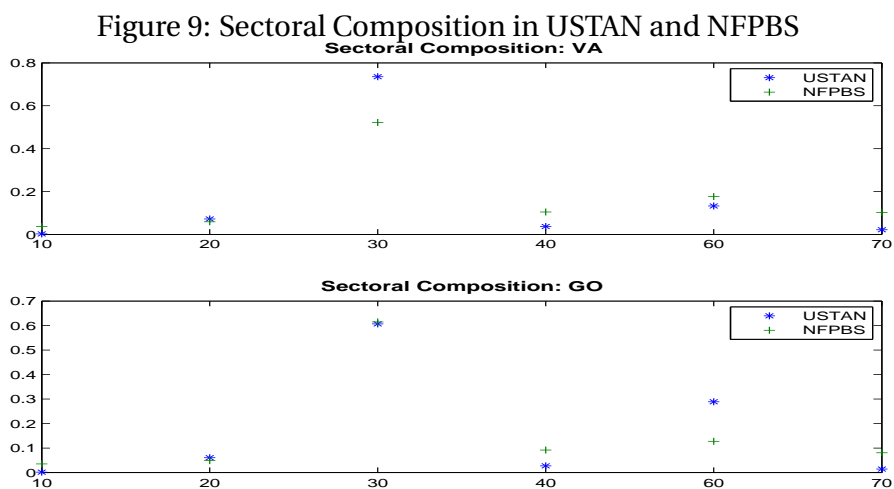
Notes: Graphs display the fraction of the sum of real value added and gross output respectively, over all firms by 1-digit sector within the USTAN sample over the NFPBS aggregate.

Figure 10 demonstrates that also the cyclical behavior of USTAN and NFPBS is close. The

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[49] To compute these statistics we only average over the data from 1973 to 1990, because from then on German national accounting does no longer report West and East Germany separately. For the business cycle statistics we use the post-reunification data, but filter separately before and after this structural break. NFPBS value added is taken from *Bruttowertschoepfung in jeweiligen Preisen*, table 3.2.1 of *VGR*, deflated year-by-year by the implicit deflator for aggregate value added, table 3.1.1 of *VGR* (we apply the same deflator to USTAN data). The base year is always 1991. We experiment also with implicit sector-specific deflators for value added from table 3.2.1 and 3.2.2 of *VGR*, and results are robust to this. NFPBS gross output is taken from *Produktionswerte in jeweiligen Preisen*, table 3.2.3 of *VGR*, deflated year-by-year by the implicit deflator for aggregate gross output, gathered from table 3.2.6 of *VGR*. NFPBS investment is *Bruttoanlageinvestitionen in jeweiligen Preisen* from table 3.2.8.1, deflated with the implicit sector-specific investment price deflators given by *Bruttoanlageinvestitionen - preisbereinigt*, a chain index, from table 3.2.9.1, *VGR*. NFPBS capital is *Nettoanlagevermoegen in Preisen von 2000* from table 3.2.19.1, *VGR*, re-chained to 1991 prices. In both the computation of investment and capital data for USTAN in the PIM we use the implicit sector and capital good specific (equipment and non-residential structures) deflators for investment: tables 3.2.8.2, 3.2.9.2., 3.2.8.3 and 3.2.9.3., *VGR*. We also experiment with deflating USTAN data with a uniform investment price deflator, the *Preisindex der Investitionsgueterproduzenten*, source: GP-X002, *Statistisches Bundesamt*. NFPBS employment is number of employed, *Arbeitnehmer*, from table 3.2.13, *VGR*. Payroll is taken from *Arbeitnehmerentgelt*, table 3.2.10., *VGR*, deflated by the same general implicit deflator for aggregate value added that we use to deflate value added numbers. Aggregate total hours comes from *Geleistete Arbeitsstunden der Arbeitnehmer*, table 2.9, *VGR*. Finally, real private consumption data are *private Konsumausgaben*, a chain index with base year in 1991, from table 3.2 in the *VGR*.

correlation of the cyclical components of value added is 0.7671 and for gross output it is 0.6049.[50]

Figure 10: Cyclical Behavior in USTAN and NFPBS
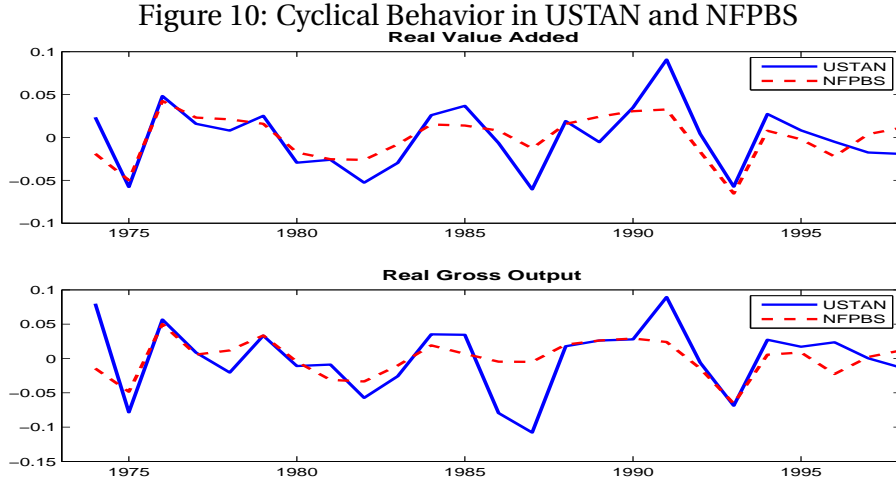
Notes: Upper panel: time series for the sum of real value added over all firms in the USTAN sample and NFBS after detrending with logarithmic first differences and a deterministic linear trend. Lower panel: time series for the sum of real gross output over all firms in the USTAN sample and NFBS after detrending with logarithmic first differences and a deterministic linear trend.

A.2 Capital Stocks

In order to obtain economically meaningful stocks of capital series for each firm, we have to re-calculate capital stocks in a Perpetual Inventory Method (PIM). The first step is to compute firm-level investment series, $i_{i,t}$, from the corporate balance sheets, which contain data only on accounting capital stocks, $k_{i,t}^a$, and accounting total depreciation, $d_{i,t}^a$. The following accumulation identity allows to back out nominal firm-level investment:[51]

$$k_{i,t+1}^a = k_{i,t}^a - d_{i,t}^a + p_t^I i_{i,t}. \tag{13}$$

The next step is to recognize that capital stocks from corporate balance sheets are not directly usable for economic analysis for two reasons: 1) accounting depreciation, $d_{i,t}^a$, in corporate balance sheets is often motivated by tax reasons and typically higher than economic

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[50]We take first differences of log value added as well as gross output and then take out for both a deterministic linear trend to remove the growth of the USTAN sample over time. The correlation between only the first differences in log value added is still 0.5348, and 0.4966, when an HP(100)-filter is applied.

[51]Specifically, $k_{i,t}^a$ is the sum of balance sheet items ap65, *Technische Anlagen und Maschinen*, and ap66, *Andere Anlagen, Betriebs-und Geschaeftsausstattung*, for equipment; and balance sheet item ap64, *Grundstuecke, Bauten*, for structures. Since balance sheet data are typically end-of-year stock data, notice that $k_{i,t}^a$ is the end-of-period capital stock in year $t-1$. $d_{i,t}^a$ is profit and loss account item ap156, *Abschreibungen auf Sachanlagen und immaterielle Vermoegensgegenstaende des Anlagevermoegens*. In contrast to $k_{i,t}^a$, $d_{i,t}^a$ is not given for each capital good separately. For the solution of this complication, see below.

depreciation, $\delta_{i,t}^e$, expressed as a rate; 2) accounting capital stocks are reported at historical prices. Both effects would lead to an underestimation of the real firm-level capital stock, if one were to simply deflate the current accounting capital stock, $k_{i,t}^a$, with a current investment price deflator, p_t^I (assuming that p_t^I increases over time). We therefore apply a Perpetual Inventory Method (PIM) to compute economic real capital stocks:

$$k_{i,1}^{(1)} = k_{i,1}^a. \tag{14}$$

$$k_{i,t+1}^{(1)} = \left(1 - \delta_t^e\right) k_{i,t}^{(1)} + \frac{p_t^I}{p_{1991}^I} i_{i,t}. \tag{15}$$

$k_{i,1}^a$ is the accounting capital stock in prices of 1991 at the beginning of an uninterrupted sequence of firm observations – if for a firm-year we have a missing investment observation, the PIM is started anew, when the firm appears again in the data set. We estimate δ_t^e for each year from national accounting data, *VGR*, separately for equipment and non-residential structures (table 3.1.3, *VGR, Nettoanlagevermoegen nach Vermoegensarten in jeweiligen Preisen, Ausruestungen und Nichtwohnbauten*; table 3.1.4, *VGR, Abschreibungen nach Vermoegensarten in jeweiligen Preisen, Ausruestungen und Nichtwohnbauten*). *VGR* contains sectoral and capital good specific depreciation data only after 1991, which is why we decided to use only capital good specific depreciation rates for the entire time horizon. For the data sources for investment price deflators see footnote 49. The drawback of this procedure is that we do not observe directly capital-good specific $d_{i,t}^a$ in the balance sheets (differently from $k_{i,t}^a$), so that (13) is not directly applicable for the two types of capital goods separately. We therefore split up $d_{i,t}^a$ according to the fraction that each capital good accounts for in the book value of total capital, weighting each capital good by its VGR depreciation rate. Creating a capital series for both capital goods this way is mainly meant to provide a better estimate for total capital for each firm, because we finally aggregate up both types of capital into a single capital good at the firm-level.

There is a final complication, which comes through relying on $k_{i,1}^a$ as the starting value of the PIM. It is typically not a good estimate of the productive real capital stock of the firm at that time. Therefore, we calculate the time-average factor ϕ (for each sector), by which $k_{i,t}^{(1)}$ is larger than $k_{i,t}^a$, and replace $k_{i,1}^a$ by $\phi k_{i,1}^a$ in the perpetual inventory method. We do this iteratively, until ϕ converges, i.e. we calculate:

$$k_{i,t+1}^{(n)} = \left(1 - \delta_t^e\right) k_{i,t}^{(n)} + \frac{p_t^I}{p_{1991}^I} i_{i,t} \tag{16}$$

$$k_{i,1}^{(n)} = \phi^{(n-1)} k_{i,1}^{(n-1)} \tag{17}$$

$$\phi^{(n)} = (NT)^{-1} \sum_{i,t} \frac{k_{i,t}^{(n)}}{k_{i,t}^{(n-1)}} \tag{18}$$

where $k_{i,t}^{(0)} = k_{i,t}^a$, $\phi^{(0)} = 1$. We stop when for each sector and each capital good category $\phi < 1.1$.

Since for our purposes we want to compute economic, i.e. productive, capital stocks, we then – as a final step – add to the capital stock series from this iterative PIM the net present value of the real expenditures for renting and leasing equipment and structures.[52]

A.3 Labor Inputs

A more particular difficulty with USTAN data is that information on the number of employees is only updated infrequently for some companies, as it is not taken directly from balance sheets, but sampled from supplementary company information. Being no balance sheet item, the employment data is not constrained by legal accounting rules and did not undergo consistency checks by Bundesbank staff. However, in order to compute firm-level Solow residuals, we need some measure of employment.

We base this measure on the payroll data ($wagebill_{i,t}$) from the profit and loss statements (item ap154, *Personalaufwand*). Payroll data is regulated by accounting standards and is checked for consistency by the Bundesbank using accounting identities. In contrast to the direct employment data, the payroll data is generally considered of high quality. Therefore, we exploit this data to construct a proxy measure for (log) employment $n_{i,t}$ as follows (with a slight abuse of notation, we use $n_{i,t}$ here for log employment).

The idea behind our proxy measure is that we can determine sectoral average wages even though firm level employment is measured with error. Since wage bargaining in Germany is highly centralized, the sectoral average wage is all we need then, since it is a good proxy for firm level wages. Therefore, dividing firm level payroll by the sectoral average wage recovers true firm level employment.

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[52] Specifically, we take item ap161, *Miet- und Pachtaufwendungen*, from the profit and loss accounts, deflate it by the implicit investment good price deflator, which we compute, in turn, from tables 3.2.8.1 and 3.2.9.1 from *VGR*, and then divide it by a measure of the user cost of capital. The latter is simply the sum of real interest rates for a given year, which - courtesy of the Bundesbank - we compute from nominal interest rates on corporate bonds and ex-post CPI inflation data (the series is available from the authors upon request), and the time-average, accounting capital-good weighted depreciation rate per firm.

Specifically, we assume that the measurement error in reported log employment, $n_{i,t}^*$,[53] is classical and additive:

$$n_{i,t}^* = n_{i,t} + \varepsilon_{i,t}. \tag{19}$$

Then we decompose the wage per employee, $\omega_{i,t}$, of firm i at time t into two effects. One is determined by a firm-time-specific wage component $w_{i,t}$, and the other one being region-, $r(i,t)$, sector-, $j(i,t)$, and size-class-specific, $s(i,t)$, where $j(i,t)$, $r(i,t)$ and $s(i,t)$ denote that firm i belongs to sector j, region r and size-class s at time t, respectively.[54] Thus, we write

$$\omega_{i,t} = \bar{w}_{j(i,t),r(i,t),s(i,t),t} + w_{i,t}. \tag{20}$$

We denote all firms that belong to sector j, region r and size-class s at time t by $I(j,r,s,t)$. Then we can estimate a sector-region-size wage component, $\bar{w}_{j,r,s,t}$, as:[55]

$$\widehat{w}_{j,r,s,t} = \frac{1}{\#I(j,r,s,t)} \sum_{i \in I(j,r,s,t)} \left[\log(wagebill_{i,t}) - n_{i,t}^* \right]. \tag{21}$$

We then use this estimate of the average wage rate to estimate employment on the basis of the firm's wage bill:

$$\hat{n}_{it} = \log wagebill_{it} - \widehat{w}_{j,r,s,t} \tag{22}$$

$$= n_{it} + \omega_{it} - \frac{1}{\#I(j,r,s,t)} \sum_{h \in I(j,r,s,t)} \left(n_{h,t} + \omega_{h,t} - \left(n_{h,t} + \varepsilon_{h,t} \right) \right) \tag{23}$$

$$= n_{it} + w_{it} - \frac{1}{\#I(j,r,s,t)} \sum_{h \in I(j,r,s,t)} \left(w_{h,t} - \varepsilon_{h,t} \right) \tag{24}$$

$$= n_{it} + w_{it} + \frac{1}{\#I(j,r,s,t)} \sum_{h \in I(j,r,s,t)} \varepsilon_{h,t}. \tag{25}$$

The second equality stems from using (19). The next to last equality holds, because one can replace ω_{it} by (20), realizing that the $\bar{w}$, which do not depend on a specific firm, cancel. The last equality holds, because, by construction, the average firm-level deviation from a sector-

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[53]We use item ap34, *Beschaeftigtenzahl im Durchschnitt des Geschaeftsjahres*, to measure $n_{i,t}^*$, where available.

[54]Specifically, for sectors we use the 2-digit classification in Table 24 in Appendix A.1. For size classes we use terciles of the capital distribution in each year. For the region-specific wage component we proceed as follows: we divide West Germany into three regions, according to zip codes: South with zip codes starting with 7,8,9, except for 98 and 99; Middle with zip codes starting with 4,5,6, except for 48 and 59; North with zip codes starting with 2,3 as well as 48 and 59. However, not all balance sheets feature zip code information, which is why we compute $\widehat{w}_{j,r,s,t}$ with and without a region component. For those firms that do not have zip code information or for those firms that are in sector-region-size bins with fewer than 50 observations in a given year, we take the estimate without the region component.

[55]To estimate $\widehat{w}_{j,r,s,t}$ we of course use only those observations, where $n_{i,t}^*$, i.e. item ap34, *Beschaeftigtenzahl im Durchschnitt des Geschaeftsjahres*, is available.

region-size bin is zero in every year. For $\#I(j,r,s,t)$ large, the average measurement error term $\left(\frac{1}{\#I(j,r,s,t)}\sum_{h\in I(j,r,s,t)}\varepsilon_{h,t}\right)$ is negligible. In addition, since wage bargaining is highly centralized in Germany, also the firm specific wage component, w_{it}, can be expected to be of lesser importance, i.e. the variance σ_w^2 is small. In particular it can be expected to be smaller than the initial measurement error in employment stocks. Therefore our measure of employment, $\hat{n}_{i,t}$, should follow real employment, $n_{i,t}$, more closely than $n_{i,t}^*$.

To corroborate this claim, we checked our procedure using data from the German social security records at the *Institut fuer Arbeitsmarkt- und Berufsforschung (IAB)*, which provide information on the wage bill and employment at the establishment level. There we observe true employment and wage bills for all plants and the time 1975-2006. Constraining ourselves to the sample period 1975-1998 and to plants with more than 12 employees, i.e. to data comparable to the one of the USTAN data, we find the correlation between $\hat{n}_{i,t}$ and $n_{i,t}$ as well as between $\Delta \hat{n}_{i,t}$ and $\Delta n_{i,t}$ to be fairly high (98% and 94%, respectively). This means that the cross-sectional variance of the firm specific wage innovations $\sigma_{\Delta w}^2$ is small (0.0026) compared to the cross-sectional variance of employment changes ($\sigma_{\Delta n}^2 = 0.0163$, $\sigma_{\Delta \hat{n}}^2 = 0.0162$). Finally, a correlation coefficient between $mean(\Delta n_{i,t})$ in the USTAN data and the log-change in aggregate NFPBS employment of 0.653 shows also the quality of our employment measure.

A.4 Solow Residual Calculation

With the estimated firm-level capital stocks and employment levels we can now compute firm-level Solow residuals from the logged production function (1). In our baseline specification we estimate the factor elasticities, v and θ, as 1-digit sector-specific median, pooled over all firm-year observations in a sector, expenditure shares.[56] Table 28 displays the estimated elasticities. Notice that for the aggregate Solow residual calculation for the baseline simulations, for which we use the data sources specified in Footnote 49 in Appendix A.1, we simply use the expenditure shares from manufacturing, as manufacturing is still the largest sector within NFPBS (had we used any weighted median of expenditure shares the result would have been the same).

To check the robustness of our results, we also try alternative specifications with predefined elasticities common across sectors. We first try a combination of v and θ, SR(2), such that a one

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[56]We use profit and loss account item ap153, *Rohergebnis*, for firm-level value added and deflate it in the baseline scenario with the aggregate value added deflator, but experiment also with sector-specific value added deflators, see Footnote 49 in Appendix A.1 for details. To compute firm-level expenditure shares, we proceed as follows: the labor share is simply total payroll divided by value added (ap154/ap153); capital expenditures, which are then again divided by value added, are the sum of the PIM capital stock and the net present value of renting and leasing expenditures multiplied by the user cost of capital as specified in Footnote 52 in Appendix A.2. For firm-level sales we use profit and loss account item ap144, *Umsatzerloese*, and deflate it with the aggregate value added deflator. For firm-level sales we use profit and loss account item ap151, *Aufwendungen fuer Roh-, Hilfs-, und Betriebsstoffe und fuer bezogene Waren*, and deflate it again with the aggregate value added deflator.

Table 28: SECTOR-SPECIFIC EXPENDITURE SHARES

ID	Sector	labor share v	capital share θ
1	Agriculture	0.2182	0.7310
2	Energy & Mining	0.3557	0.5491
3	Manufacturing	0.5565	0.2075
4	Construction	0.6552	0.1771
6	Trade	0.4536	0.2204
7	Transport & Communication	0.4205	0.2896

third share of capital in a CRTS production function and a reduced form revenue elasticity of capital of 0.5 are compatible. This implies an implicit markup of 1.33, if one were to rationalize DRTS with a CRTS production function and monopolistic competition. We then either fix the revenue elasticity of capital (SR5, SR3) or fix the one third share of capital in a CRTS production function (SR6, SR4) and lower the markup. Table 29 summarizes the scenarios.

Table 29: OVERVIEW OVER THE DIFFERENT SOLOW RESIDUAL SPECIFICATIONS

Specification	labor share v	capital share θ	CRTS capital share $\frac{\theta}{\theta+v}$	markup $\frac{1}{\theta+v}$	revenue elasticity $\frac{\theta}{1-v}$
Baseline	0.5565	0.2075	0.31	1.32	0.47
SR2	0.5	0.25	0.33	1.33	0.5
SR3	0.6667	0.1667	0.2	1.20	0.5
SR4	0.5556	0.2778	0.33	1.20	0.63
SR5	0.6	0.2	0.25	1.25	0.5
SR6	0.5333	0.2667	0.33	1.25	0.57

A.5 Cross-sectional Dispersion Data

Table 30: CROSS-SECTIONAL DISPERSION DATA FOR THE LOG-CHANGES IN SOLOW RESIDUALS, VALUE ADDED AND SALES IN THE BASELINE EMPIRICAL SCENARIO

Year	$std(\Delta \log \epsilon_{i,t})$	$std(\Delta \log y_{i,t})$	$std(\Delta \log sales_{i,t})$
1973	0.12204	0.14743	0.17619
1974	0.12784	0.15719	0.20029
1975	0.13017	0.16057	0.20948
1976	0.12285	0.14851	0.2013
1977	0.11902	0.14056	0.18989
1978	0.1182	0.13888	0.19715
1979	0.11875	0.13843	0.18751
1980	0.11945	0.13881	0.18311
1981	0.12397	0.14369	0.18629
1982	0.12395	0.14519	0.18548
1983	0.12137	0.14378	0.18798
1984	0.12086	0.14251	0.18383
1985	0.12065	0.14491	0.18567
1986	0.1212	0.14179	0.1901
1987	0.12028	0.14024	0.18593
1988	0.11673	0.13571	0.18348
1989	0.1143	0.1328	0.17782
1990	0.11868	0.1401	0.18643
1991	0.12155	0.14548	0.18557
1992	0.12014	0.14218	0.18209
1993	0.12166	0.14537	0.19942
1994	0.12075	0.1408	0.18527
1995	0.1184	0.13602	0.18021
1996	0.11722	0.13396	0.17517
1997	0.11203	0.12965	0.176
1998	0.11017	0.13004	0.17971

Notes: std: cross-sectional standard deviation of the within-transformed data. No detrending.

B   Appendix B - Robustness

B.1   Further Robustness on the Empirical Results

Table 31 shows that the three main empirical findings of this paper – high idiosyncratic uncertainty and countercyclicality as well as small volatility of idiosyncratic uncertainty – are robust to both different pre-specified and uniform-across-sectors factor elasticities in the production function as well as different lag structures in the production factors: Baselinevar1 assumes predetermined labor in addition to predetermined capital, whereas Baselinevar2 has both labor and capital productive instantaneously. Tables 32 to 37 show the analogous results of Tables 5 to 9 and Table 12 in Section 2.3 for $std(\Delta \log y_{i,t})$ and $std(\Delta \log sales_{i,t})$, respectively.

Table 31: ROBUSTNESS FOR DIFFERENT SOLOW RESIDUAL SPECIFICATIONS

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100) - Y)$	$\mu(\cdot)$
$std(\Delta \log \epsilon_{i,t})_{\text{Baseline}}$	2.67%	-0.481	0.120
$std(\Delta \log \epsilon_{i,t})_{\text{SR2}}$	2.71%	-0.491	0.122
$std(\Delta \log \epsilon_{i,t})_{\text{SR3}}$	2.39%	-0.517	0.120
$std(\Delta \log \epsilon_{i,t})_{\text{SR4}}$	2.56%	-0.481	0.123
$std(\Delta \log \epsilon_{i,t})_{\text{SR5}}$	2.51%	-0.510	0.120
$std(\Delta \log \epsilon_{i,t})_{\text{SR6}}$	2.62%	-0.486	0.122
$std(\Delta \log \epsilon_{i,t})_{\text{Baselinevar1}}$	3.64%	-0.591	0.154
$std(\Delta \log \epsilon_{i,t})_{\text{Baselinevar2}}$	2.63%	-0.501	0.117

Notes: std: cross-sectional standard deviation, linearly detrended. *cv*: time series coefficient of variation. *ρ*: time series correlation coefficient.

μ: time series average. *$HP(\lambda) - Y$*: Cyclical component of GDP after HP-filtering using smoothing parameter λ.

Table 32: COUNTERCYCLICALITY OF IDIOSYNCRATIC UNCERTAINTY - ROBUSTNESS TO CYCLICAL INDICATOR

Cyclical Indicator	$\rho(std(\Delta \log y_{i,t}), \cdot)$
Baseline: HP(100)-Y	-0.451
HP(6.25)-Y	-0.474
Log-diff-Y	-0.482
$mean(\Delta \log y_{i,t})$	-0.490
Log-diff-N	-0.548
Log-diff-Solow Residual	-0.337

Cyclical Indicator	$\rho(std(\Delta \log sales_{i,t}), \cdot)$
Baseline: HP(100)-Y	-0.405
HP(6.25)-Y	-0.563
Log-diff-Y	-0.375
$mean(\Delta \log sales_{i,t})$	-0.410
Log-diff-N	-0.436
Log-diff-Solow Residual	-0.228

Notes: See notes to Table 31. *N* refers to aggregate employment.

Table 33: MORE ROBUSTNESS

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100) - Y)$	$\mu(\cdot)$
$iqr(\Delta \log y_{i,t})$	6.13%	-0.413	0.150
$std(\Delta \log y_{i,t})_{\text{raw}}$	3.95%	-0.394	0.153
$std(\Delta \log y_{i,t})_{1973-1990}$	3.72%	-0.650	0.143
$std(\Delta \log y_{i,t})_{1977-1998}$	3.32%	-0.235	0.140
$iqr(\Delta \log sales_{i,t})$	8.22%	-0.343	0.162
$std(\Delta \log sales_{i,t})_{\text{raw}}$	4.05%	-0.306	0.199
$std(\Delta \log sales_{i,t})_{1973-1990}$	3.90%	-0.432	0.189
$std(\Delta \log sales_{i,t})_{1977-1998}$	2.78%	-0.094	0.185

Notes: See notes to Table 31. iqr stands for interquartile range, which is linearly detrended.

Table 34: RESULTS BY SECTOR

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100) - Y)$	$\rho(\cdot, HP(100) - Y^{Sect.})$	$\mu(\cdot)$
$std(\Delta \log y_{i,t})_{\text{Aggregate}}$	3.73%	-0.451	-	0.142
$std(\Delta \log y_{i,t})_{\text{AGR}}$	4.24%	0.044	-0.324	0.176
$std(\Delta \log y_{i,t})_{\text{MIN/ENE}}$	11.39%	-0.221	0.144	0.116
$std(\Delta \log y_{i,t})_{\text{MAN}}$	5.03%	-0.579	-0.323	0.140
$std(\Delta \log y_{i,t})_{\text{CON}}$	4.77%	-0.614	-0.442	0.150
$std(\Delta \log y_{i,t})_{\text{TRD}}$	3.48%	-0.057	-0.335	0.141
$std(\Delta \log y_{i,t})_{\text{TRA/COM}}$	4.30%	0.117	0.177	0.151
$std(\Delta \log sales_{i,t})_{\text{Aggregate}}$	3.82%	-0.405	-	0.187
$std(\Delta \log sales_{i,t})_{\text{AGR}}$	10.14%	-0.060	0.029	0.217
$std(\Delta \log sales_{i,t})_{\text{MIN/ENE}}$	18.12%	0.047	0.049	0.118
$std(\Delta \log sales_{i,t})_{\text{MAN}}$	4.66%	-0.506	-0.293	0.170
$std(\Delta \log sales_{i,t})_{\text{CON}}$	6.99%	-0.064	-0.084	0.328
$std(\Delta \log sales_{i,t})_{\text{TRD}}$	5.93%	-0.267	-0.223	0.176
$std(\Delta \log sales_{i,t})_{\text{TRA/COM}}$	8.06%	-0.27	-0.237	0.174

Notes: See notes to Table 31. AGR: Agriculture; MIN/ENE: Mining & Energy; MAN: Manufacturing; CON: Construction; TRD: Trade (Retail & Wholesale); TRA/COM: Transportation & Communication. Y refers to the output of the aggregate of the non-financial private business sector. $Y^{Sect.}$ refers to the output of the corresponding sector.

Table 35: RESULTS BY FIRM SIZE

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100) - Y)$	$\mu(\cdot)$
$std(\Delta\log\epsilon_{i,t})_{\text{Aggregate}}$	2.67%	-0.481	0.120
Firm Size By Capital			
$std(\Delta\log\epsilon_{i,t})_{\text{First Quartile}}$	1.81%	-0.487	0.136
$std(\Delta\log\epsilon_{i,t})_{\text{Second Quartile}}$	2.54%	-0.297	0.120
$std(\Delta\log\epsilon_{i,t})_{\text{Third Quartile}}$	3.76%	-0.485	0.114
$std(\Delta\log\epsilon_{i,t})_{\text{Fourth Quartile}}$	4.13%	-0.479	0.109
$std(\Delta\log\epsilon_{i,t})_{\text{Largest 5\%}}$	4.71%	-0.464	0.106
Firm Size By Value Added			
$std(\Delta\log\epsilon_{i,t})_{\text{First Quartile}}$	1.95%	-0.438	0.138
$std(\Delta\log\epsilon_{i,t})_{\text{Second Quartile}}$	2.77%	-0.462	0.119
$std(\Delta\log\epsilon_{i,t})_{\text{Third Quartile}}$	3.32%	-0.436	0.114
$std(\Delta\log\epsilon_{i,t})_{\text{SMALL95}}$	4.15%	-0.424	0.107
$std(\Delta\log\epsilon_{i,t})_{\text{Largest 5\%}}$	4.90%	-0.436	0.100

Notes: See notes to Table 31. First Quartile refers to the smallest 25% firms in a given year, Second Quartile to the firms with size in the 25%-50% range, etc., if size is measured in capital stock and real value added, respectively.

Table 36: RESULTS BY LEGAL FORM

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100) - Y)$	$\mu(\cdot)$
$\sigma(\Delta\log y_{i,t})_{\text{Aggregate}}$	3.73%	-0.451	0.142
$std(\Delta\log y_{i,t})_{\text{Publicly Traded}}$	6.12%	-0.322	0.126
$std(\Delta\log y_{i,t})_{\text{Limited Liability}}$	4.29%	0.484	0.145
$std(\Delta\log y_{i,t})_{\text{Unlimited Liability}}$	3.42%	-0.396	0.138
$\sigma(\Delta\log sales_{i,t})_{\text{Aggregate}}$	3.82%	-0.405	0.187
$std(\Delta\log sales_{i,t})_{\text{Publicly Traded}}$	17.18%	0.086	0.157
$std(\Delta\log sales_{i,t})_{\text{Limited Liability}}$	4.72%	-0.414	0.193
$std(\Delta\log sales_{i,t})_{\text{Unlimited Liability}}$	4.18%	-0.172	0.178

Notes: See notes to Tables 2 and 3. Publicly Traded refers incorporated entities (AG and KGaA), Limited Liability refers to limited liability companies (GmbH and GmbH&Co.), Unlimited Liability refers to fully liable partnerships, unincorporated associations and municipal agencies (OHG, KG, e.V., KöR), where the latter two categories are only a small fraction.

Table 37: RESULTS BY MATERIAL USAGE

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100) - Y)$	$\mu(\cdot)$
$std(\Delta \log y_{i,t})_{\text{Aggregate}}$	3.73%	-0.451	0.142
$std(\Delta \log y_{i,t})_{\text{Material Intensity const.}}$	5.94%	-0.270	0.117
$std(\Delta \log sales_{i,t})_{\text{Aggregate}}$	3.82%	-0.405	0.187
$std(\Delta \log sales_{i,t})_{\text{Material Intensity const.}}$	6.64%	-0.277	0.124

Notes: See notes to Table 31. Material Intensity const. refers to firms whose material expenditures over sales $\frac{m_{i,t}}{sales_{i,t}}$ have changed by less then one percentage point in absolute terms.

We next provide further evidence for the robustness of the three empirical findings of this paper to *sample selection* and *variable construction*. First, we use sector-specific price deflators for value added instead of the aggregate one (for sources see Footnote 49 in Appendix A.1). Second, we use an aggregate price deflator for investment goods (see Footnote 49 in Appendix A.1 for details) in the perpetual inventory method instead of sectoral deflators separately for equipment and structures. Third, we employ a stricter outlier removal criterion of 2.5 standard deviations around the firm- and year-specific mean in Solow residual and value added innovations, as well as investment rates and employment changes. Fourth, we use a more liberal outlier criterion using 5 and 10 standard deviations instead of 3.[57] Fifth, we employ a specification, where we assume that an outlier above 3 standard deviations means a merger and, subsequently, treat these firms as new firms in addition to removing them in the year, where the outlier occurs. Sixth, we use all the firms that we observe at least twice with first differences.[58] Finally, we carry out a more standard perpetual inventory method that simply uses the reported capital stocks in the first year of observation for a firm, instead of solving a fixed point problem in correction factors (see Appendix A.2 for details). As one can see from Table 38, the results are robust to all these alternative sampling procedures.

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[57]The latter variant lowers the number of dropped firm-year observations due to outliers in factor and value added changes from 41,453 to 4,240, and the ones due to outliers in Solow residual changes from 18,978 to 1,486. This leaves the total number of firm-year observations at 934,315 and the total number of firms in the sample at 78,092.

[58]This lowers the number of dropped firm-year observations due to not satisfying the minimum observation requirement from 417,550 to 158,950. This leaves the total number of firm-year observations at 971,308 and the total number of firms in the sample at 114,528.

Table 38: MORE ROBUSTNESS

Cross-sectional Moment	$cv(\cdot)$	$\rho(\cdot, HP(100)-Y)$	$\mu(\cdot)$
$std(\Delta\log\epsilon_{i,t})_{\text{Baseline}}$	2.67%	-0.481	0.120
$std(\Delta\log\epsilon_{i,t})_{\text{Sectoral deflators for VA}}$	2.66%	-0.483	0.120
$std(\Delta\log\epsilon_{i,t})_{\text{Uniform price index for I-goods}}$	2.68%	-0.480	0.120
$std(\Delta\log\epsilon_{i,t})_{\text{Stricter outlier removal}}$	2.51%	-0.499	0.109
$std(\Delta\log\epsilon_{i,t})_{\text{Looser outlier removal}}$	2.88%	-0.476	0.150
$std(\Delta\log\epsilon_{i,t})_{\text{Very loose outlier removal}}$	5.10%	-0.578	0.195
$std(\Delta\log\epsilon_{i,t})_{\text{Stricter merger criterion}}$	2.69%	-0.485	0.118
$std(\Delta\log\epsilon_{i,t})_{\text{Shorter in sample}}$	2.46%	-0.485	0.122
$std(\Delta\log\epsilon_{i,t})_{\text{Standard Perpetual Inventory}}$	2.64%	-0.492	0.119

Notes: See notes to Table 31.

Figure 11: Time Series of the Dispersions of Firm-Level Value Added and Sales Innovations (Normalized by the Average Dispersion) and the Cyclical Component of GDP
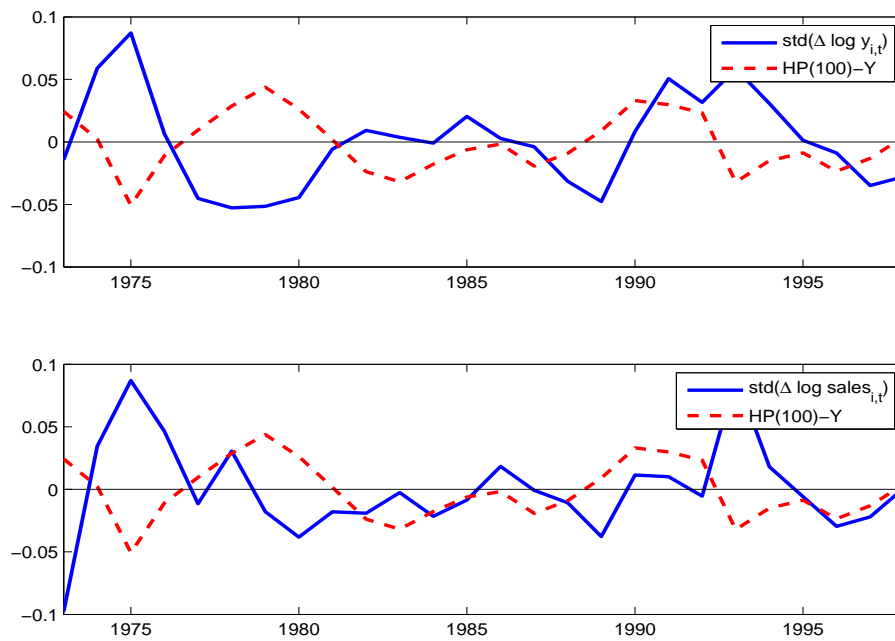
Notes: See notes to Table 31.

Figure 12: Scatter Plot between the Cross-sectional Dispersion of Firm-Level Solow Residual Innovations and the Cyclical Component of GDP
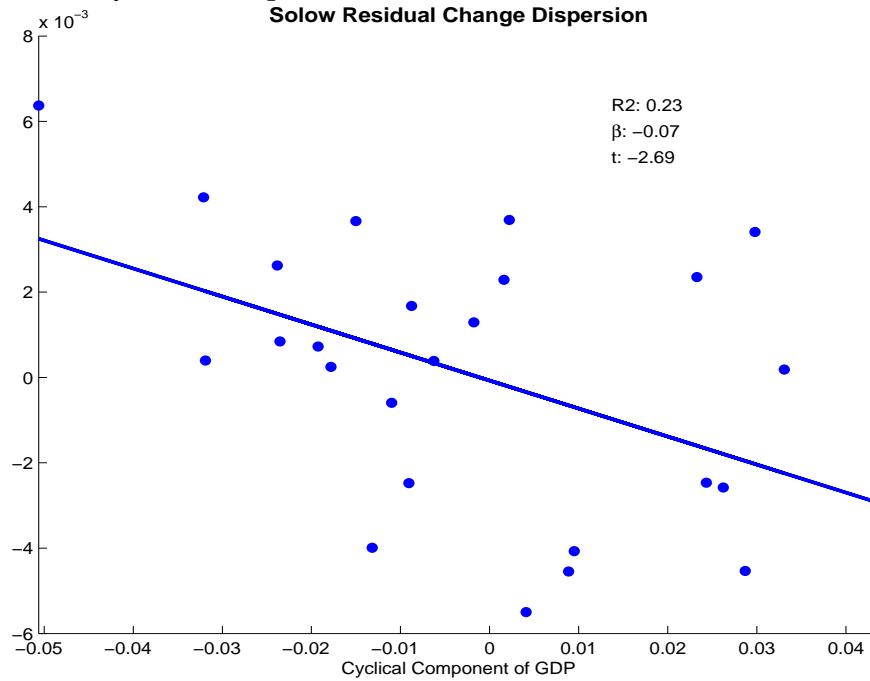

Figure 13: Scatter Plots between the Cross-sectional Dispersion of Firm-Level Value Added/Sales Innovations and the Cyclical Component of GDP
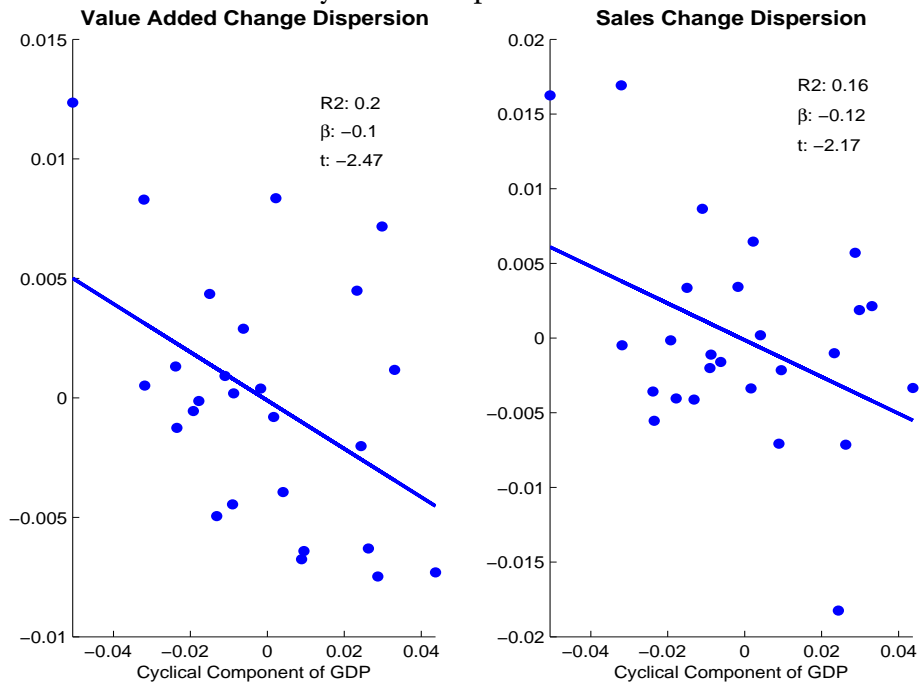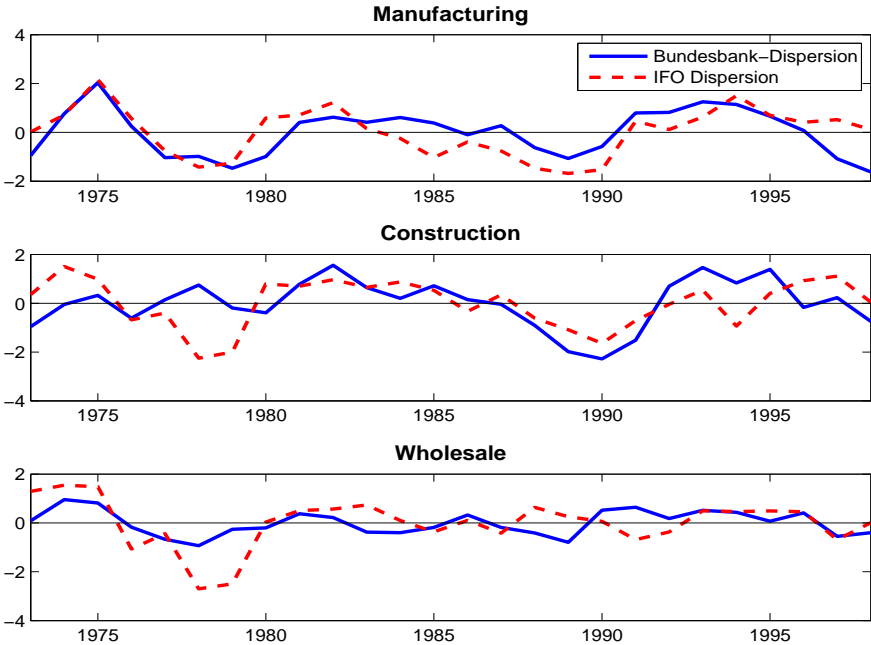
Figure 14: Time Series of the Dispersion of Firm-Level Solow Residual Innovations and the IFO Uncertainty Index

Notes: Both time series are linearly detrended and normalized by their time-series standard deviation.

B.2   Further Robustness on the Simulation Results

In this subsection, we provide one further robustness on our three quantitative results: the inability of the pure 'Risk Model' to produce realistic business cycles; the small changes in the second moment analysis that adding uncertainty shocks to a standard first moment shock 'RBC Model' brings; and the interpretation of these changes as largely due to a bad news effect that uncertainty shocks entail. Specifically, (Table 39), we more than triple the adjustment costs parameter, $\bar{\xi}$, and set it to unity. This is to check whether higher nonconvex adjustment costs could give the wait-and-see effect are larger role. We find that this is not the case.

Finally, (Table 40), we compare the baseline results with the ones from a seemingly unrelated regressions (SUR) specification for the aggregate driving force, (2), i.e. a specification, where the off-diagonal elements of ϱ_A are restricted to zero. This is interesting for two reasons: first, this specification mitigates somewhat the obvious problems with estimating a bivariate VAR with 7 independent parameters from 2×26 data points. We show that both the actual estimates of the remaining VAR parameters as well as the implications for our simulation results are remarkably robust. Secondly, Table 40 shows that most of the changes between the 'RBC Model' and the 'Full/News Model' are mainly due to the interaction effect between aggregate Solow residuals and firm-level uncertainty, that in the unrestricted baseline specification is embodied in the negative VAR coefficient of tomorrow's (but known today) firm-level uncertainty on tomorrow's aggregate Solow residual, an effect that we identified as a bad news effect.

Table 39: AGGREGATE BUSINESS CYCLE STATISTICS - HIGHER ADJUSTMENT COSTS ($\bar{\xi} = 1$)

	Full Model	News Model	RBC Model	Risk Model	Data
Volatility of Output	3.86%	4.07%	3.06%	0.29%	2.30%
Volatility of aggregate variables relative to output volatility					
Consumption	0.36	0.34	0.42	0.78	0.78
Investment	4.40	4.50	4.13	7.00	1.90
Employment	0.70	0.72	0.66	1.47	0.78
Persistence					
Output	0.42	0.46	0.31	0.53	0.48
Consumption	0.63	0.62	0.50	0.49	0.67
Investment	0.35	0.41	0.26	0.27	0.42
Employment	0.33	0.41	0.26	0.25	0.61
Contemporaneous Correlation with Aggregate Output					
Consumption	0.89	0.90	0.90	-0.05	0.66
Investment	0.98	0.98	0.98	0.85	0.83
Employment	0.97	0.98	0.96	0.81	0.68
Contemporaneous Correlation with Aggregate Consumption					
Investment	0.80	0.81	0.79	-0.57	0.60
Employment	0.76	0.79	0.75	-0.63	0.36

Notes:

Business cycle statistics of aggregate output, Y, consumption C, investment I and employment N. N in the model includes the amount of labor used to adjust the firms' capital stocks. All variables are logged and then HP-filtered with a smoothing parameter of 100. The first numbers in a column refer to a simulation of the model over $T = 1500$ periods.

Table 40: AGGREGATE BUSINESS CYCLE STATISTICS - SUR

	Full Model	Full Model SUR	News Model	News Model SUR	RBC Model
Volatility of Output	4.04%	3.11%	4.24%	3.18%	3.17%
Volatility of aggregate variables relative to output volatility					
Consumption	0.32	0.44	0.30	0.38	0.38
Investment	4.71	4.40	4.77	4.45	4.36
Employment	0.74	0.70	0.76	0.71	0.70
Persistence					
Output	0.42	0.21	0.46	0.23	0.31
Consumption	0.66	0.58	0.65	0.54	0.54
Investment	0.34	0.11	0.41	0.17	0.26
Employment	0.33	0.10	0.40	0.17	0.25
Contemporaneous Correlation with Aggregate Output					
Consumption	0.84	0.80	0.85	0.85	0.86
Investment	0.98	0.95	0.98	0.97	0.97
Employment	0.97	0.93	0.98	0.96	0.96
Contemporaneous Correlation with Aggregate Consumption					
Investment	0.73	0.59	0.75	0.71	0.73
Employment	0.69	0.52	0.72	0.67	0.69

Notes: See notes to Table **??**. SUR means a specification of the aggregate driving force, where the off-diagonal elements of ϱ_A in (2) are restricted to zero. $\varrho_A = \begin{pmatrix} 0.4717 & 0 \\ 0 & 0.7335 \end{pmatrix}$ and $\Omega = \begin{pmatrix} 0.0186 & 0.1707 \\ 0.1707 & 0.0026 \end{pmatrix}$. Recall that the autocorrelation for the 'RBC Model' is 0.5259, and the standard deviation of the innovation is 0.0182.