Applied Data Analytics

Statistics — Miscellaneous topics

Index numbers and inflation

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Proportional numbers

One number divided by another, non-zero number.

1. Structural numbers: Divide total into parts

   Imports by good, students by school type, employees by sector

2. Relationship Numbers: Divide two different, but related numbers

   All flows, arithmetic mean, return on investment

3. Measurement numbers: Divide by the same number at different times

   Price of a good in 2024 relative to 2010, GDP in 2000 relative to 1000

Index numbers

- Aggregate multiple numbers into one, typically a (weighted) sum
- Comparisons only make sense for proportional numbers
- There is an OECD-EU JRC handbook on constructing such numbers
- Most common: Inflation
  - Basket of goods & services × associated prices
  - Different baskets → different inflation rates

    CPI, PPI, core inflation, …

Price and quantity data

Good	Period	Quantity	Price
A	0	83	2
	t	73	3
B	0	45	20
	t	60	19
C	0	2500	0.1
	t	1800	0.2

Price and quantity data: notation

Good	Period	Quantity	Price
A	0	$q_{A,0}$	$p_{A,0}$
	t	$q_{A,t}$	$p_{A,t}$
B	0	$q_{B,0}$	$p_{B,0}$
	t	$q_{B,t}$	$p_{B,t}$
C	0	$q_{C,0}$	$p_{C,0}$
	t	$q_{C,t}$	$p_{C,t}$

- Periods $s \in (1, 2, \ldots, t)$
- Goods $i \in \{1, 2, \ldots, n\}$
- $t \cdot n$ quantities $q_{i,s} > 0$
- $t \cdot n$ prices $p_{i,s} > 0$

Composite price index by period

$$P_s = \sum_{i=1}^{n} p_{i,s} \cdot q_{i,s}$$

- For comparing two periods: What to do with Δq?
- Just dividing P_t by P_0 would yield growth in (total) expenditures

$$\frac{\sum_{i=1}^{n} p_{i,t} \cdot q_{i,t}}{\sum_{i=1}^{n} p_{i,0} \cdot q_{i,0}}$$

- So would like to hold q constant

Laspeyres index

$$
\begin{aligned}
P_{t,0}^{\text{Laspeyres}} &= \sum_{i=1}^{n} \frac{p_{i,t}}{p_{i,0}} \cdot g(i,\ p_{1,0}, p_{2,0}, \ldots, p_{n,0},\ q_{1,0}, q_{2,0}, \ldots, q_{n,0}) \\
&= \sum_{i=1}^{n} \frac{p_{i,t}}{p_{i,0}} \cdot \frac{p_{i,0} \cdot q_{i,0}}{\sum_{j=1}^{n} p_{j,0} \cdot q_{j,0}} \\
&= \frac{\sum_{i=1}^{n} p_{i,t} \cdot q_{i,0}}{\sum_{i=1}^{n} p_{i,0} \cdot q_{i,0}}
\end{aligned}
$$

Paasche index

$$P_{t,0}^{\text{Paasche}} = \sum_{i=1}^{n} \frac{p_{i,t}}{p_{i,0}} \cdot g\left(i,\ p_{1,0}, p_{2,0}, \ldots, p_{n,0},\ q_{1,t}, q_{2,t}, \ldots, q_{n,t}\right)$$

$$= \sum_{i=1}^{n} \frac{p_{i,t}}{p_{i,0}} \cdot \frac{p_{i,0} \cdot q_{i,t}}{\sum_{j=1}^{n} p_{j,0} \cdot q_{j,t}}$$

$$= \frac{\sum_{i=1}^{n} p_{i,t} \cdot q_{i,t}}{\sum_{i=1}^{n} p_{i,0} \cdot q_{i,t}}$$