Discussion Paper Series – CRC TR 224

Discussion Paper No. 487
Project B 04

Transparency in Sequential Common-Value Trade

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December 2023

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Support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 is gratefully acknowledged.

Collaborative Research Center Transregio 224 - www.crctr224.de
Rheinische Friedrich-Wilhelms-Universität Bonn - Universität Mannheim

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This version: November 18, 2022

Abstract

We consider the sale of a single indivisible common-value good in a dynamic market where short-lived buyers arrive over time. Buyers observe private signals about the value. The seller is initially uninformed and proposes the terms of trade. As time passes, all players learn about the value from delay in trade. Importantly, this learning process depends on what is made public about buyer-seller interactions. We compare the division of surplus across three transparency regimes that differ with respect to whether buyers observe the seller's past actions or time-on-the-market. When the seller's time-on-the-market but not the seller's past actions are observable, and if buyers' signals are sufficiently rich, then there is no perfect Bayesian equilibrium where the seller extracts the full surplus. In the other two regimes, where buyers observe either everything or nothing about the seller's past actions and time-on-the-market, the seller extracts the full surplus in at least one equilibrium, no matter the signal structure.

Keywords: Common-value, dynamic trade, transparency, learning, division of surplus

*We thank Francesc Dilmé, Daniel Krähmer, Stephan Lauermann, David McAdams, Moritz Meyer-ter-Vehn, Benny Moldovanu, Axel Niemeyer, and Aniko Öry for their comments. Preusser gratefully acknowledges funding by the German Research Foundation (DFG) through CRC TR 224 (Project B04). Preusser is grateful for the hospitality of Yale University, where parts of this paper were written.

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1 Introduction

In many markets, participants learn over time. A buyer who arrives to the market at a late date may learn payoff-relevant information from the fact that earlier buyers bargained with the seller but chose not to buy. The precise inference depends on the information that is made public about past interactions: how many earlier buyers bargained with the seller, and what terms of trade were they offered? We analyze the welfare implications of this kind of transparency in a dynamic market for a common-value good.

Existing work focuses on situations where the seller and no one else is initially informed about the good's value, and where the short-lived buyers propose the terms of trade (for example, Fuchs et al., 2016; Hörner and Vieille, 2009; Kim, 2017). Yet, it is also plausible that that information is revealed only gradually to all market participants, and that the seller proposes the terms of trade.

To motivate these modifications, consider that when the seller proposes they can affect the flow of information to the market, which is an idea that goes back at least to Taylor (1999). Indeed, a buyer's rejecting a low price is more informative than rejecting a high price. The market's transparency affects what later buyers learn from these rejections, and, therefore, the seller's incentives for charging high prices in early periods. We take a step towards understanding this interaction between learning and transparency.

In our model, there is a long-lived seller and a sequence of short-lived buyers. The seller has a single indivisible good for which the buyers have a common value. The value takes one of two values. The seller solicits buyers one-by-one. While the seller is uninformed about the value, the buyers observe informative signals from a discrete signal structure. Conditional on the value, these signals are independent and identically distributed.

In each period, the seller makes a recommendation that identifies for which signal realizations a buyer should buy the object. An intermediary then picks a price that implements the seller's recommendation. We mainly think of this formulation of the trading procedure as a convenient modelling choice that permits us to focus on the flow of information to the market; we give an interpretation once we have presented the model.

We compare three transparency regimes:

(1) The seller's past recommendations and time-on-the-market are both observable.

(2) Past recommendations are unobservable, but time-on-the-market is observable.

(3) Past recommendations and time-on-the-market are both unobservable.

For each regime, we ask whether the seller extracts the full surplus from trade. There are commonly known gains from trade, meaning that the object is traded with certainty in all equilibria. We can thus focus on the way the surplus is divided between the players.

Our main insight is that, in a sense to be made precise momentarily, the seller's ability to extract the full surplus is smallest in the intermediate regime (2). The seller would benefit from passing to either of the two more extreme regimes (1) and (3).

In all regimes, the seller extracts the full surplus if and only if buyers accrue zero information rents. This, in turn, happens if and only if trade is certain to take place with a buyer observing the most optimistic signal. When recommendations and time-on-the-market are both observable, we establish as a benchmark that the seller indeed extracts the full surplus in the unique perfect Bayesian equilibrium. It is as if the seller had commitment power.

Our first main result concerns the game with unobservable past recommendations but observable time-on-the-market. We show that, if the private signals of buyers are sufficiently rich, then the seller's utility is bounded away from the full surplus across all perfect Bayesian equilibria. By sufficiently rich we mean that for each value realization the conditional signal distribution approximates a continuous strictly positive density.

The argument is as follows: If the seller deviates (from a candidate equilibrium strategy) when meeting the first buyer, all later buyers fail to account for this deviation in their beliefs. For certain deviations, later buyers' beliefs will be too optimistic about the value relative to the correct Bayesian posterior—they are fooled into overpaying. The downside for the seller from the deviation is that if the first buyer trades, then this buyer accrues information rents. We show that this downside is dominated in rich signal structures. Using such a deviation, we argue that the unique strategy profile that would let the seller appropriate the full surplus cannot be sustained in equilibrium. (However, an equilibrium exists.)

For our second main result, we turn to the third regime where neither past recommendations nor time-on-the-market are observable. A deviation (from a candidate equilibrium strategy) can now signal that the seller has failed to trade with many buyers. Failing to trade is an indicator of poor value. Thus a deviation entails trading at terms that are quite unfavorable for the seller. Building on this intuition, we show that the seller extracts the full surplus in a sequential equilibrium. The same idea can be used construct sequential equilibria where buyers are left some surplus. In these equilibria (that may or may not leave surplus to buyers), the seller makes a constant recommendation for many early periods. The price is constant across these periods, and the good will trade with overwhelming probability at this constant price. We derive these results in a modified game where the number of buyers is finite but large, and where the seller incurs small costs for soliciting new buyers. Therefore, these sequential equilibria are sustained even in the presence of (small) incentives for trading quickly.

Since our result on the failure of full surplus extraction in the second regime assumes that signals are rich, a natural follow-up is to ask whether the seller would benefit from coarser signal structures. This is indeed the case, in the following sense: if buyers' signals about the value are binary, then buyer surplus is zero in all sequential equilibria of all three regimes.

The paper is organized as follows. In Section 2 we study the model with unobservable recommendations and observable time-on-the-market. The regime where everything is observable is presented as a benchmark in this section. In Section 3, we consider the game where neither the seller's recommendations nor time-on-the-market are observable. Section 4 discusses the literature, and Section 5 concludes. All proofs are in the appendices.

2 Observable time-on-the-market

2.1 Model

We consider a game between a seller, and countably infinitely-many buyers and intermediaries. The seller is long-lived. All other players are short-lived and arrive to the market in a pre-determined order.

Environment The seller (she) owns a single indivisible good which she values at 0. Buyers have a common value for the good that depends on an unobservable state. The state has two possible realizations, ℓ and h, with associated values v_ℓ and v_h. We assume $0 < v_\ell < v_h$, and so it is common knowledge that there are gains from trade. Let $\alpha_{\omega,0} \in (0,1)$ be the common prior that the state is $\omega \in \{\ell, h\}$. It will frequently be more convenient to represent beliefs via the likelihood ratio of state h vs. ℓ. We denote the prior likelihood ratio by $\pi_0 = \alpha_{h,0}/\alpha_{\ell,0}$.[1]

At the start of the game, the seller is uninformed about the state. Each buyer (he) is endowed with a private signal from a finite set S. Conditional on state ω, the signals of (each finite subset of) the buyers are independent draws from a distribution f_ω that has support S.

Since the state is binary, it is without loss to order signals according to their likelihood ratios; that is, we assume

$$\forall_{s,s' \in S} \quad s < s' \Rightarrow \frac{f_h(s)}{f_\ell(s)} < \frac{f_h(s')}{f_\ell(s')}. \tag{MLRP}$$

Given $s \in S$ and $\pi \in (0, \infty)$, let

$$\hat{v}(s, \pi) = v_\ell + (v_h - v_\ell)\frac{\pi \frac{f_h(s)}{f_\ell(s)}}{\pi \frac{f_h(s)}{f_\ell(s)} + 1}. \tag{2.1}$$

The value is $\hat{v}(s, \pi)$ is the posterior value for a buyer who observes a signal realization s starting at a belief π. The prior value of the good, termed the *full surplus*, is denoted $\hat{v}_0$ and given by

$$\hat{v}_0 = v_\ell + (v_h - v_\ell)\frac{\pi_0}{\pi_0 + 1}. \tag{2.2}$$

Trading protocol The game unfolds in discrete time, indexed by $\mathbb{N} = \{1, 2, \ldots\}$. In period i, the seller, buyer i, and intermediary i are active. First, the seller picks an element σ_i of S. We think of this as the seller recommending that buyers with a signal above σ_i buy the object. Accordingly, we refer to this as the recommended cutoff. The recommendation is observed by intermediary i, who then posts a price p_i. Next, buyer i arrives to the market with probability $\lambda \in (0, 1)$. If he arrives, he learns σ_i, p_i, and the realization of his private signal. He then decides whether to buy at p_i. If he does, the object is traded and the game ends. If buyer i does not arrive to the market or does not trade, the game moves to the next period.

[1] So, a belief of 0 means that the state is sure to be ℓ. A belief of ∞ means the state is sure to be h. All relevant Bayesian posteriors in our model will lead to beliefs in $(0, \infty)$.

Note that the seller's time-on-the-market is observable in the sense that each buyer i is in the market in period i or not at all.

The seller's payoff is the price at which the object is traded, if at all. Buyer i's payoff in state ω is $v_\omega - p_i$ if he trades; else his payoff is 0. As for the intermediaries we assume the following: If the seller recommends σ_i and buyer i arrives with a signal s_i such that $\sigma_i \leq s_i$ but buyer i ends up not buying the good, then the payoff of intermediary i is $-\infty$. In all other cases, the intermediary's payoff is p_i. (We interpret the intermediaries further below.) The solution concept is perfect Bayesian equilibrium.

Equilibrium prices Buyer i's beliefs about the state depend on his private signal s_i, the seller's recommendation σ_i, and the fact that he finds the good to not have been sold in previous periods;[2] we comment in more detail below what exactly one can infer from the recommendation. Let $\pi_i(\sigma_i)$ denote i's belief (expressed as the likelihood ratio of h vs. ℓ) after learning that the game has reached round i and learning the seller's action σ_i, but before learning his private signal s_i. We will refer to π simply as buyers' beliefs. This is a slight abuse of language as π does not include a buyer's inference from his private signal, but no confusion should arise.

Once buyer i learns s_i, his valuation for the good updates to $\hat{v}(s_i, \pi_i(\sigma_i))$. Thus he is willing to accept a price p_i if and only if

$$\hat{v}(s_i, \pi_i(\sigma_i)) \geq p_i.$$

The MLRP implies that $\hat{v}(s_i, \pi_i(\sigma_i))$ is increasing in s_i. In equilibrium, the intermediary, acting according to his preferences, sets the price as large as possible subject to the constraint that buyer i accepts if s_i is weakly above σ_i, Hence the intermediary sets $p_i = \hat{v}(\sigma_i, \pi_i(\sigma_i))$ whenever the principal recommends σ_i. Buyer i will accept after a recommendation of σ_i if and only if i's private signal is weakly above σ_i.

These observations let us simplify the description of equilibrium: It suffices to specify the recommendations of the seller, and buyers' beliefs π.

Formally, the set of pure strategies of the seller is the set S^∞ of sequences in S.

[2] The belief could in principle also depend on the intermediary's price. Note however, that the buyer knows as much as the intermediary about the history, and hence we can safely omit this dependence.

The set of mixed strategies of the seller is the set $\Delta(S^\infty)$ of distributions over S^∞.[3] Generic elements of S^∞ and $\Delta(S^\infty)$, respectively, are denoted σ and μ, respectively. Buyers' beliefs are given by a function $\pi\colon \mathbb{N} \times S \to [0, \infty]$. Given (μ, π), we denote the seller's utility by $V(\mu, \pi)$. (See the appendix for general formulas for the seller's expected utility and buyers' posteriors.)

Definition 1. A pair (μ, π) is an **equilibrium** if μ maximizes $V(\cdot, \pi)$ across $\Delta(S^\infty)$, and π satisfies all of the following:

- For all $s \in S$, we have $\pi_1(s) = \pi_0$.
- For all $i \geq 2$ and all $s \in S$, if s is played by μ with non-zero probability in period i (meaning $\mu(\{\sigma \in S^\infty : \sigma_i = s\}) > 0$), then $\pi_i(s)$ is derived from μ via Bayes' rule.

The first condition on the beliefs requires that the seller cannot signal what she does not: buyer 1, who is the first to interact with the seller, draws no inference from the seller's recommded cutoff in period 1 as the seller is initially uninformed about the state. The second condition states that all other beliefs are derived from Bayes' rule where possible: all periods i are reached with non-zero probability (since with non-zero probability buyer i is the first to arrive to the market), and hence $\pi_i(s)$ can be derived from Bayes' rule if and only if μ plays s with non-zero probability in period i.

What can buyers in periods 2 and onwards infer from past play and the seller's cutoff? Along the equilibrium path, trade happens if and only if the active buyer's signal is weakly greater than the cutoff. When buyer i arrives to the market and finds that the good has not been sold, he therefore infers that all earlier buyers who arrived to the market drew signals strictly below the cutoffs recommended in the past. The inference from this event depends on the values of these cutoffs. If the seller's strategy is mixed, learning her current cutoff lets buyer i draw inference about past cutoffs, and hence about past buyers' signals, and hence about the value of the good.

Sequential equilibria Some of our results concern sequential equilibria.[4]

[3] The finite set S has the discrete metric, and S^∞ has the product metric. A distribution over S^∞ means a Borel probability-measure on S^∞.

[4] The definition is at a slight abuse of language as we do not consider perturbations of the buyers' or intermediaries' strategies.

Definition 2. A strategy of the seller is **fully mixed** if in all periods it recommends all cutoffs with non-zero probability.

An equilibrium (μ, π) is a **sequential equilibrium** if there is a sequence $\{\mu_k\}_{k\in\mathbb{N}}$ of fully mixed strategies and a sequence $\{\pi_k\}_{k\in\mathbb{N}}$ of beliefs satisfying both of the following:

(1) The sequence $\{\mu_k\}_{k\in\mathbb{N}}$ weak-$*$ converges to μ, and the sequence $\{\pi_k\}_{k\in\mathbb{N}}$ converges to π pointwise.

(2) For all k, the beliefs π_k are derived from μ_k via Bayes' rule.

Lemma 2.1. *There exists a sequential equilibrium.*

Interpreting the intermediaries We think of the seller as recommending the cutoff above which a buyer should buy the good. In order to actually implement this cutoff via a price, the seller relies on intermediaries. This could be because intermediaries have greater expertise than the seller for interacting with buyers. For concreteness, suppose the seller writes a contract that rewards the intermediary with a share ρ of the price, provided the price is low enough to implement the seller's recommendation. If there are multiple intermediaries in each period, they compete the share ρ down to 0.

That said, we mostly view the intermediaries as a convenient modelling tool. The advantage of our formulation is that we can focus on the cutoff at which trade happens. In each period, the cutoff determines the information rents that the present buyer accrues in the event of trade, and it determines what the market learns about the value in the event of no-trade. Hence the cutoffs are key to determining the division of surplus.

2.2 The full surplus is an upper bound

We begin our analysis by showing that the full surplus is an upper bound on the seller's equilibrium expected utility. For expositional purposes, let us assume $\lambda = 1$, meaning that buyers are sure to arrive to the market (but the results are stated for arbitrary $\lambda \in (0, 1)$). For $\lambda = 1$, the game reaches period i if and only if all preceding buyers had signals strictly below the seller's cutoff. Let $\tilde{s}_i$ denote buyer i's random signal, and let $\tilde{\sigma}_i$ denote the (possibly random) cutoff of the seller in period i. We therefore identify the event

$$\{\tilde{s}_1 < \tilde{\sigma}_1, \ldots, \tilde{s}_{i-1} < \tilde{\sigma}_{i-1}, \tilde{s}_i \geq \tilde{\sigma}_i\}$$

with the event that buyer i ends up buying the object.

Consider an equilibrium (μ, π). The seller's strategy μ together with the distribution of states and signals induces some joint distribution of recommended cutoffs, states, and signals. We denote the probability- and expectation-operators with respect to this distribution by $\mathbb{P}$ and $\mathbb{E}$.

Recall that if buyer i buys at a cutoff σ_i, he will pay $\hat{v}_i(\sigma_i, \pi_i(\sigma_i))$. This is his posterior valuation conditional on the game reaching period i, the seller recommending σ_i, and his signal being *equal* to σ_i; let us denote this valuation by

$$\mathbb{E}\left[v | \tilde{s}_1 < \tilde{\sigma}_1, \ldots, \tilde{s}_{i-1} < \tilde{\sigma}_{i-1}, \tilde{s}_i = \sigma_i\right]$$

The seller's equilibrium expected utility is therefore

$$\sum_{i=1}^{\infty} \Bigg(\mathbb{P}\left(\tilde{s}_1 < \tilde{\sigma}_1, \ldots, \tilde{s}_{i-1} < \tilde{\sigma}_{i-1}, \tilde{s}_i \geq \tilde{\sigma}_i\right) \times \mathbb{E}\left[v | \tilde{s}_1 < \tilde{\sigma}_1, \ldots, \tilde{s}_{i-1} < \tilde{\sigma}_{i-1}, \tilde{s}_i = \tilde{\sigma}_i\right] \Bigg). \tag{2.3}$$

The MLRP implies that this is no greater than

$$\sum_{i=1}^{\infty} \Bigg(\mathbb{P}\left(\tilde{s}_1 < \tilde{\sigma}_1, \ldots, \tilde{s}_{i-1} < \tilde{\sigma}_{i-1}, \tilde{s}_i \geq \tilde{\sigma}_i\right) \times \mathbb{E}\left[v | \tilde{s}_1 < \tilde{\sigma}_1, \ldots, \tilde{s}_{i-1} < \tilde{\sigma}_{i-1}, \tilde{s}_i \geq \tilde{\sigma}_i\right] \Bigg). \tag{2.4}$$

By iterated expectations, the sum in (2.4) is nothing but the prior value of the good, namely the full surplus $\hat{v}_0$. In fact, the MLRP implies that (2.3) is strictly less than (2.4) if, with non-zero μ-probability, a period is reached where the cutoff is strictly below the largest signal in S. That is, the seller leaves information rents unless she is certain to trade with the highest possible signal.

To state this formally, let us denote by $\bar{s}$ the largest signal in S. Let $\bar{\sigma}$ be the sequence of cutoffs that is constantly equal to $\bar{s}$.

Lemma 2.2. *In all equilibria, the seller's utility is at most $\hat{v}_0$. If in an equilibirum the seller's utility is $\hat{v}_0$, then in this equilibrium the seller's strategy is $\bar{\sigma}$.*

Note that even if the seller's recommendation is constantly equal to $\bar{s}$, the price is strictly decreasing over time. Indeed, in this case the price in period i is given by

$$\mathbb{E}\left[v \middle| \tilde{s}_1 < \bar{s}, \ldots, \tilde{s}_{i-1} < \bar{s}, \tilde{s}_i = \bar{s}\right].$$

As i increases, this expectation conditions on a larger number of signals being below $\bar{s}$, which depresses beliefs.[5] Put differently, as the good is not being sold, the intermediaries are decreasing prices at just the right rate to keep buyer with signals equal to $\bar{s}$ indifferent between buying.

Lemma 2.2 does not say that the strategy $\bar{\sigma}$ is actually sustained in equilibrium. Before investigating whether this can happen, let us make good on discussing the promised benchmark where past recommendation are observable.

2.3 Full surplus extraction with observable recommendations

Suppose for a moment that the seller's recommended cutoffs were observable. We claim that in this case she can extract the full surplus in equilibrium by playing the pure strategy $\bar{\sigma}$. In fact, this is the only equilibrium. To see this, suppose the seller uses some pure strategy σ.[6] Since the seller's actions are observable, buyer i makes the correct inference from play; that is, his belief agrees with the Bayesian posterior induced by σ. The price which buyer i is offered must therefore correctly account for the Bayesian inference from reaching period i. Since this is true for all periods i, the

[5]Precisely, the conditional expectation reads

$$v_\ell + (v_h - v_\ell)\frac{\pi_0 \frac{f_h(\bar{s})}{f_\ell(\bar{s})}\left(\frac{1-\lambda f_h(\bar{s})}{1-\lambda f_\ell(\bar{s})}\right)^{i-1}}{\pi_0 \frac{f_h(\bar{s})}{f_\ell(\bar{s})}\left(\frac{1-\lambda f_h(\bar{s})}{1-\lambda f_\ell(\bar{s})}\right)^{i-1} + 1}.$$

which, by the MLRP, is strictly decreasing in i.

[6]Since her actions are observable, pure strategies are without loss.

seller's utility from σ is (assuming $\lambda = 1$)

$$\sum_{i=1}^{\infty} \Bigg(\mathbb{P}\left(\tilde{s}_1 < \sigma_1, \ldots, \tilde{s}_{i-1} < \sigma_{i-1}, \tilde{s}_i \geq \sigma_i\right) \\ \times \mathbb{E}\left[v | \tilde{s}_1 < \sigma_1, \ldots, \tilde{s}_{i-1} < \sigma_{i-1}, \tilde{s}_i = \sigma_i\right] \Bigg)$$

The arguments from the previous section imply that, if $\sigma \neq \bar{\sigma}$, then this utility is strictly less than $\hat{v}_0$, and hence strictly less than the utility from $\bar{\sigma}$.

The argument from the previous paragraph does not apply in the game with unobservable recommendations since, following a deviation in some period, later buyers do not revise their beliefs. Exploiting these incorrect beliefs, we show the seller may profitably deviate from $\bar{\sigma}$ by exploiting these incorrect beliefs to obtain a utility strictly above the prior value $\hat{v}_0$.

2.4 No full surplus with rich signals

In this section, we show that if signals are sufficiently rich, then the seller cannot extract the full surplus in equilibrium. We first make precise what we mean by a rich signal structure. Fixing a pair of cdfs (G_h, G_ℓ) on $[0, 1]$ and an integer k, consider S_k, $f_{\ell,k}$ and $f_{h,k}$ defined as follows:

$$S_k = \left\{0, \frac{1}{k}, \ldots, 1 - \frac{1}{k}\right\}$$
$$\forall_{s \in S_k}, \quad f_{\omega,k}(s) = G_\omega\left(s + \frac{1}{k}\right) - G_\omega(s).$$

We say the sequence $\{(S_k, f_{h,k}, f_{\ell,k})\}_{k \in \mathbb{N}}$ *converges* to (G_h, G_ℓ).

Our result asserts that if (G_h, G_ℓ) admit well-behaved densities, then, fixing a signal structure far enough along the sequence, the seller cannot extract the full surplus. Note that the full surplus $\hat{v}_0 = v_\ell + (v_h - v_\ell)\frac{\pi_0}{\pi_0 + 1}$ does not depend on the signal structure.

Proposition 2.3. *Let (G_h, G_ℓ) be a pair of cdfs on $[0, 1]$. Let $\{(S_k, f_{h,k}, f_{\ell,k})\}_{k \in \mathbb{N}}$ be a sequence of finite signal structures converging to (G_h, G_ℓ).*

If G_h and G_ℓ admit continuous and strictly positive densities g_h and g_ℓ on $[0, 1]$ such that $\frac{g_h}{g_\ell}$ is strictly increasing, then the following holds for all except finitely many

k: If the signal structure is given by $(S_k, f_{h,k}, f_{\ell,k})$, then the seller's utility is bounded away from $\hat{v}_0$ across all equilibria.

For the proof, it suffices to show that the pure strategy that always plays the largest signal fails to be an equilibrium (Lemma 2.2). For expositional purposes, let $\lambda = 1$. Suppressing the dependence on k, let $\bar{s} = \bar{s}_k$ denote the largest signal in the k'th signal structure.

Suppose towards a contradiction that there is an equilibrium where the seller's utility is $\hat{v}_0$ and she plays $\bar{s}$ in all periods. We consider a one-time deviation in period 1 to a cutoff strictly below $\bar{s}$. Let $s^\circ = s^\circ_k$ denote this cutoff (where we again suppress the dependence on k). The deviation will have two effects, with opposing implications for the seller's utility. The upside from the deviation is that if buyer 1 does not end up trading, then all later buyers will hold incorrect beliefs. In particular, since $s^\circ < \bar{s}$, rejecting a cutoff of s° is a *stronger* signal in favor of the *bad* state ℓ than rejecting $\bar{s}$. Therefore, all later buyers will hold a belief that is too optimistic; their willingness to pay will be too high relative to the true Bayesian posterior. The downside from the deviation is that if buyer 1 has a private signal strictly above s°, he will trade and accrue information rents.

Let us spell this out in more detail. Since buyer 1's belief does not react to the seller's action in round 1, the contribution from buyer 1 to the utility from the deviation is

$$\mathbb{P}(\tilde{s}_1 \geq s^\circ)\mathbb{E}[v|\tilde{s}_1 = s^\circ].$$

Now consider buyer $i > 1$. The probability that he will trade under the deviation is

$$\mathbb{P}\left(\tilde{s}_1 < s^\circ, \tilde{s}_2 < \bar{s}, \ldots, \tilde{s}_{i-1} < \bar{s}, \tilde{s}_i = \bar{s}\right)$$

Buyer i receives his on-path recommendation, and hence his belief equals his on-path belief. Since the candidate equilibrium has the seller recommend $\bar{s}$ in all periods, the price that buyer i pays, if he trades, is

$$\mathbb{E}\left[v|\tilde{s}_1 < \bar{s}, \tilde{s}_2 < \bar{s}, \ldots, \tilde{s}_{i-1} < \bar{s}, \tilde{s}_i = \bar{s}\right].$$

The seller's utility from the deviation is therefore

$$\begin{aligned}
&\mathbb{P}(\tilde{s}_1 \geq s^\circ)\mathbb{E}[v|\tilde{s}_1 = s^\circ] \\
&+\sum_{i=2}^{\infty}\Bigg(\mathbb{P}\left(\tilde{s}_1 < s^\circ, \tilde{s}_2 < \bar{s}, \dots, \tilde{s}_{i-1} < \bar{s}, \tilde{s}_i = \bar{s}\right) \\
&\qquad\qquad \times \mathbb{E}\left[v|\tilde{s}_1 < \bar{s}, \tilde{s}_2 < \bar{s}, \dots, \tilde{s}_{i-1} < \bar{s}, \tilde{s}_i = \bar{s}\right]\Bigg).
\end{aligned}$$

Let us compare this to the full surplus $\hat{v}_0$ (which is the seller's utility from constantly recommending $\bar{s}$). By iterated expectations, we may write $\hat{v}_0$ as

$$\begin{aligned}
&\mathbb{P}(\tilde{s}_1 \geq s^\circ)\mathbb{E}[v|\tilde{s}_1 \geq s^\circ] \\
&+\sum_{i=2}^{\infty}\Bigg(\mathbb{P}\left(\tilde{s}_1 < s^\circ, \tilde{s}_2 < \bar{s}, \dots, \tilde{s}_{i-1} < \bar{s}, \tilde{s}_i = \bar{s}\right) \\
&\qquad\qquad \times \mathbb{E}\left[v|\tilde{s}_1 < s^\circ, \tilde{s}_2 < \bar{s}, \dots, \tilde{s}_{i-1} < \bar{s}, \tilde{s}_i = \bar{s}\right]\Bigg).
\end{aligned}$$

Thus the deviation is profitable if and only if

$$\begin{aligned}
&\mathbb{P}(\tilde{s}_1 \geq s^\circ)\left(\mathbb{E}[v|\tilde{s}_1 = s^\circ] - \mathbb{E}[v|\tilde{s}_1 \geq s^\circ]\right) \\
&+\sum_{i=2}^{\infty}\Bigg(\mathbb{P}\left(\tilde{s}_1 < s^\circ, \tilde{s}_2 < \bar{s}, \dots, \tilde{s}_{i-1} < \bar{s}, \tilde{s}_i = \bar{s}\right) \\
&\qquad \times \Big(\mathbb{E}\left[v|\tilde{s}_1 < \bar{s}, \tilde{s}_2 < \bar{s}, \dots, \tilde{s}_{i-1} < \bar{s}, \tilde{s}_i = \bar{s}\right] \\
&\qquad\qquad - \mathbb{E}\left[v|\tilde{s}_1 < s^\circ, \tilde{s}_2 < \bar{s}, \dots, \tilde{s}_{i-1} < \bar{s}, \tilde{s}_i = \bar{s}\right]\Big)\Bigg)
\end{aligned}$$

is strictly positive. The difference

$$\mathbb{E}[v|\tilde{s}_1 = s^\circ] - \mathbb{E}[v|\tilde{s}_1 \geq s^\circ]$$

is strictly negative, as we infer from the MLRP; this is the information rent left to buyer 1. Each term inside the infinite sum, however, is strictly positive. To see this, note that

$$\mathbb{E}\left[v|\tilde{s}_1 < \bar{s}, \tilde{s}_2 < \bar{s}, \dots, \tilde{s}_{i-1} < \bar{s}, \tilde{s}_i = \bar{s}\right]$$

conditions on $(i-1)$-many buyers failing to trade at $\bar{s}$. However,

$$\mathbb{E}\left[v|\tilde{s}_1 < s^\circ, \tilde{s}_2 < \bar{s}, \ldots, \tilde{s}_{i-1} < \bar{s}, \tilde{s}_i = \bar{s}\right]$$

conditions on one buyer failing to trade at s° and $(i-2)$-many buyers failing to trade at $\bar{s}$. Since not trading at s° is a stronger signal for the bad state than not trading at $\bar{s}$, each term inside the infinite sum is strictly positive.

The seller thus benefits from the deviation if late buyers' wrong beliefs outweigh buyer 1's information rents. The proof of Proposition 2.3 shows that this happens whenever the signal structure is sufficiently rich. If we choose $s^\circ = s^\circ_k$ ever closer to 1 as $k \to \infty$, the fact that the likelihood ratio $\frac{g_h}{g_\ell}$ is continuous implies that both the loss due information rents as well as the gain due to incorrect beliefs vanish. By using that the likelihood ratio is bounded at the top (which is implied by the fact that the densities are continuous and strictly positive), we show that the loss vanishes more rapidly than the gain for a suitable choice of s°_k. In particular, this is the case when s°_k converges an order of magnitude more slowly to 1 than $\bar{s} = \bar{s}_k$.

2.5 Full surplus with binary signals

Since Proposition 2.3 concerns rich signals, a natural follow-up question asks how the surplus is divided when signals are coarse. The next result shows that when signals are binary we reach a conclusion starkly different from Proposition 2.3.

Proposition 2.4. *Let signals be binary, meaning $|S| = 2$. If (μ, π) is a sequential equilibrium, then the seller's strategy is the pure strategy $\bar{\sigma}$, and her expected utility is the full surplus $\hat{v}_0$.*

The key observation is that, for binary signals, no strategy induces more pessimistic beliefs than constantly playing the highest cutoff $\bar{s}$. Let $\underline{s}$ denote the smallest signal, which here simply means the only signal different from $\bar{s}$. A recommendation of $\underline{s}$ is accepted whenever a buyer arrives to the market. Failing to trade at $\underline{s}$ therefore reveals that no buyer arrived to the market. Since this event contains no information about the value, the belief remains unchanged. Conversely, whenever $\bar{s}$ does not lead to a trade, the posterior that the state is h decreases.

For a sequential equilibrium, the observation from the previous paragraph implies that the posterior induced by $\bar{\sigma}$ is also a lower bound on buyers' off-path beliefs.

Hence a lower bound on equilibrium utility is given by the utility from deviating to $\bar{\sigma}$ and forming the induced prices using the beliefs induced by $\bar{\sigma}$. But this lower utility is nothing but $\hat{v}_0$, as we infer from the discussion in Section 2.2.[7]

3 Unobservable time-on-the-market

3.1 Model

In this section, we consider another game. Its defining property is that buyers observe neither the seller's past recommendations nor the seller's time-on-the-market. That is, relative to the game of the previous section, a buyer is now also unaware of the label of the period in which he is asked to make a move.

The number n of buyers and intermediaries is now finite. At the beginning of the game, Nature picks a permutation of $\{1, \ldots, n\}$ according to the uniform distribution. The realized permutation is not observed by any player, and it determines the order in which buyers and intermediaries arrive to the market. When asked to make a move, each buyer observes his private signal, the intermediary's price, and the seller's recommendation. Each intermediary observes the seller's current recommendation. When in period i the seller recommends a cutoff σ_i, the buyer's posterior belief (expressed as the likelihood ratio of h vs. ℓ) is denoted $\pi^{\emptyset}(\sigma_i)$.[8] In the same situation, the intermediary will find it optimal to choose a price of $\hat{v}(\sigma_i, \pi^{\emptyset}(\sigma_i))$. As before, a buyer finds it optimal to accept σ_i at a private signal s if and only if s is weakly greater than σ_i.

On the seller's side, we now assume that she incurs a cost c whenever the game moves to the next period. This cost can be interpreted as costs for soliciting new buyers, and we assume $c \in [0, \lambda v_\ell]$.[9]

A mixed strategy of the seller is now a distribution μ over the set S^n of finite

[7]The argument sketched here uses the assumption that arrivals to the market are probabilistic, meaning $\lambda \in (0, 1)$. Suppose that arrivals are certain, $\lambda = 1$. Playing the lowest signal now means trading with probability one. In a sequential equilibrium, the beliefs of buyers who are reached with probability zero along the path of play must therefore equal the beliefs induced by $\bar{\sigma}$. A similar argument thus shows that the deviation to $\bar{\sigma}$ must still yield $\hat{v}_0$.

[8]Mnemonically, the superscript $\emptyset$ indicates that buyers know neither the seller's past actions nor her time-on-the-market.

[9]The assumption that c is in $[0, \lambda v_\ell]$ implies that the seller will always find it optimal to keep searching until the pool of buyers is exhausted. Specifically, she can always recommend the lowest signal as a cutoff, leading to trade at a price of at least v_ℓ when a buyer arrives with probability λ.

cutoff sequences. Buyers' beliefs are represented by $\pi^{\emptyset}\colon S \to [0,\infty]$. The seller's profit from this pair is denoted $V^{\emptyset}(\mu,\pi^{\emptyset},n,c)$. (The appendix presents formulas for the seller's utility and buyers' Bayesian posteriors.) Let $\Gamma^{\emptyset}(n,c)$ denote the game described here.

Definition 3. A pair $(\mu,\pi^{\emptyset})$ is an **equilibrium** of $\Gamma^{\emptyset}(n,c)$ if μ is a maximizer of $V^{\emptyset}(\cdot,\pi^{\emptyset},n,c)$, and $\pi^{\emptyset}$ satisfies the following: For all $s \in S$, if s is played by μ with non-zero probability in some period (meaning $\sum_{i=1}^{n} \mu(\{\sigma \in S^n \colon \sigma_i = s\}) > 0$), then $\pi^{\emptyset}(s)$ is derived from μ via Bayes' rule.

A strategy of the seller is **fully mixed** if for all cutoffs there is at least one period in which the cutoff is played with non-zero probability; that is, all $s \in S$ satisfy $\sum_{i=1}^{n} \mu(\{\sigma \in S^n \colon \sigma_i = s\}) > 0$.[10]

An equilibrium $(\mu,\pi^{\emptyset})$ is a **sequential equilibrium** if there is a sequence $\{\mu_k\}_{k\in\mathbb{N}}$ of fully mixed strategies and a sequence $\{\pi^{\emptyset}_k\}_{k\in\mathbb{N}}$ of beliefs satisfying both of the following.

(1) The sequence $\{\mu_k\}_{k\in\mathbb{N}}$ converges to μ, and the sequence $\{\pi^{\emptyset}_k\}_{k\in\mathbb{N}}$ converges to $\pi^{\emptyset}$.[11]
(2) For all k, the beliefs $\pi^{\emptyset}_k$ are derived from μ_k via Bayes' rule.

Lemma 3.1. *For all $n \in \mathbb{N}$ and $c \in [0,\lambda v_\ell]$ there exists a sequential equilibrium of $\Gamma^{\emptyset}(n,c)$.*

3.2 Signaling calendar time

Our aim in this section is to show that, along a certain sequence of sequential equilibria, the seller can extract the full surplus as the number of buyers grows large and solicitation costs vanish. However, not all sequences of sequential equilibria have this property. To state the result formally, given $s \in S$, let $\bar{F}_\omega(s)$ denote the probability of observing a signal weakly above s. Recall also that $\underline{s}$ denotes the smallest signal.

Proposition 3.2. *Let $s^* \in S \setminus \{\underline{s}\}$. Let $\{c_n\}_{n\in\mathbb{N}}$ be a sequence in $[0,\lambda v_\ell]$ converging to 0. For all $n \in \mathbb{N}$, there exists μ_n, $\pi^{\emptyset}_n$, and an integer j_n such that the sequence $\{(\mu_n,\pi^{\emptyset}_n,j_n)\}_{n\in\mathbb{N}}$ satisfies all of the following:*

[10]Notice that this notion of a fully mixed strategy differs from the regime with observable time-on-the-market.

[11]All strategies and beliefs are viewed as elements of Euclidean space.

(1) *For all but finitely many $n \in \mathbb{N}$, the pair $(\mu_n, \pi_n^{\emptyset})$ is a sequential equilibrium of $\Gamma^{\emptyset}(n, c_n)$.*
(2) *For all $n \in \mathbb{N}$, the seller using μ_n plays s^* in the first j_n rounds with probability one; that is, we have $\mu_n(\{\sigma \in S^n \colon (\sigma_1, \ldots, \sigma_{j_n}) = (s^*, \ldots, s^*)\}) = 1$.*
(3) *The sequence $\{j_n\}_{n \in \mathbb{N}}$ diverges to ∞.*
(4) *Along the sequence, the good is traded with probability converging to 1. The seller's expected utility and the price at which the good is traded converge almost surely to*

$$v_\ell + (v_h - v_\ell) \frac{\pi \frac{f_h(s^*)}{f_\ell(s^*)} \frac{\bar{F}_\ell(s^*)}{\bar{F}_h(s^*)}}{\pi \frac{f_h(s^*)}{f_\ell(s^*)} \frac{\bar{F}_\ell(s^*)}{\bar{F}_h(s^*)} + 1}. \tag{3.1}$$

In the special case $s^* = \bar{s}$ we have $\bar{F}_\omega(\bar{s}) = f_\omega(\bar{s})$ for all ω, and hence the term in (3.1) equals $\hat{v}_0$. That is, the seller gets the full surplus along this sequence of equilibria. Whenever s^* is different from $\bar{s}$, however, the (MLRP) implies

$$\frac{f_h(s^*)}{f_\ell(s^*)} \frac{\bar{F}_\ell(s^*)}{\bar{F}_h(s^*)} < 1.$$

Thus, for s^* different from $\bar{s}$, the seller's equilibrium utility converges to a value strictly below $\hat{v}_0$.

The basic observation that we use for the proof of Proposition 3.2 is that the seller's recommendation contains information about her time-on-the-market. Let us sketch the proof idea for the case $s^* = \bar{s}$. Suppose for a moment that for some integer j the seller uses a pure strategy σ that recommends $\bar{s}$ in all of the first j periods, and never thereafter. When the seller recommends to a buyer an on-path cutoff different from $\bar{s}$, this reveals that the seller has unsuccessfully tried to sell the object for at least j rounds. Since failing to trade the object depresses beliefs, picking a cutoff different from $\bar{s}$ thus leads to a price approximately equal to v_ℓ, provided j is sufficiently large. Let us compare to this to the price from recommending $\bar{s}$. Since $\bar{s}$ is on-path under σ, a buyer's belief $\pi^{\emptyset}(\bar{s}, \sigma)$ after arriving to the market and being recommended $\bar{s}$ can be computed via Bayes' rule. This belief is given by

$$\pi^{\emptyset}(\bar{s},\sigma)=\pi_0\frac{\sum_{i=1}^{n}\frac{1}{n}\mathbf{1}_{(\sigma_i=\bar{s})}(1-\lambda f_h(\bar{s}))^{i-1}}{\sum_{i=1}^{n}\frac{1}{n}\mathbf{1}_{(\sigma_i=\bar{s})}(1-\lambda f_\ell(\bar{s}))^{i-1}}=\pi_0\frac{\sum_{i=1}^{j}(1-\lambda f_h(\bar{s}))^{i-1}}{\sum_{i=1}^{j}(1-\lambda f_\ell(\bar{s}))^{i-1}}.$$

Evaluating the geometric sums shows that, for large j, this belief approximately equals $\pi_0\frac{f_\ell(\bar{s})}{f_h(\bar{s})}$. The price $\hat{v}(\bar{s},\pi^{\emptyset}(\bar{s},\sigma))$ after $\bar{s}$ equals the posterior valuation conditional on a private signal of $\bar{s}$ and conditional on arriving to the market and being recommended $\bar{s}$. Hence this price approximately equals

$$v_\ell+(v_h-v_\ell)\frac{\pi_0\frac{f_\ell(\bar{s})}{f_h(\bar{s})}\frac{f_h(\bar{s})}{f_\ell(\bar{s})}}{\pi_0\frac{f_\ell(\bar{s})}{f_h(\bar{s})}\frac{f_h(\bar{s})}{f_\ell(\bar{s})}+1}=v_\ell+(v_h-v_\ell)\frac{\pi_0}{\pi_0+1}.$$

This is nothing but the full surplus $\hat{v}_0$.

In summary, a cutoff of $\bar{s}$ yields a price of $\hat{v}_0$, whereas deviations from $\bar{s}$ yields a price of v_ℓ (approximately, when j is large). This suggests that the seller's offering $\bar{s}$ for a large number periods j can actually be sustained in equilibrium, leading to trade with overwhelming probability at $\hat{v}_0$, and hence to the seller's extracting the full surplus. A complication in this argument is that the seller also incurs costs for solicitng new buyers and that the pool of buyers is finite. Since $\bar{s}$ leads to the smallest per-period probability of trade, the seller has an incentive to deviate from $\bar{s}$ to save on costs or, when the pool of buyers is almost exhausted, to ensure a last minute sale of the object. Hence we have to consider the possibility that the seller plays signals other than $\bar{s}$ along the equilibrium path. This complicates the construction of equilibrium; care has to be taken to let j (the number of initial periods in which the seller constantly recommends $\bar{s}$) diverge to ∞, but not too rapidly.

The proof for general $s^* \in S \setminus \{\underline{s}\}$ is similar. To understand why, suppose the seller's strategy is to play s^* for the first j rounds. As long as s^* is not the lowest signal $\underline{s}$, failing to trade at s^* depresses beliefs.[12] Hence the earlier reasoning implies that deviating from s^* leads to a price approximately equal to v_ℓ, while s^* leads to a strictly higher price (namely the price in (3.1)).

[12] As remarked at an earlier point, when $\underline{s}$ does not lead to a trade in some round, the Bayesian inference is that no buyer arrived to the market in that round. Non-arrivals reveal nothing about the state of the world, and hence playing $\underline{s}$ for many rounds will not depress beliefs towards zero.

3.3 Full surplus with binary signals

Proposition 3.2 implies that the players may fail to coordinate on a seller-optimal equilibrium whenever there are at least three signals. We conclude this section by addressing the case of binary signals. In parallel to Proposition 2.4, we find that the seller extracts the full surplus along all sequences of sequential equilibria.

Proposition 3.3. *Let signals be binary, meaning $|S| = 2$. Let $\{c_n\}_{n\in\mathbb{N}}$ be a sequence in $[0, \lambda v_\ell]$ converging to 0. For all $n \in \mathbb{N}$, let $(\mu_n, \pi_n^\emptyset)$ be a sequential equilibrium of $\Gamma^\emptyset(n, c_n)$.*[13] *As $n \to \infty$, the seller's utility along the sequence of equilibria converges to $\hat{v}_0$.*

Our proof uses the same ideas as our proof of Proposition 2.4. Namely, with binary signals, the belief $\pi_n^\emptyset(\bar{s})$ must be bounded below by the posterior induced by $\bar{\sigma}$ (verifying this in the present game is more complicated than in the game with observable time). It is then easy to verify that the utility from deviating to $\bar{\sigma}$ admits a lower bound which converges $\hat{v}_0$. Since the utility from this deviation is itself a lower bound on equilibrium utility and since equilibrium utility is bounded above by $\hat{v}_0$, the claim follows.

4 Related literature

This paper is related to the literature on transparency in dynamic markets with adverse selection (Fuchs et al., 2016; Hörner and Vieille, 2009; Kaya and Liu, 2015; Kaya and Roy, 2022a,b,c; Kim, 2017). The paper of Kim (2017) is perhaps closest. In Kim's model, uninformed buyers make private offers to the seller of a single unit. The seller's costs and the object's value are the seller's private information. Kim compares two regimes that differ in whether buyers observe the seller's time-on-the-market. (Kim also studies a version of the model with an inflow of new sellers and buyers.) When time-on-the-market is unobservable, buyers play stationary strategies. When it is observable, buyers update on the fact that high types of the seller are more willing to wait for favorable terms, leading buyers to offer higher prices as the game progresses. Hence transparency affects the seller's incentives to delay trade so as to be offered a high price. In contrast, in our model, delay in trade is a result of the

[13]As we recall, Lemma 3.1 implies that $\Gamma^\emptyset(n, c_n)$ admits a sequential equilibrium for all n.

seller's setting high prices (indirectly, by recommending the highest signal cutoff). A further difference is that the seller is initially uninformed, and buyers learn through their private signals and delay in trade. Transparency affects this learning process, and hence the seller's incentives to delay trade.

In several other papers on dynamic markets with learning, the seller is initially informed about the value of the good, but additional signals arrive to the market over time (as in our paper). In Kaya and Kim (2018), Lauermann and Wolinsky (2016), and Zhu (2012), buyers observe private signals about the value. In Daley and Green (2012), buyers' signals are public. We differ from these papers in that, in our model, there is no initial information asymmetry, but asymmetry develops endogenously through delay in trade and buyers' private signals.[14] Note that the lack of initial asymmetry matters: in the game with observable time and unobservable actions, the proof of Proposition 2.3 uses that the first buyer does not revise his beliefs after observing the seller's deviation. One can show that the deviation used in the proof is not profitable if we let this buyer revise his beliefs arbitrarily.

In the regime with unobservable time, the seller's action can signal calendar time, and hence the good's value. The idea that the seller's actions signal information about value is not new. For example, Barsanetti and Camargo (2022) recently explore this idea in a model where the seller is informed about the value. Lauermann and Wolinsky (2016), who study a regime with unobservable time, also discuss this idea as part of a robustness check. A subtle distinction is that in our model the seller is *not* initially informed about the value. The informational content of an action depends on what the seller learns along the equilibrium path.

Taylor (1999) considers a two-period model that is related to ours. In each period, buyers bid for the good and the seller sets a reserve price. Taylor discusses the effects of the reserve price on the speed of learning: as in our benchmark model with observable actions, the seller gains from setting high initial prices to keep future beliefs high. Taylor further notes that high types of the seller benefit from public records. We instead focus on the seller's ability to extract the full surplus in a different informational setup and with a large number of buyers.

Bose et al. (2006, 2008) study a model close to ours. Namely, a version of our benchmark with observable actions but where the seller has an infinite number of

[14] In this regard, we are similar to Hwang (2018), in whose model there is no initial asymmetry, but asymmetry grows as the *seller* observes an exogenous private signal.

units (and chooses prices, rather than recommendations). The history of prices and sales is public. Bose et al. (2006) study whether the monopolist's strategy triggers herding behavior. Bose et al. (2008) characterize optimal offers when signals are binary. Their results have no immediate counterparts in our benchmark model with observable actions as we consider the sale of a single unit. Specifically, since selling a unit is good news about the value, their model admits belief dynamics that are absent in ours.

In an earlier working paper, Bose et al. (2002) also consider unobservable price offers when signals are binary and sales are observable. Their Lemma 10 shows that the seller may be unable to commit to trading exclusively with the most optimistic signal. The result is driven by the fact that selling a unit is good news about the value, and hence accelerating trades makes later buyers more optimistic. As noted above, this effect is absent in our model with a single unit. Indeed, their Lemma 10 sharply contrasts our results for binary signals.

There are further papers investigating other notions of transparency in more distant settings. The following are some examples. In the bilateral bargaining of Hwang and Li (2017), the focus is on transparency of on one party's outside option. In the multilateral bargaining game of Krasteva and Yildirim (2012), the focus is on transparency of the negotiation sequence and prices. Chaves (2019) studies how the transparency of on-going negotiations affects the incentives of third parties to interrupt these negotiations. Dilmé (2022) studies imperfect signals about a long-lived players actions in repeated bargaining. In the reputation models of Pei (2022a,b), the question is how limited observability of the long-lived player's actions affect that players ability to build a reputation.

5 Conclusion

In a dynamic market for a common value good, we have uncovered a sense in which hiding information about the seller's actions but disclosing information about her time-on-the-market is beneficial to buyers. For future work, it is interesting to consider what changes if the seller has multiple objects for sale (as in the work of Bose et al. (2006, 2008)) or if there are multiple sellers whose goods have correlated values. With multiple objects, one can investigate how the transparency of sales affect equilibrium outcomes. A different intriguing direction could attempt to endogenize buyers' arrival to the market. Our results use that the (random) order in which buyers arrive to the market is exogenous. What would change if buyers could strategically time when to solicit an offer from the seller? Lastly, our results suggest interesting open questions for information design. While we have shown that binary signals are optimal for the seller in the limit game, it is open what signal structures minimize the seller's revenue. Relatedly, what signal structure would maximize or minimize the overall surplus when there are frictions?

Appendices

Appendix A Observable time-on-the-market

A.1 Definitions and notation

This part of the appendix derives the expressions for buyers' posteriors belief and the seller's expected utility.

Since S is finite, the set S^∞ of sequences in S is compact and metric (in the product metric). This renders $\Delta(S^\infty)$ a compact metrizable space (Aliprantis and Border, 2006, Theorem 15.11). Let Π denote the set of functions from $\mathbb{N} \times S$ to $[0, \pi_0]$. As a countable product of compact intervals, the set Π is a compact metric space when equipped with the product metric.

In the main text, we initially introduced buyers beliefs as functions mapping to $[0, \infty]$. As we will see, on-path beliefs always lie in $[0, \pi_0]$. Restricting off-path beliefs to $[0, \pi_0]$ does not eliminate equilibria (since the prices, and hence the seller's utility, are increasing in buyers' beliefs). Hence there is no loss in viewing beliefs as element of Π.

Let F_ω denote the cdf. of the signals in state ω. For all s in S, let us define $\underline{F}_\omega = F_\omega(s) - f_\omega(s)$ as the probability of observing a signal strictly below s. Further, let $\bar{F}_\omega(s) = 1 - \underline{F}_\omega(s)$ denote the probability of observing a signal weakly above s.

A.1.1 The seller's expected utility

Let $\pi \in \Pi$ and $\sigma \in S^\infty$. When the seller uses the pure strategy σ, then in state ω the game reaches period i with probability $\prod_{j=1}^{i-1} \big(1 - \lambda \bar{F}_\omega(\sigma_j)\big)$. Conditional on reaching period i, buyer i ends up buying the object with probability $\lambda \bar{F}_\omega(\sigma_i)$; in that case, given beliefs π, he pays $\hat{v}(\sigma_i, \pi_i(\sigma_i))$. So the seller's expected utility equals

$$V(\sigma, \pi) = \sum_{i=1}^{\infty} \sum_{\omega \in \{\ell, h\}} \alpha_{\omega,0} \lambda \bar{F}_\omega(\sigma_i) \left(\prod_{j=1}^{i-1} \big(1 - \lambda \bar{F}_\omega(\sigma_j)\big) \right) \hat{v}(\sigma_i, \pi_i(s)). \tag{A.1}$$

The infinite sum is well-defined since for all $s \in S$ we have

$$1 - \lambda \bar{F}_\omega(s) \leq 1 - \lambda f_\ell(\bar{s}) < 1, \tag{A.2}$$

meaning that $\prod_{j=1}^{i-1} \big(1 - \lambda \bar{F}_\omega(\sigma_j)\big)$ is bounded above by $(1 - \lambda f_\ell(\bar{s}))^{i-1}$.

Using the bound in (A.2) and finiteness of S, a routine argument shows that $V(\sigma, \pi)$ is continuous in (σ, π). Hence V is bounded. Thus it makes sense to define the seller's expected utility from a mixed strategy μ as

$$V(\mu, \pi) = \int_{\sigma \in S^\infty} V(\sigma, \pi)\, d\mu(\sigma).$$

Using the bound in (A.2) and finiteness of S once again, we also find that V is continuous on $\Delta(S^\infty) \times \Pi$.

A.1.2 Buyers' beliefs

Given $\mu \in \Delta(S^\infty)$ and an integer i, let $S(i, \mu)$ denote the set of signals $s \in S$ satisfying

$$\int_{\sigma \in S^\infty} \mathbf{1}_{(\sigma_i = s)}\, d\mu(\sigma) > 0.$$

These are the signals s which μ plays with non-zero probability in round i. For all $s \in S(i, \mu)$, let

$$\hat{\pi}_i(s, \mu) = \pi_0 \frac{\int_{\sigma \in S^\infty} \mathbf{1}_{(\sigma_i = s)} \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_h(\sigma_j)\right) \, d\mu(\sigma)}{\int_{\sigma \in S^\infty} \mathbf{1}_{(\sigma_i = s)} \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_\ell(\sigma_j)\right) \, d\mu(\sigma)} \tag{A.3}$$

denote the Bayesian posterior likelihood of h versus ℓ conditional on reaching period i and the seller then offering a cutoff of s.

It is not difficult to see that if $s \in S(i, \mu)$ and $\{\mu_k\}_k$ is a sequence that weak-$*$ converges to μ, then $s \in S(i, \mu_k)$ holds for all but finitely many k. In the same situation, the posterior $\hat{\pi}_i(s, \mu_k)$ is well-defined for all but finitely many k and converges to $\hat{\pi}_i(s, \mu)$.

A.2 Equilibrium Existence

Proof of Lemma 2.1. Let μ_0 denote the strategy with the property that all $i \in \mathbb{N}$ and $s \in S$ satisfy $\mu_0\{\sigma \in S^\infty \colon \sigma_i = s\} = \frac{1}{|S|+1}$. That is, the seller randomizes uniformly over S in each period. This strategy μ_0 exists as one may verify, say, via an application of Ionescu-Tulcea's theorem (Bogachev, 2007, Theorem 10.7.3).

Given a strategy μ' and an integer k, note that $(1 - \frac{1}{k})\mu' + \frac{1}{k}\mu_0$ is fully mixed. Hence the Bayesian posterior belief as defined (A.3) induced by $(1 - \frac{1}{k})\mu' + \frac{1}{k}\mu_0$ is well-defined. Let us denote this belief by $\hat{\pi}(\cdot|\mu', k)$.[15] Consider the correspondence

$$\mu' \mapsto \arg\max_{\mu \in \Delta(S^\infty)} V(\mu, \hat{\pi}(\cdot|\mu', k))$$

As observed in Appendix A.1.2, buyer's beliefs are continuous in the seller's strategy when the strategy is fully mixed. That is $\mu' \mapsto \hat{\pi}(\cdot|\mu', k)$ is continuous. We further noted in Appendix A.1.1 that V is jointly continuous in the seller's strategy and beliefs. An application of Berge's Maximum Theorem (Aliprantis and Border, 2006,

[15] That is, $\hat{\pi}(\cdot|\mu', k)$ is defined for all i and s by

$$\hat{\pi}_i(s|\mu', k) = \hat{\pi}_i\left(s, \left(1 - \frac{1}{k}\right)\mu' + \frac{1}{k}\mu_0)\right).$$

Theorem 17.31) implies that the above arg max-correspondence is non-empty and compact-valued, and upper-hemicontinuous. Since V is linear in the seller's strategy, the arg max-correpondence is convex-valued, too. We thus infer from the Kakutani-Fan-Glicksberg Theorem (see e.g. Corollary 17.55 of Aliprantis and Border (2006, p. 583)) that for all k there exists a strategy μ_k^* satisfying

$$\mu_k^* \in \arg\max_{\mu \in \Delta(S^\infty)} V(\mu, \hat{\pi}(\cdot|\mu_k^*, k)).$$

Let π_k^* denote the belief $\hat{\pi}(\cdot|\mu_k^*, k)$.

By compactness of $\Delta(\Sigma^\infty)$ and Π, the sequence $\{\mu_k^*, \pi_k^*\}_{k\in\mathbb{N}}$ admits a convergent subsequence. Let this be the sequence itself, and let (μ^*, π^*) denote the limit. We claim that (μ^*, π^*) is a sequential equilibrium. To that end, we note that μ^* is the limit of the sequence

$$\left\{ \left(1 - \frac{1}{k}\right) \mu_k^* + \frac{1}{k}\mu_0 \right\}_{k\in\mathbb{N}}.$$

For all k, the strategy $\left(1 - \frac{1}{k}\right) \mu_k^* + \frac{1}{k}\mu_0$ is fully mixed and the belief π_k^* is obtained from $\left(1 - \frac{1}{k}\right) \mu_k^* + \frac{1}{k}\mu_0$ via Bayes' rule. Therefore, to show that (μ^*, π^*) is a sequential equilibrium, it suffices to show that μ^* maximizes $V(\cdot, \pi^*)$ across $\Delta(S^\infty)$. Letting μ be an arbitrary strategy, we know that for all k we have $V(\mu_k^*, \pi_k^*) \geq V(\mu, \pi_k^*)$. Taking $k \to \infty$ and using continuity of V, we infer that $V(\mu^*, \pi^*) \geq V(\mu, \pi^*)$ holds, as promised.

□

A.3 Failure of surplus extraction

A.3.1 Auxiliary results

Proof of Lemma 2.2. Let (μ, π) be an equilibrium. Let $i \in \mathbb{N}$ and $\sigma_i \in S(i, \mu)$. If trade happens at (i, σ_i), the price equals

$$\hat{v}(\sigma_i, \pi_i(\sigma_i)) = v_\ell + (v_h - v_\ell)\frac{\pi_i(\sigma_i)\frac{f_h(\sigma_i)}{f_\ell(\sigma_i)}}{\pi_i(\sigma_i)\frac{f_h(\sigma_i)}{f_\ell(\sigma_i)} + 1}.$$

Since $\sigma_i \in S(i,\mu)$, the belief $\pi_i(\sigma_i)$ is derived from Bayes' rule. Using (A.3), one may verify that $\pi_i(\sigma_i) \in (0,\infty)$ holds. By the MLRP, the price in the previous display is no greater than

$$v_\ell + (v_h - v_\ell)\frac{\pi_i(\sigma_i)\frac{\bar{F}_h(\sigma_i)}{\bar{F}_\ell(\sigma_i)}}{\pi_i(\sigma_i)\frac{\bar{F}_h(\sigma_i)}{\bar{F}_\ell(\sigma_i)} + 1}.$$

This is the posterior valuation conditional on the joint event that a signal above σ_i realizes, the game reaches period i, and the sellers recommends σ_i; let us denote this event by $E_i(\sigma_i)$. The posterior valuation conditional on $E_i(\sigma_i)$ is $\mathbb{E}[v|E_i(\sigma_i)]$. Note that since $\pi_i(\sigma_i) \in (0,\infty)$ we have $\hat{v}(\sigma_i, \pi_i(\sigma_i)) < \mathbb{E}[v|E_i(\sigma_i)]$ whenever $\sigma_i < \bar{s}$ holds.

Trade happens at (i,σ_i) if and only if the event $E_i(\sigma_i)$ occurs. We know from the bound in (A.2) that the probability of not trading within the first i rounds converges to 0 as $i \to \infty$, uniformly across all strategies of the seller. Put differently, as $i \to \infty$, the probability that the event

$$\bigcup_{(\sigma_1,\ldots,\sigma_i)\in S^i} \left(\bigcup_{j=1}^{i} E_j(\sigma_j) \right)$$

does not occur converges to 0. It follows from the Law of Iterated Expectations that the seller's profit is at most the prior valuation $\hat{v}_0$, with equality if and only if the induced cutoff in each period is $\bar{s}$ with probability one. The unique strategy for which this can hold is therefore the pure strategy $\bar{\sigma}$. □

Lemma A.1. *Let (g_h, g_ℓ) and $\{(S_k, f_{h,k}, f_{\ell,k})\}_{k\in\mathbb{N}}$ be as in the hypothesis of Proposition 2.3. For all k, let $\bar{s}_k = 1 - 1/k$. There exists a sequence $\{s_k^\circ\}_{k\in\mathbb{N}}$ such that for all except finitely-many k we have $s_k^\circ \in S_k$, and such that (the following limits exist and satisfy)*

$$\infty > \lim_{k\to\infty} \frac{f_{h,k}(\bar{s}_k)}{f_{\ell,k}(\bar{s}_k)} > 1, \tag{A.4a}$$

$$\lim_{k\to\infty} \frac{f_{h,k}(\bar{s}_k)}{1 - \underline{F}_{\ell,k}(s_k^\circ)} = 0, \tag{A.4b}$$

$$\lim_{k\to\infty} \frac{f_{h,k}(\bar{s}_k)}{f_{\ell,k}(\bar{s}_k)} \frac{f_{\ell,k}(s_k^\circ)}{f_{h,k}(s_k^\circ)} = 1. \tag{A.4c}$$

Proof of Lemma A.1. For all k, let $s_k^\circ = \max\{s \in S_k \colon s \le 1 - 1/\sqrt{k}\}$. Note that we have $\bar{s}_k = 1 - 1/k$.

Considering (A.4a), we note that $\frac{f_{h,k}(\bar{s}_k)}{f_{\ell,k}(\bar{s}_k)} = \frac{1-G_h(1-1/k)}{1-G_\ell(1-1/k)}$ converges to $g_h(1)/g_\ell(1)$, as an application of L'Hôpital's rule shows. This limit is strictly greater than one.

Next consider (A.4b). The ratio $\frac{f_{h,k}(\bar{s}_k)}{1-\underline{F}_{\ell,k}(s_k^\circ)}$ equals $\frac{1-G_h(1-1/k)}{1-G_\ell(1-1/\sqrt{k})}$ approximately. Another application of L'Hôpital's rule shows that the limit of the latter is zero.

Turning to (A.4c), we have:[16]

$$
\begin{aligned}
&\frac{f_{h,k}(\bar{s}_k)}{f_{\ell,k}(\bar{s}_k)} \frac{f_{\ell,k}(s_k^\circ)}{f_{h,k}(s_k^\circ)} \\
\approx &\left(\frac{1-G_h(1-1/k)}{1-G_\ell(1-1/k)}\right)\left(\frac{G_\ell(1+1/k-1/\sqrt{k})-G_\ell(1-1/\sqrt{k})}{G_h(1+1/k-1/\sqrt{k})-G_h(1-1/\sqrt{k})}\right).
\end{aligned}
$$

An application of L'Hôpital's rule shows that the limit of this term equals the limit of

$$
\left(\frac{g_h(1)}{g_\ell(1)}\right)\left(\frac{g_\ell(1+1/k-1/\sqrt{k})(1-\sqrt{k}/2)+g_\ell(1-1/\sqrt{k})\sqrt{k}/2}{g_h(1+1/k-1/\sqrt{k})(1-\sqrt{k}/2)+g_h(1-1/\sqrt{k})\sqrt{k}/2}\right).
$$

Since g_h and g_ℓ are continuous, this term converges to one, as desired. $\square$

Lemma A.2. *If s and s' are signals in S satisfying $f_h(s) \geq f_\ell(s)$ and $s' > s$, then*

$$
\frac{1-\lambda \bar{F}_h(s')}{1-\lambda \bar{F}_\ell(s')} > \frac{1-\lambda \bar{F}_h(s)}{1-\lambda \bar{F}_\ell(s)}
$$

holds.

Proof of Lemma A.2. It suffices to verify this for the case where s' is the signal directly above s. In that case, we have $\underline{F}_\omega(s) + f_\omega(s) = \underline{F}_\omega(s')$. Standard algebraic manipulations show

$$
\begin{aligned}
&\frac{1-\lambda \bar{F}_h(s')}{1-\lambda \bar{F}_\ell(s')} - \frac{1-\lambda \bar{F}_h(s)}{1-\lambda \bar{F}_\ell(s)} \\
=&\frac{\lambda\left(f_h(s)(1-\lambda \bar{F}_\ell(s))-f_\ell(s)1-\lambda \bar{F}_h(s)\right)}{\left(1-\lambda \bar{F}_\ell(s')\right)\left(1-\lambda \bar{F}_\ell(s)\right)}.
\end{aligned}
$$

[16] When $\{x_k\}_{k\in\mathbb{N}}$ and $\{y_k\}_{k\in\mathbb{N}}$ are sequences of real numbers, we write $x_k \approx y_k$ to mean $\lim_{k\to\infty} x_k/y_k = 1$.

The (MLRP) implies $(1 - \lambda \bar{F}_\ell(s)) \geq (1 - \lambda \bar{F}_h(s)$, and we have $f_h(s) \geq f_\ell(s)$ by assumption. □

A.3.2 Proof of Proposition 2.3

Proof of Proposition 2.3. We will prove that, for large enough k, there does not exist an equilibrium in which the seller's expected utility equals $\hat{v}_0$. This implies that her utility is bounded away from $\hat{v}_0$ across all equilibria. For, otherwise, compactness of $\Delta(S^\infty) \times \Pi$ lets us extract a convergent subsequence of equilibria along which her utility converges to $\hat{v}_0$; the limit of this subsequence will be an equilibrium in which her utility equals $\hat{v}_0$, and we have a contradiction.

For all k, let $\bar{s}_k = 1 - 1/k$, and let s_k° be as in the conclusion of Lemma A.1. In what follows, we will suppress the dependence of k from the notation by writing $(S, f_h, f_\ell, s^\circ, \bar{s})$ instead of $(S_k, f_{h,k}, f_{\ell,k}, s_k^\circ, \bar{s}_k)$. No confusion should arise.

In light of Lemma 2.2, we can show that there is no equilibrium where the seller's expected utility equals $\hat{v}_0$ by showing that the pure strategy $\bar{\sigma}$ is not an equilibrium. Towards a contradiction, suppose $\bar{\sigma}$ is supported in equilibrium by some beliefs π of the buyers. We will argue that, for all except finitely-many k, the following strategy constitutes a profitable deviation from $\bar{\sigma}$ for the seller: In the first period, the seller recommends s°; in all later periods i, the seller recommends $\bar{s}$. Let σ denote this sequence of recommendations.

In equilibrium, the first buyer's beliefs do not depend on the seller's action. Thus the prices induced by the deviation are $\hat{v}(s^\circ, \pi_0) = \mathbb{E}[v | \tilde{s} = s^\circ]$ in period 1 and $\hat{v}(\bar{s}, \hat{\pi}_i(\bar{s}, \bar{\sigma}))$ for all $i \geq 2$. To economize on notation, let

$$x_\omega^\circ = 1 - \lambda \bar{F}_\omega(\bar{s}) \qquad \text{and} \qquad \bar{x}_\omega = 1 - \lambda \bar{F}_\omega(\bar{s}).$$

Thus x_ω° and $\bar{x}_\omega$, respectively, denote the probabilities of not trading after recommending cutoffs s° and $\bar{s}$, respectively, within a given period.

We can now write the seller's utility from the deviation to σ as

$$\begin{aligned} &\mathbb{E}[v | \tilde{s} = s^\circ] \sum_{\omega \in \{\ell, h\}} \lambda \alpha_{\omega,0} \bar{F}_\omega(\bar{s}) \\ &+ \sum_{i=2}^{\infty} \left(\hat{v}(\bar{s}, \hat{\pi}_i(\bar{s}, \bar{\sigma})) \sum_{\omega \in \{\ell, h\}} \lambda \alpha_{\omega,0} \bar{F}_\omega(\bar{s}) x_\omega^\circ \bar{x}_\omega^{i-2} \right). \end{aligned} \tag{A.5}$$

We complete the proof by arguing that, for all but finitely-many k, the term in the previous expression is strictly greater than $\hat{v}_0$.

Consider the following equality for the expected utility from the deviation (the first expression is simply a restatement of the expected utility from the deviation; the equality adds a zero):

$$\begin{aligned}
&\hat{v}(s^\circ, \pi_0) \sum_{\omega \in \{\ell, h\}} \lambda \alpha_{\omega,0} \bar{F}_\omega(s^\circ) \\
&\quad + \sum_{i=2}^{\infty} \hat{v}(\bar{s}, \hat{\pi}_i(\bar{s}, \bar{\sigma})) \sum_{\omega \in \{\ell, h\}} \lambda \alpha_{\omega,0} f_\omega(\bar{s}) x_\omega^\circ \bar{x}_\omega^{i-2} \\
&= (\mathbb{E}[v|\tilde{s} = s^\circ] - \mathbb{E}[v|\tilde{s} \geq s^\circ]) \sum_{\omega \in \{\ell, h\}} \lambda \alpha_{\omega,0} \bar{F}_\omega(s^\circ) && \text{(A.6)} \\
&\quad + \sum_{i=2}^{\infty} \Bigg((\hat{v}(\bar{s}, \hat{\pi}_i(\bar{s}, \bar{\sigma})) - \hat{v}(\bar{s}, \hat{\pi}_i(\bar{s}, \sigma))) \\
&\qquad\qquad \times \sum_{\omega \in \{\ell, h\}} \lambda \alpha_{\omega,0} f_\omega(\bar{s}) x_\omega^\circ \bar{x}_\omega^{i-2} \Bigg) && \text{(A.7)} \\
&\quad + \mathbb{E}[v|\tilde{s} \geq s^\circ] \sum_{\omega \in \{\ell, h\}} \lambda \alpha_{\omega,0} \bar{F}_\omega(s^\circ) && \text{(A.8)} \\
&\quad + \sum_{i=2}^{\infty} \hat{v}(\bar{s}, \hat{\pi}_i(\bar{s}, \sigma)) \sum_{\omega \in \{\ell, h\}} \lambda \alpha_{\omega,0} f_\omega(\bar{s}) x_\omega^\circ \bar{x}_\omega^{i-2} && \text{(A.9)}
\end{aligned}$$

Iterated expectations show that the sum of (A.8) and (A.9) equals the prior value $\hat{v}_0$.[17] Thus, to show that utility from the deviation is strictly greater than $\hat{v}_0$, it suffices to show that the sum of (A.6) and (A.7) is strictly positive.

Several lines of algebra establish the following identities:

$$\begin{aligned}
&\mathbb{E}[v|\tilde{s} = s^\circ] - \mathbb{E}[v|\tilde{s} \geq s^\circ] \\
&= \frac{(v_h - v_\ell)\alpha_h \alpha_\ell \left(f_h(s^\circ)\bar{F}_\ell(s^\circ) - f_\ell(s^\circ)\bar{F}_h(s^\circ) \right)}{\left(\sum_{\omega \in \{\ell, h\}} \alpha_{\omega,0} f_\omega(s^\circ) \right) \left(\sum_{\omega \in \{\ell, h\}} \alpha_{\omega,0} \bar{F}_\omega(s^\circ) \right)},
\end{aligned}$$

[17] For each $i \geq 2$, the summand in (A.9) is the probability that trade happens in period i under the sequence σ. multiplied by the posterior value conditional on said event. In (A.8), we note that $\mathbb{E}[v|\tilde{s} \geq s^\circ] \sum_{\omega \in \{\ell, h\}} \lambda \alpha_{\omega,0} \bar{F}_\omega(s^\circ)$ is precisely that trade happens in period 1 under σ multiplied by the posterior value on that event.

and

$$
\begin{aligned}
&\hat{v}(\bar{s}, \hat{\pi}_i(\bar{s}, \bar{\sigma})) - \hat{v}(\bar{s}, \hat{\pi}_i(\bar{s}, \sigma)) \\
=&\frac{(v_h - v_\ell)\alpha_h\alpha_\ell f_h(\bar{s}) f_\ell(\bar{s}) \bar{x}_h^{i-2}\bar{x}_\ell^{i-2}\left(\bar{x}_h x_\ell^\circ - \bar{x}_\ell x_h^\circ\right)}{\left(\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(\bar{s})\bar{x}_\omega^{i-1}\right)\left(\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(\bar{s}) x_\omega^\circ \bar{x}_\omega^{i-2}\right)}.
\end{aligned}
$$

Notice that the positive term $(v_h - v_\ell)\alpha_h\alpha_\ell$ appears in both of the previous two identities. For the purposes of evaluating the sign of the sum of (A.6) and (A.7), we may ignore this term. If we now plug the previous two identities back into (A.6) and (A.7), it follows that we must verify that the following sum is strictly positive sufficiently far enough along the sequence of signal structures:

$$
\frac{\left(f_h(s^\circ)\bar{F}_\ell(s^\circ) - f_\ell(s^\circ)\bar{F}_h(s^\circ)\right)}{\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(s^\circ)} \tag{A.10}
$$

$$
+\sum_{i=2}^{\infty} \frac{f_h(\bar{s}) f_\ell(\bar{s}) \bar{x}_h^{i-2}\bar{x}_\ell^{i-2}\left(\bar{x}_h x_\ell^\circ - \bar{x}_\ell x_h^\circ\right)}{\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(\bar{s})\bar{x}_\omega^{i-1}}. \tag{A.11}
$$

For convenience, let us restate the implications of Lemma A.1 (the dependence on k being suppressed in the notation).

$$
\infty > \lim_{k\to\infty} \frac{f_h(\bar{s})}{f_\ell(\bar{s})} > 1, \tag{A.12a}
$$

$$
\lim_{k\to\infty} \frac{f_h(\bar{s})}{1 - \underline{F}_\ell(s^\circ)} = 0, \tag{A.12b}
$$

$$
\lim_{k\to\infty} \frac{f_h(\bar{s})}{f_\ell(\bar{s})}\frac{f_\ell(s^\circ)}{f_h(s^\circ)} = 1. \tag{A.12c}
$$

We continue by establishing a lower bound on the term in (A.11). Consider the difference

$$
\begin{aligned}
&\bar{x}_h x_\ell^\circ - \bar{x}_\ell x_h^\circ \\
=&(1 - \lambda f_h(\bar{s}))(1 - \lambda \bar{F}_\ell(s^\circ)) - (1 - \lambda f_\ell(\bar{s}))(1 - \lambda \bar{F}_h(s^\circ)).
\end{aligned} \tag{A.13}
$$

We claim that this difference is strictly positive for all large k. We know from (A.12a) that $f_h(\bar{s})/f_\ell(\bar{s})$ is strictly greater than one and eventually bounded away from one.

Further, $\frac{f_h(\bar{s})}{f_\ell(\bar{s})}\frac{f_\ell(s^\circ)}{f_h(s^\circ)}$ approaches one. Thus $f_h(s^\circ)/f_\ell(s^\circ)$ must be strictly larger than one for all sufficiently large k. It now follows from Lemma A.2 that (A.13) is strictly positive for such k.

Next, consider the ratio

$$\frac{\bar{x}_\ell^{i-2}}{\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(\bar{s})\bar{x}_\omega^{i-1}}. \tag{A.14}$$

Recall the definition $x_\omega(s) = 1 - \lambda(1 - \underline{F}_\omega(s))$. The (MLRP) implies $\bar{x}_h^{i-1}/\bar{x}_\ell^{i-2} \leq 1$, and hence the following is a lower bound on (A.14):

$$\frac{\bar{x}_\ell^{i-2}}{\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(\bar{s})\bar{x}_\omega^{i-1}} \geq \frac{1}{\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(\bar{s})}. \tag{A.15}$$

The fact that the term in (A.13) is strictly positive and the inequality in (A.15) together imply that the following is a lower bound on (A.11):

$$\begin{aligned}
&\sum_{i=2}^{\infty} \frac{f_h(\bar{s})f_\ell(\bar{s})\bar{x}_h^{i-2}\bar{x}_\ell^{i-2}\left(\bar{x}_h x_\ell^\circ - \bar{x}_\ell x_h^\circ\right)}{\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(\bar{s})\bar{x}_\omega^{i-1}} \\
\geq &\frac{f_h(\bar{s})f_\ell(\bar{s})}{\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(\bar{s})} \left(\bar{x}_h x_\ell^\circ - \bar{x}_\ell x_h^\circ\right) \sum_{i=2}^{\infty} \bar{x}_h^{i-2} \\
= &\frac{f_h(\bar{s})f_\ell(\bar{s})}{\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(\bar{s})} \left(\bar{x}_h x_\ell^\circ - \bar{x}_\ell x_h^\circ\right) \frac{1}{1-\bar{x}_h}.
\end{aligned}$$

If we plug back in the definition $x_\omega(s) = 1 - \lambda(1 - \underline{F}_\omega(s)) = 1 - \lambda\bar{F}_\omega(s)$, we obtain

$$f_\ell(\bar{s})\frac{(1-\lambda f_h(\bar{s}))(1-\lambda\bar{F}_\ell(s^\circ)) - (1-\lambda f_\ell(\bar{s}))(1-\lambda\bar{F}_h(s^\circ))}{\lambda\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(\bar{s})}. \tag{A.16}$$

To summarize: We may complete the proof by verifying that the sum of (A.10) and (A.16) is strictly positive for all sufficiently large k. This sum reads

$$\frac{\big(f_h(s^\circ)\bar{F}_\ell(s^\circ) - f_\ell(s^\circ)\bar{F}_h(s^\circ)\big)}{\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(s^\circ)} \tag{A.17}$$

$$+ f_\ell(\bar{s})\frac{(1-\lambda f_h(\bar{s}))(1-\lambda \bar{F}_\ell(s^\circ)) - (1-\lambda f_\ell(\bar{s}))(1-\lambda \bar{F}_h(s^\circ))}{\lambda \sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(\bar{s})}. \tag{A.18}$$

It is useful to rearrange the sum of (A.17) and (A.18) before proceeding.

Dividing the sum of (A.17) and (A.18) by

$$\frac{f_\ell(\bar{s})}{\bar{F}_\ell(s^\circ) \sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(\bar{s})}$$

leaves its sign unchanged. Rearranging the resulting terms further via standard algebraic manipulations, we find that the sign of the sum of (A.17) and (A.18) is the sign of

$$\left(\frac{f_h(s^\circ)}{f_\ell(\bar{s})} \frac{\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(\bar{s})}{\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(s^\circ)} - 1\right)$$

$$- \frac{\bar{F}_h(s^\circ)}{\bar{F}_\ell(s^\circ)} \left(\frac{f_\ell(s^\circ)}{f_\ell(\bar{s})} \frac{\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(\bar{s})}{\sum\limits_{\omega\in\{\ell,h\}} \alpha_{\omega,0} f_\omega(s^\circ)} - 1\right)$$

$$+ \frac{f_\ell(\bar{s})(1-\lambda \bar{F}_h(s^\circ)) - f_h(\bar{s})(1-\lambda \bar{F}_\ell(s^\circ))}{\bar{F}_\ell(s^\circ)}$$

$$= \left(\pi_0 \left(\frac{f_h(\bar{s})}{f_\ell(\bar{s})} - 1\right) + 1 - \frac{f_\ell(s^\circ)}{f_h(s^\circ)}\right) \left(\frac{f_\ell(s^\circ)}{f_h(s^\circ)} + \pi_0\right)^{-1} \tag{A.19}$$

$$- \pi_0 \frac{\bar{F}_h(s^\circ)}{\bar{F}_\ell(s^\circ)} \frac{f_h(s^\circ)}{f_\ell(s^\circ)} \left(\frac{f_h(\bar{s})}{f_\ell(\bar{s})} \frac{f_\ell(s^\circ)}{f_h(s^\circ)} - 1\right) \left(1 + \pi_0 \frac{f_h(s^\circ)}{f_\ell(s^\circ)}\right)^{-1} \tag{A.20}$$

$$+ \frac{f_\ell(\bar{s})(1-\lambda \bar{F}_h(s^\circ)) - f_h(\bar{s})(1-\lambda \bar{F}_\ell(s^\circ))}{\bar{F}_\ell(s^\circ)}. \tag{A.21}$$

We complete the proof by arguing that, along the sequence of signal structures, the term in (A.19) is positive (far enough along the sequence) and bounded away from 0, whereas the sum of (A.20) and (A.21) admits a lower bound that converges to 0.

Beginning with (A.19) we infer from (A.12a) and (A.12b) that $\frac{f_h(\bar{s})}{f_\ell(\bar{s})} - 1$ and $1 - \frac{f_\ell(s^\circ)}{f_h(s^\circ)}$ are eventually and bounded away from 0. We also know from (A.12a) that $\left(\frac{f_\ell(s^\circ)}{f_h(s^\circ)} + \pi_0\right)^{-1}$ is bounded. Hence (A.19) is eventually positive and bounded away from 0.

Turning to (A.19), we infer from (A.21) that $\left(\frac{f_h(\bar{s})}{f_\ell(\bar{s})}\frac{f_\ell(s^\circ)}{f_h(s^\circ)} - 1\right)$ converges to 0. The ratio $\frac{\bar{F}_h(s^\circ)}{\bar{F}_\ell(s^\circ)}$ is bounded (it converges to the ratio of the densities at 1.) Simultaneously, we know from (A.12a) and (A.12c) that all others terms in (A.19) are bounded along the sequence. Thus (A.20) converges to 0.

Lastly, turning to (A.21), we have the following lower bound on (A.21):

$$\frac{f_\ell(\bar{s})(1 - \lambda\bar{F}_h(s^\circ)) - f_h(\bar{s})(1 - \lambda\bar{F}_\ell(s^\circ))}{\bar{F}_\ell(s^\circ)} \geq -\frac{f_h(\bar{s})(1 - \lambda\bar{F}_\ell(s^\circ))}{\bar{F}_\ell(s^\circ)}.$$

We conclude from (A.12c) that this lower bound converges to 0. □

A.4 Surplus extraction with binary signals

Proof of Proposition 2.4. We proceed along a number of claims.

Claim A.3. *If $\mu' \in \Delta(S^\infty)$ is a fully mixed mixed strategy, then for all i and s we have $\hat{\pi}_i(s, \mu') \geq \hat{\pi}_i(\bar{s}, \bar{\sigma})$*

Proof of Claim A.3. Given an integer i and a pure strategy σ, let $N(i, \sigma) = |\{j \in \{1, \ldots, i-1\} \colon \sigma_j = \bar{s}\}|$. That is, $N(i, \sigma)$ is the number of times σ plays $\bar{s}$ in rounds 1 to $i - 1$.

Since signals are binary, we may write the posterior $\hat{\pi}_i(s,\mu')$ as follows:

$$
\begin{aligned}
\hat{\pi}_i(s,\mu') =& \pi_0 \frac{\sum_{\sigma\in S^n} \mathbf{1}_{(\sigma_i=s)}\mu'(\sigma)\prod_{j=1}^{i-1}(1-\lambda(1-\underline{F}_h(\sigma_j)))}{\sum_{\sigma\in S^n} \mathbf{1}_{(\sigma_i=s)}\mu'(\sigma)\prod_{j=1}^{i-1}(1-\lambda(1-\underline{F}_\ell(\sigma_j)))} \\
=& \pi_0 \frac{\sum_{\sigma\in S^n} \mathbf{1}_{(\sigma_i=s)}\mu'(\sigma)\,(1-\lambda)^{(i-1)-N(i,\sigma)}\,(1-\lambda f_h(\bar{s}))^{N(i,\sigma)}}{\sum_{\sigma\in S^n} \mathbf{1}_{(\sigma_i=s)}\mu'(\sigma)\,(1-\lambda)^{(i-1)-N(i,\sigma)}\,(1-\lambda f_\ell(\bar{s}))^{N(i,\sigma)}} \\
=& \pi_0 \frac{\sum_{\sigma\in S^n} \mathbf{1}_{(\sigma_i=s)}\mu'(\sigma)\left(\frac{1-\lambda f_h(\bar{s})}{1-\lambda}\right)^{N(i,\sigma)}}{\sum_{\sigma\in S^n} \mathbf{1}_{(\sigma_i=s)}\mu'(\sigma)\left(\frac{1-\lambda f_\ell(\bar{s})}{1-\lambda}\right)^{N(i,\sigma)}}.
\end{aligned}
$$

The belief $\hat{\pi}_i(\bar{s},\bar{\sigma})$ is given by

$$
\hat{\pi}_i(\bar{s},\bar{\sigma}) =\pi_0\left(\frac{1-\lambda f_h(\bar{s})}{1-\lambda f_\ell(\bar{s})}\right)^{i-1}.
$$

Hence the sign of difference $\hat{\pi}_i(s,\mu') - \hat{\pi}_i(\bar{s},\bar{\sigma})$ is the sign of

$$
\begin{aligned}
&\sum_{\sigma\in S^n} \mathbf{1}_{(\sigma_i=s)}\mu'(\sigma)\Bigg((1-\lambda f_\ell(\bar{s}))^{i-1}\left(\frac{1-\lambda f_h(\bar{s})}{1-\lambda}\right)^{N(i,\sigma)} \\
&\qquad\qquad - (1-\lambda f_h(\bar{s}))^{i-1}\left(\frac{1-\lambda f_\ell(\bar{s})}{1-\lambda}\right)^{N(i,\sigma)}\Bigg) \\
=& \sum_{\sigma\in S^n}\Bigg(\frac{\mathbf{1}_{(\sigma_i=s)}\mu'(\sigma)}{(1-\lambda)^{N(i,\sigma)}}\left(\frac{(1-\lambda f_h(\bar{s}))(1-\lambda f_\ell(\bar{s}))}{1-\lambda}\right)^{N(i,\sigma)} \\
&\qquad \times\left((1-\lambda f_\ell(\bar{s}))^{i-1-N(i,\sigma)} - (1-\lambda f_h(\bar{s}))^{i-1-N(i,\sigma)}\right)\Bigg).
\end{aligned}
$$

This sum is weakly positive since $f_\ell(\bar{s}) < f_h(\bar{s})$ holds and since $N(i,\sigma)$ is no greater than $i-1$. ☐

Claim A.4. *If (μ,π) is a sequential equilibrium of $\Gamma(\infty,0)$, then the seller's equilibrium expected utility is $\mathbb{E}[v|\pi_0]$, and μ is the pure strategy $\bar{\sigma}$*

Proof of Claim A.4. Since (μ,π) is a sequential equilibrium, the belief π is the pointwise limit of a sequence of Bayesian posteriors derived from fully-mixed strategies. Thus Claim A.3 implies that, for arbitrary $i \in \mathbb{N}$ and $s \in S$, the belief $\pi_i(s)$ is bounded below by $\hat{\pi}_i(\bar{s}, \bar{\sigma})$. The seller's utility is pointwise-increasing in the beliefs. Hence, given beliefs π, and the seller's utility from the pure strategy $\bar{\sigma}$ is at least $V(\bar{\sigma}, \hat{\pi}(\cdot, \bar{\sigma}))$. Since $\hat{\pi}_i(\bar{s}, \bar{\sigma})$ is the posterior induced by $\bar{\sigma}$, the reasoning of Lemma 2.2 via iterated expectations shows that $V(\bar{\sigma}, \hat{\pi}(\cdot, \bar{\sigma}))$ equals $\hat{v}_0$. Thus $\hat{v}_0$ is a lower bound on the seller's equilibrium utility. We know from Lemma 2.2 that $\hat{v}_0$ is also an upper bound on the seller's equilibrium utility. Hence another application of Lemma 2.2 shows that the seller's utility is $\hat{v}_0$ and that her strategy is $\bar{\sigma}$. □

□

Appendix B Unobservable time-on-the-market

B.1 Definitions and notation

In this section we derive expressions for the seller's expected utility and buyers' posterior beliefs in the game of Section 3.

Let $n \in \mathbb{N}$ and let $c \in [0, \lambda v_\ell]$. A mixed strategy of the seller is an element μ_n of $\Delta(S^n)$. Buyers' beliefs are represented by a function $\pi_n^\emptyset \colon S \to [0, \pi_0]$. (In the main text, we introduced beliefs as a function mapping to $[0, \infty]$, but, as in Appendix A.1, it is without loss to focus on beliefs in $[0, \pi_0]$.)

B.1.1 The seller's expected utility

Given $\pi_n^\emptyset$, the seller's expected utility from a mixed strategy μ_n is

$$V^\emptyset(\mu_n, \pi_n^\emptyset, n, c) = \sum_{\sigma \in S^n} \mu_n(\sigma) \sum_{i=1}^{n} \sum_{\omega \in \{\ell, h\}} \Bigg(\alpha_\omega \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_\omega(\sigma_j)\right) \\ \times \left(\lambda \bar{F}_\omega(\sigma_i) \hat{v}(\sigma_i, \pi^\emptyset(\sigma_i)) - c\right) \Bigg). \tag{B.1}$$

B.1.2 Buyers' inference

Given a mixed strategy μ_n, let $S_n(\mu_n)$ denote the subset of signals that μ_n plays with non-zero probability in at least one of the n periods. That is, s is in $S_n(\mu_n)$ if and only if

$$\sum_{i=1}^{n} \sum_{\sigma \in S^n} \mathbf{1}_{(\sigma_i = s)} \mu_n(\sigma) > 0. \tag{B.2}$$

Given μ_n and $s \in S_n(\mu_n)$, the Bayesian posterior conditional on arriving to the market on being recommended s is well-defined. We denote it by $\hat{\pi}_n^{\emptyset}(s, \mu_n)$. It is given by

$$\hat{\pi}_n^{\emptyset}(s, \mu_n) = \pi_0 \frac{\sum_{\sigma \in S^n} \sum_{i=1}^{n} \mathbf{1}_{(\sigma_i = s)} \mu_n(\sigma) \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_{\omega}(\sigma_j)\right)}{\sum_{\sigma \in S^n} \sum_{i=1}^{n} \mathbf{1}_{(\sigma_i = s)} \mu_n(\sigma) \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_{\omega}(\sigma_j)\right)}. \tag{B.3}$$

A mixed strategy μ_n is *fully mixed* if $S_n(\mu_n) = S$; that is, if each cutoff is recommended with non-zero probability in at least one period.

B.2 Auxiliary Results

This part of the appendix presents some auxiliary results.

Lemma B.1. *Let $n \in \mathbb{N}$, $c \in [0, \lambda v_\ell]$, and $\mu_n \in \Delta(S^n)$. Let $\pi_n^{\emptyset} \colon S \to [0,1]$ be a function that agrees with $\hat{\pi}_n^{\emptyset}(\cdot, \mu_n)$ at all s in $S_n(\mu_n)$. Then we have*

$$V^{\emptyset}(\mu_n, \pi_n^{\emptyset}, n, c) \leq \hat{v}_0.$$

The proof is analogous to that of Lemma 2.2 and is omitted.

The next result is chiefly used in the upcoming proof of Proposition 3.2; the reader may prefer to skip the result for now returning to it as needed. Consider an auxiliary fixed-point problem in which, for some given integer j and signal s^*, the seller is restricted to randomizing over pure strategies in which s^* is played in all of the first j rounds. Formally, given $n \in \mathbb{N}$ and $j \in \mathbb{N} \cup \{0\}$ such that $n - 1 \geq j$, let

$$\Sigma_{n,j,s^*} = \left\{(\sigma_1, \ldots, \sigma_n) \in S^n \colon \left(\forall_{k' \colon 1 \leq k' \leq j},\, \sigma_{k'} = s^*\right)\right\}.$$

The set of probability distributions over Σ_{n,j,s^*} is denoted by $\Delta(\Sigma_{n,j,s^*})$. As a convention, for $j = 0$, the set Σ_{n,j,s^*} means the set S^n.

Lemma B.2. *Let $c \in [0, v_\ell]$. Let $n \in \mathbb{N}$ and $j \in \mathbb{N} \cup \{0\}$ be such that $n-1 \geq j$. Let $s^* \in S$. There exists a sequence $\{\mu_k\}_{k\in\mathbb{N}}$ in $\Delta(\Sigma_{n,j,s^*})$, a strategy $\mu_{n,j}$ in $\Delta(\Sigma_{n,j,s^*})$, and a belief $\pi^{\emptyset}_{n,j}$ satisfying all of the following:*

(1) We have

$$\mu_{n,j} \in \underset{\mu' \in \Delta(\Sigma_{n,j,s^*})}{\arg\max}\, V^{\emptyset}(\mu', \pi^{\emptyset}_{n,j}, n, c). \tag{B.4}$$

(2) For all k, the strategy μ_k is fully mixed.

(3) The sequence $\{\mu_k\}_{k\in\mathbb{N}}$ converges to $\mu_{n,j}$ as $k \to \infty$.

(4) The sequence of induced beliefs $\{\hat{\pi}^{\emptyset}_n(\cdot, \mu_k)\}_k$ converges to $\pi^{\emptyset}_{n,j}$ as $k \to \infty$.

The proof proceeds via routine arguments and is omitted.

As an immediate corollary, we find that $\Gamma^{\emptyset}(n, c)$ admits some sequential equilibrium. In the main text, this was stated as Lemma 3.1.

Proof of Lemma 3.1. Invoke Lemma B.2 with $j = 0$. □

The next auxiliary lemma will be useful for the upcoming proof of Proposition 3.2; the reader may prefer to skip the result for now returning to it as needed. It characterizes the beliefs which are induced by a strategy in $\Delta(\Sigma_{n,j,s^*})$ for large n and j. Verbally, all on-path cutoffs different from s^* lead to a belief that the state is ℓ with overwhelming probability. Conversely, the belief at s^* is approximately $\pi_0 \frac{\bar{F}_\ell(s^*)}{\bar{F}_h(s^*)}$.

Lemma B.3. *Let $s^* \in S \setminus \{\underline{s}\}$. Let $\{j_n\}_{n\in\mathbb{N}}$ be a sequence of integers. For all n, let μ_n be a mixed strategy in $\Delta(\Sigma_{n,j_n,s^*})$. If the sequence $(j_n)_{n\in\mathbb{N}}$ diverges to $+\infty$, then for every $\varepsilon > 0$ there exists $n_\varepsilon \in \mathbb{N}$ such that for all n greater than n_ε all of the following are true:*

(1) If s is in $S_n(\mu_n) \setminus \{s^*\}$, then $\hat{\pi}^{\emptyset}_n(s^*, \mu_n) < \varepsilon$ holds.

(2) We have

$$\left| \hat{\pi}^{\emptyset}_n(s^*, \mu_n) - \pi_0 \frac{\bar{F}_\ell(s^*)}{\bar{F}_h(s^*)} \right| < \varepsilon. \tag{B.5}$$

Proof of Lemma B.3. Let $\varepsilon > 0$. Turning to the first claim, let n be arbitrary and consider a signal s in $S_n(\mu_n) \setminus \{s^*\}$. The posterior $\hat{\pi}^{\emptyset}_n(s, \mu_n)$ is defined to be

$$\hat{\pi}^{\emptyset}_n(s, \mu_n) = \frac{\sum\limits_{\sigma \in S^n} \sum\limits_{i=1}^{n} \mathbf{1}_{(\sigma_i = s)} \mu_n(\sigma) \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_h(\sigma_j)\right)}{\sum\limits_{\sigma \in S^n} \sum\limits_{i=1}^{n} \mathbf{1}_{(\sigma_i = s)} \mu_n(\sigma) \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_\ell(\sigma_j)\right)}.$$

By definition of $\Delta(\Sigma_{n,j_n,s^*})$, the distribution μ_n assigns positive probability to a pure strategy σ only if σ is in Σ_{n,j_n,s^*}. Accordingly, conditional on seeing a signal different from s^*, a buyer can be sure that at least j_n rounds have passed in which s^* was not accepted. This implies the following identity for arbitrary ω:

$$\begin{aligned} &\sum_{\sigma \in S^n} \sum_{i=1}^{n} \mathbf{1}_{(\sigma_i = s)} \mu_n(\sigma) \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_\omega(\sigma_j)\right) \\ &= \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{j_n} \sum_{\sigma \in \Sigma_{n,j_n,s^*}} \sum_{i=j_n+1}^{n} \mathbf{1}_{(\sigma_i = s)} \mu_n(\sigma) \prod_{j=j_n+1}^{i-1} \left(1 - \lambda \bar{F}_\omega(\sigma_j)\right). \end{aligned}$$

Hence the posterior belief $\hat{\pi}^{\emptyset}_n(s, \mu_n)$ reads

$$\hat{\pi}^{\emptyset}_n(s, \mu_n) = \left(\frac{1 - \lambda \bar{F}_h(s^*)}{1 - \lambda \bar{F}_\ell(s^*)}\right)^{j_n - 1} \frac{\sum\limits_{\sigma \in \Sigma_{n,j_n,s^*}} \sum\limits_{i=j_n}^{n} \mathbf{1}_{(\sigma_i = s)} \mu_n(\sigma) \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_h(\sigma_j)\right)}{\sum\limits_{\sigma \in \Sigma_{n,j_n,s^*}} \sum\limits_{i=j_n}^{n} \mathbf{1}_{(\sigma_i = s)} \mu_n(\sigma) \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_\ell(\sigma_j)\right)}.$$

The (MLRP) implies that the second fraction in this expression is less than 1. Moreover, since s^* is not $\underline{s}$, we infer from the (MLRP) that $1 - \lambda \bar{F}_h(s^*) < 1 - \lambda \bar{F}_\ell(s^*)$ holds. Thus there is some integer j'_ε satisfying

$$j_n \geq j'_\varepsilon \quad \Rightarrow \quad \left(\frac{1 - \lambda \bar{F}_h(s^*)}{1 - \lambda \bar{F}_\ell(s^*)}\right)^{j_n - 1} < \varepsilon.$$

In particular, for such j_n above j'_ε, the belief $\hat{\pi}^{\emptyset}_n(s, \mu_n)$ is less than ε for all $s \in S_n(\mu_n)$. Keeping this in mind, let us turn to the second part of the claim.

Consider the probability that a buyer assigns to following joint event: He arrives to the market when the object has not yet been traded and is then offered a signal of s^*. Conditional on the state being ω, we denote this probability by $q_{\omega,n}$; it is given by

$$q_{\omega,n} = \frac{1}{n} \sum_{\sigma \in S^n} \sum_{i=1}^{n} \mathbf{1}_{(\sigma_i = s^*)} \mu_n(\sigma) \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_\omega(\sigma_j)\right).$$

Using that μ_n is in $\Delta(\Sigma_{n,j_n,s^*})$, we find that $q_{\omega,n}$ equals

$$\frac{1}{n}\Bigg(\sum_{i=1}^{j_n} \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i-1} + \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{j_n} \sum_{\sigma \in \Sigma_{n,j_n,s^*}} \sum_{i=j_n+1}^{n} \mu_n(\sigma) \mathbf{1}_{(\sigma_i = s^*)} \prod_{j=j_n+1}^{i-1} \left(1 - \lambda \bar{F}_\omega(\sigma_j)\right) \Bigg)$$

A moment's thought reveals that the following are lower and upper bounds, respectively, on $q_{\omega,n}$:

$$\begin{aligned} &\frac{1}{n} \sum_{i=1}^{j_n} \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i-1} \\ \leq & q_{\omega,n} \\ \leq & \frac{1}{n} \left(\sum_{i=1}^{j_n} \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i-1} \right) + \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{j_n} \sum_{i=j_n+1}^{n} (1 - \lambda \bar{F}_\omega(\bar{s}))^{i-(j_n+1)}. \end{aligned}$$

Recall that $j_n \to \infty$ as $n \to \infty$. Hence we have $n q_{\omega,n} \to 1/(\lambda \bar{F}_\omega(s^*)$ as $n \to \infty$. The posterior belief $\hat{\pi}_n^{\emptyset}(s^*, \mu_n)$ is equal to the ratio $\pi_0 q_{h,n}/q_{\ell,n}$. Hence, there is an integer j''_ε such that $\hat{\pi}_n^{\emptyset}(s^*, \mu_n)$ is within ε of $\pi_0 \bar{F}_\ell(s^*)/\bar{F}_h(s^*)$ if j_n is greater than j''_ε .

Let $j_\varepsilon = \max(j'_\varepsilon, j''_\varepsilon)$. The preceding arguments show that all desired inequalities hold for μ_n if j_n is above j_ε. Recalling that $j_n \to \infty$ as $n \to \infty$, the claim follows. □

B.3 Proof of Proposition 3.2

Before delving into the details, let us sketch the idea. Let $s^* \in S \setminus \{\underline{s}\}$. We begin by defining a sequence $(j_n)_{n \in \mathbb{N}}$ of integers. Think of this as a sequence that diverges to ∞, but not too rapidly. In the game with n buyers, we then consider a restricted notion of equilibrium in which the seller is required to play s^* in the first j_n rounds (including in deviations). Lemma B.2 from Appendix B.2 shows that such an "equilibrium" exists. As long as j_n diverges to ∞, Lemma B.3 from Appendix B.2 then implies that all signals different from s^* induce buyers to update their beliefs such that their willingness to pay is approximately v_ℓ; this step requires that s^* be different from $\underline{s}$. Moreover, their willingness to pay after s^* is bounded away from v_ℓ; it is approximately the expression for the seller's limit utility as given in (3.1). We can then show that, far enough along the sequence, the seller will indeed find it optimal to only play s^* in the first, say, i_n rounds. To complete the proof, it is therefore sufficient to check that i_n is eventually larger than j_n, i.e. that the earlier constraint on the seller's strategy is eventually non-binding. For this final step, we require that j_n not diverge too quickly. Intuitively, the seller may have a motive to save on search costs even if this entails trading at an undesirable price. Thus the speed at which costs c_n vanish needs to be taken into account when letting j_n diverge.

Proof of Proposition 3.2. We shall first define the sequence of integers $\{j_n\}_{n\in\mathbb{N}}$. Our candidates for $(\mu_n, \pi_n^\emptyset)$ will then be derived from Lemma B.2.

For all n let j_n denote the largest integer (or zero) smaller than

$$\min\left(\frac{1}{2}\frac{\ln v_\ell - \ln c_n}{-\ln(1-\lambda\bar{F}_h(s^*))}, \frac{n}{2}\right). \tag{B.6}$$

(If $c_n = 0$, we understand the minimum to equal $n/2$.) The motivation for this obscure choice of j_n will reveal itself in the final step of the proof. For the moment, we only note that $n-1 \geq j_n$ holds, and that j_n and $n-j_n$ both go to infinity as n goes to infinity.

For arbitrary n, we may appeal to Lemma B.2 with j_n in the role of j to assert the following:

Claim B.4. *For all n, there exists a strategy μ_n in Σ_{n,j_n,s^*}, a belief $\pi_n^\emptyset$, and a sequence $\{\mu_{n,k}\}_{k\in\mathbb{N}}$ in $\Delta(\Sigma_{n,j_n,s^*})$ such that all of the following are true:*

(1) We have

$$\mu_n \in \underset{\mu'\in\Delta(\Sigma_{n,j_n,s^*})}{\arg\max}\ V^\emptyset(\mu', \pi_{n,j_n}^\emptyset, n, c). \tag{B.7}$$

(2) For all k, the strategy $\mu_{n,k}$ is completely mixed, i.e. the sets $S_n(\mu_{n,k})$ and S are equal.

(3) The sequence $(\mu_{n,k})_{k\in\mathbb{N}}$ converges to μ_n as $k\to\infty$.

(4) The sequence of induced beliefs $(\hat{\pi}_n^\emptyset(\cdot,\mu_{n,k}))_k$ converges to $\pi_n^\emptyset$ as $k\to\infty$.

In what follows, we understand that for all n, the tuple $(\mu_n, \pi_n^\emptyset, \{\mu_{n,k}\}_{k\in\mathbb{N}})$ is as in the conclusion of Equation (B.7). We will prove that the sequence $\{\mu_n, \pi_n^\emptyset, j_n\}_{n\in\mathbb{N}}$ satisfies all desired properties.

As a first step, we use Lemma B.3 to characterize the beliefs $\pi_n^\emptyset$. Note that Lemma B.3 is silent on the beliefs at off-path cutoffs. We will make use of the fact that, by construction, μ_n and $\pi_n^\emptyset$ are well-approximated by $\mu_{n,k}$ and $\hat{\pi}_n^\emptyset(\mu_{n,k})$, respectively. Since $\mu_{n,k}$ is completely mixed, we may then use Lemma B.3 to characterize the beliefs at all cutoffs.

Claim B.5. *For all $\varepsilon > 0$ there exists $n_\varepsilon \in \mathbb{N}$ such that for all n greater than n_ε all of the following are true:*

(1) If s is in $S \setminus \{s^*\}$, then $\hat{\pi}_n^\emptyset(s, \mu_n) < \varepsilon$ holds.

(2) We have

$$\left|\hat{\pi}_n^\emptyset(s^*, \mu_n) - \pi_0 \frac{\bar{F}_\ell(s^*)}{\bar{F}_h(s^*)}\right| < \varepsilon.$$

Proof Claim B.5. Let $\varepsilon > 0$. Part 4. of Claim B.4 implies that for all n we may find k_n such that $|\hat{\pi}^\emptyset(s, \mu_{n,k_n}) - \pi_n^\emptyset(s)| < \varepsilon/2$ holds for all $s \in S$. For later reference, note that μ_{n,k_n} is fully mixed.

Consider the sequence $\{\mu_{n,k_n}\}_{n\in\mathbb{N}}$ thus defined. Since j_n diverges to ∞, we may appeal to Lemma B.3 to find an integer n_ε such that for all n above n_ε all of the following are true:

(1) If s is in $S_n(\mu_{n,k_n}) \setminus \{s^*\}$, then $\hat{\pi}^\emptyset(s, \mu_{n,k_n}) < \varepsilon/2$.

(2) We have

$$\left|\hat{\pi}^\emptyset(s^*, \mu_{n,k_n}) - \pi_0 \frac{\bar{F}_\ell(s^*)}{\bar{F}_h(s^*)}\right| < \varepsilon/2.$$

Since μ_{n,k_n} is fully mixed, the sets $S_n(\mu_{n,k_n})$ and S are equal. We also recall that the inequality

$$|\hat{\pi}^\emptyset(s, \mu_{n,k_n}) - \pi_n^\emptyset(s)| < \varepsilon/2$$

holds for all $s \in S$. The claim follows from the above inequalities. □

The previous step allows an easy comparison of the prices that the seller can hope to obtain under $\pi_n^\emptyset$. To keep some of more algebraic steps readable, we simplify notation. For all $s \in S$ let

$$u_n(s) = \hat{v}(s, \pi_n^{\emptyset}(s))$$

denote the price that the seller would obtain from trading at a cutoff of s when beliefs are $\pi_n^{\emptyset}$. An immediate implication of Claim B.5 is that $u_n(s)$ converges to v_ℓ for all s different from s^*. Moreover, we have

$$u_n(s^*) \xrightarrow{n\to\infty} \frac{1}{\pi_0 \frac{f_h(s^*)}{f_\ell(s^*)} \frac{\bar{F}_\ell(s^*)}{\bar{F}_h(s^*)} + 1} \left(v_h \pi_0 \frac{f_h(s^*)}{f_\ell(s^*)} \frac{\bar{F}_\ell(s^*)}{\bar{F}_h(s^*)} + v_\ell \right).$$

In particular, for all sufficiently large values of n and all s different from s^*, we may assert that $u_n(s^*) - u_n(s)$ is positive and bounded away from zero. These inequalities and the limit for $u_n(s^*)$ are the only properties of $\pi_n^{\emptyset}$ that will be relevant in the remainder of the proof.

In what follows, if σ_n in S^n is some pure strategy for some n, then $\sigma_{n,i}$ means the i'th entry of σ_n, i.e. the seller's action in period i.

Claim B.6. *There exists $n^* \in \mathbb{N}$ such that for all n greater than n^*, if $\sigma_n$satisfies*

$$\sigma \in \arg\max_{\sigma' \in S^n} V^{\emptyset}(\sigma', \pi_n^{\emptyset}, n, c_n),$$

then the following is true: For all i and i', if $\sigma_{n,i} = s^*$ and $i' < i$, then $\sigma_{n,i'} = s^*$.

In other words, eventually, every pure best response to $\pi_n^{\emptyset}$ will admit a cutoff-structure: If s^* is played, then it is played up to some integer-cutoff, and never afterwards. (This integer-cutoff may be different for each best response. The cutoff may also equal n, in which case the strategy plays s^* in all periods.)

Proof Claim B.6. Recall that $u_n(s^*) - u_n(s)$ is positive and bounded away from zero for all sufficiently large values of n and all $s \in S \setminus \{s^*\}$. Recall also that c_n converges to zero. Hence we may find an integer n^* such if n is greater than n^* and s is in $S \setminus \{s^*\}$, then for all ω the inequality

$$u_n(s^*) - u_n(s) > \frac{c_n}{\lambda} \frac{\underline{F}_\omega(s^*) - \underline{F}_\omega(s)}{\bar{F}_\omega(s)\bar{F}_\omega(s^*)}$$

holds.

Fix an integer n greater than n^*, and let σ be in $\in \arg\max_{\sigma' \in S^n} V^{\emptyset}(\sigma', \pi_n^{\emptyset}, n, c_n)$. Towards a contradiction, suppose the claim was false. Then there exists an index i satisfying $\sigma_{n,i+1} = s^*$ and $\sigma_{n,i} \neq s^*$. Let $v_{i+2,\omega}$ denote the seller's expected payoff from period $i+2$ onwards under σ, conditional on state ω. Conditional on state ω, her expected payoff from period i onwards under σ is thus given by

$$\begin{aligned} & u_n(\sigma_{n,i})\lambda \bar{F}_\omega(\sigma_{n,i}) \\ & + \left(u_n(s^*)\lambda \bar{F}_\omega(s^*) - c_n\right)\left(1 - \lambda \bar{F}_\omega(\sigma_{n,i})\right) \\ & + v_{i+2,\omega}\left(1 - \lambda \bar{F}_\omega(\sigma_{n,i})\right)\left(1 - \lambda \bar{F}_\omega(s^*)\right). \end{aligned} \tag{B.8}$$

Consider the strategy σ' in which the seller picks s^* in period i, picks s in period $i+1$, and otherwise acts as under σ. The contribution of periods before i as well as the probability of reaching period i under σ' is clearly the same as under σ. Conditional on period i being reached in state ω, the probability that period $i+2$ is reached under σ' is $\left(1 - \lambda \bar{F}_\omega(s^*)\right)\left(1 - \lambda \bar{F}_\omega(\sigma_{n,i})\right)$; this is the same as under σ. The continuation $v_{i+2,\omega}$ from period $i+2$ onwards in state ω is also unchanged by the deviation since her behaviour in periods $i+2$ onwards does not change. Thus we may evaluate the profit from the deviation in state ω by comparing the expression in (B.8) to the following:

$$\begin{aligned} & u_n(s^*)\lambda \bar{F}_\omega(s^*) \\ & + \left(u_n(\sigma_{n,i})\lambda \bar{F}_\omega(\sigma_{n,i}) - c_n\right)\left(1 - \lambda \bar{F}_\omega(s^*)\right) \\ & + v_{i+2,\omega}\left(1 - \lambda \bar{F}_\omega(s^*)\right)\left(1 - \lambda \bar{F}_\omega(\sigma_{n,i})\right). \end{aligned} \tag{B.9}$$

The deviation to σ' is profitable if (B.9) is strictly larger than (B.8). By rearranging, we find that deviation is profitable in state ω if and only if

$$u_n(s^*) - u_n(\sigma_{n,i}) > \frac{c_n}{\lambda} \frac{\underline{F}_\omega(s^*) - \underline{F}_\omega(\sigma_{n,i})}{\bar{F}_\omega(\sigma_{n,i})\bar{F}_\omega(s^*)}$$

holds. This inequality is implied by our choice of n^* and the assumption that $\sigma_{n,i}$ is a cutoff different from s^*. Thus the deviation to σ' is profitable in both states of the world, and so we have a contradiction to the fact that σ is a best response to $\pi_n^{\emptyset}$. $\square$

Claim B.7. *There exists an integer n^{**} such that if $n \geq n^{**}$, then $(\mu_n, \pi_n^{\emptyset})$ is a sequential equilibrium of $V^{\emptyset}(n, c_n)$.*

Proof Claim B.7. Recalling the construction of $(\mu_n, \pi_n^{\emptyset})$ in Claim B.4, it suffices to verify that μ_n is a best response to $\pi_n^{\emptyset}$ for all but finitely many n. Towards a contradiction, suppose not. Then there is a subsequence such that for each of its members there exists a profitable deviation from μ_n. By possibly relabelling, let this subsequence be the sequence itself. The expected utility of the seller is a linear function of her mixed strategy. Thus the assumption implies that for all n there exists a pure strategy σ_n such that

$$\begin{aligned} V^{\emptyset}(\sigma_n, \pi_n^{\emptyset}, n, c_n) &= \max_{\sigma'' \in S^n} V^{\emptyset}(\sigma'', \pi_n^{\emptyset}, n, c_n) \\ &> V^{\emptyset}(\mu_n, \pi_n^{\emptyset}, n, c_n). \end{aligned} \tag{B.10}$$

(The maximum is attained since S^n is finite.)

Let n^* be as in Claim B.6. For integers n above n^*, we infer that there must exist i_n in $\{0, \ldots, n\}$ such that for all $i \in \{1, \ldots, n\}$ the following equivalence holds:

$$\sigma_{n,i} = s^* \qquad \Leftrightarrow \qquad i \leq i_n. \tag{B.11}$$

That is, the strategy σ plays s^* exactly up to some last period i_n, possibly never. In particular, we conclude that σ_n belongs to the set Σ_{n,i_n,s^*}.

Recall our construction of $(\mu_n, \pi_n^{\emptyset})$. In particular, according to (B.7), we have

$$V^{\emptyset}(\mu_n, \pi_n^{\emptyset}, n, c_n) = \sup_{\sigma'' \in \Sigma_{n,j_n,s^*}} V^{\emptyset}(\sigma'', \pi_n^{\emptyset}, n, c_n). \tag{B.12}$$

Note that Σ_{n,j_n,s^*} contains Σ_{n,i,s^*} whenever i is an integer greater than j_n; for Σ_{n,i,s^*} contains exactly those pure strategies which play s^* for at least i periods, whereas the set Σ_{n,j_n,s^*} contains those strategies which play s^* for at least j_n periods. We have already argued that σ_n is in Σ_{n,i_n,s^*}. We therefore conclude from (B.10) that, for all n, the integer i_n from (B.11) is less than j_n.

Consider the pure strategy $\sigma_n^* = (s^*, \ldots, s^*)$, i.e. the strategy that constantly plays s^*. Note that σ_n^* is in Σ_{n,j_n,s^*}, so that (B.12) implies

$$V^{\emptyset}(\mu_n, \pi_n^{\emptyset}, n, c_n) \geq V^{\emptyset}(\sigma_n^*, \pi_n^{\emptyset}, n, c_n). \tag{B.13}$$

Using the inequality $i_n \leq j_n$, we shall argue that for sufficiently large values of n we have

$$V^{\emptyset}(\sigma_n^*, \pi_n^{\emptyset}, n, c_n) > V^{\emptyset}(\sigma_n, \pi_n^{\emptyset}, n, c_n);$$

in light of (B.10) and (B.13), this yields a contradiction.

Consider the expected utility from σ_n^*, first. It is given by

$$\begin{aligned}
& u_n(s^*) \sum_{\omega \in \{\ell,h\}} \sum_{i=1}^{n} \alpha_\omega \lambda \bar{F}_\omega(s^*) \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i-1} \\
& \qquad - \lambda c_n \sum_{\omega \in \{\ell,h\}} \sum_{i=1}^{n} \alpha_\omega \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i-1} \\
& = u_n(s^*) \sum_{\omega \in \{\ell,h\}} \alpha_\omega \left(1 - \left(1 - \lambda \bar{F}_\omega(s^*)\right)^n\right) \\
& \qquad - c_n \sum_{\omega \in \{\ell,h\}} \alpha_\omega \frac{1 - \left(1 - \lambda \bar{F}_\omega(s^*)\right)^n}{\bar{F}_\omega(s^*)}.
\end{aligned} \tag{B.14}$$

Now consider the expected utility from σ_n. We recall that σ_n selects s^* exactly up to some period i_n, where $i_n \leq j_n$. By ignoring solicitation costs, we obtain an upper bound on the expected utility from σ_n. Verbally, a further upper bound is obtained in the following hypothetical scenario: If the seller does not trade within i_n periods, she gets a price of $\max_{s \in S \setminus \{s^*\}} (u_n(s))$ as soon she reaches period $i_n + 1$. Formally,

$$
\begin{aligned}
& V^{\emptyset}(\sigma_n, \pi_n^{\emptyset}, n, c_n) \\
=& u_n(s^*) \sum_{\omega \in \{\ell, h\}} \sum_{i=1}^{i_n} \alpha_\omega \lambda \bar{F}_\omega(s^*) \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i-1} \\
& + \max_{s \in S \setminus \{s^*\}} (u_n(s)) \sum_{\omega \in \{\ell, h\}} \Bigg(\alpha_\omega \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i_n} \\
& \qquad \times \sum_{i=i_n+1}^{n} \lambda \bar{F}_\omega(\sigma_{n,i}) \prod_{j=i_n+1}^{i-1} \left(1 - \lambda \bar{F}_\omega(\sigma_{n,j})\right) \Bigg) \\
\leq & u_n(s^*) \sum_{\omega \in \{\ell, h\}} \alpha_\omega \left(1 - \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i_n}\right) \\
& + \max_{s \in S \setminus \{s^*\}} (u_n(s)) \sum_{\omega \in \{\ell, h\}} \alpha_\omega \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i_n} \\
=& u_n(s^*) \sum_{\omega \in \{\ell, h\}} \alpha_\omega \left(1 - \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i_n}\right) \\
& + \max_{s \in S \setminus \{s^*\}} (u_n(s)) \sum_{\omega \in \{\ell, h\}} \alpha_\omega \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i_n} \\
=& u_n(s^*) + \left(\max_{s \in S \setminus \{s^*\}} (u_n(s)) - u_n(s^*) \right) \sum_{\omega \in \{\ell, h\}} \alpha_\omega \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i_n}. \qquad \text{(B.15)}
\end{aligned}
$$

We complete the argument by showing that (B.14) is greater than (B.15) for sufficiently large n. The difference (B.14) minus (B.15) is given by

$$
\begin{aligned}
& u_n(s^*) \sum_{\omega \in \{\ell, h\}} \alpha_\omega \left(1 - \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{n}\right) - c_n \sum_{\omega \in \{\ell, h\}} \alpha_\omega \frac{1 - \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{n}}{\bar{F}_\omega(s^*)} \\
& - u_n(s^*) - \left(\max_{s \in S \setminus \{s^*\}} (u_n(s)) - u_n(s^*) \right) \sum_{\omega \in \{\ell, h\}} \alpha_\omega \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i_n} \\
=& \sum_{\omega \in \{\ell, h\}} \alpha_\omega \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i_n} \Bigg(u_n(s^*) - \max_{s \in S \setminus \{s^*\}} (u_n(s)) - \frac{1}{\bar{F}_\omega(s^*)} \frac{c_n}{\left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i_n}} \\
& \qquad - \left(1 - \lambda \bar{F}_\omega(s^*)\right)^{n - i_n} \left(u_n(s^*) - \frac{c_n}{\bar{F}_\omega(s^*)} \right) \Bigg)
\end{aligned}
$$

We know that $n - i_n$ is greater than $n - j_n$, and we know that the latter diverges as

$n \to \infty$. Hence, for all ω, the term

$$\left(1 - \lambda \bar{F}_\omega(s^*)\right)^{n-i_n} \left(u_n(s^*) - \frac{c_n}{\bar{F}_\omega(s^*)}\right)$$

converges to 0 as $n \to \infty$. We also recall that $u_n(s^*) - \max_{s \in S \setminus \{s^*\}}$ is positive and bounded away from 0 as $n \to \infty$. To prove that (B.14) minus (B.15) is strictly positive for sufficiently large n, it therefore suffices to show that

$$\frac{c_n}{\left(1 - \lambda \bar{F}_\omega(s^*)\right)^{i_n}}$$

converges to 0 as $n \to \infty$. Again using that i_n is less than j_n, it suffices to check that

$$\frac{c_n}{\left(1 - \lambda \bar{F}_\omega(s^*)\right)^{j_n}} \tag{B.16}$$

converges to zero. Recall that definition of j_n as

$$j_n = \min\left(\frac{1}{2} \frac{\ln v_\ell - \ln c_n}{-\ln(1 - \lambda \bar{F}_h(s^*))}, \frac{n}{2}\right)$$

where we understand the minimum to be $n/2$ if $c_n = 0$. Therefore,

$$\frac{c_n}{\left(1 - \lambda \bar{F}_\omega(s^*)\right)^{j_n}} \leq c_n^{1/2} v_\ell^{1/2}.$$

Since $c_n \to 0$ as $n \to \infty$, we conclude that (B.16) converges to 0, as promised. □

In view of Claim B.7, the next claim completes the proof.

Claim B.8. *All of the following are true:*

(1) For all $n \in \mathbb{N}$, we have $\mu_n\{\sigma \in S^n : (\sigma_1, \ldots, \sigma_{j_n}) = (s^*, \ldots, s^*)\} = 1$.

(2) The sequence $\{j_n\}_{n \in \mathbb{N}}$ diverges to ∞.

(3) Along the sequence (μ_n, π_n), the good is traded with probability converging to 1. The seller's expected utility and the price at which the good is traded converge almost surely to

$$v_\ell + (v_h - v_\ell) \frac{\pi \frac{f_h(s^*)}{f_\ell(s^*)} \frac{\bar{F}_\ell(s^*)}{\bar{F}_h(s^*)}}{\pi \frac{f_h(s^*)}{f_\ell(s^*)} \frac{\bar{F}_\ell(s^*)}{\bar{F}_h(s^*)} + 1}. \tag{B.17}$$

Proof of Claim B.8. Part (1) is immediate from the fact that μ_n is in Σ_{n,j_n,s^*}. Part (2) follows from the definition of j_n. Turning to part (3), note that the good is traded at a price of $u_n(s^*)$ whenever at least one of the first j_n buyers who arrives to the market has a signal equal to s^*. Conditional on state ω, the probability of this event is $(1 - \lambda f_\omega(s^*))^{j_n}$. Since $u_n(s^*)$ converges to (B.17), we conclude from here that good is traded with probability converging to 1, and that the realized price conditional on trade converges almost surely to (B.17). It is clear that the seller's expected solicitation costs converge to 0, and hence the seller's expected utiltiy also converges to (B.17). $\square$

$\square$

B.4 Surplus extraction with binary signals

Proof of Proposition 3.3. Let $\bar{\sigma}_n$ denote the pure strategy that plays $\bar{s}$ in all periods. Given a strategy σ_n in S^n, we denote its i'th entry by $\sigma_{n,i}$.

Given $n \in \mathbb{N}$ and $m \in \{0, \ldots, n\}$, let $\sigma_n^{(m)}$ be the strategy which plays $\bar{s}$ in all rounds up to and including round m, and which plays $\underline{s}$ in all later rounds. Let us also abbreviate $\bar{x}_\omega = 1 - \lambda f_\omega(\bar{s})$.

Claim B.9. *For all $m \in \{0, \ldots, n\}$ we have $\hat{\pi}_n^\emptyset(\bar{s}, \sigma_n^{(m)}) \geq \hat{\pi}_n^\emptyset(\bar{s}, \bar{\sigma}_n)$.*

Proof Claim B.9. We will show that $\hat{\pi}_n^\emptyset(\bar{s}, \sigma_n^{(m)}) \geq \hat{\pi}_n^\emptyset(\bar{s}, \sigma_n^{(m+1)})$ holds for arbitrary $m \in \{1, \ldots, n-1\}$. This proves the claim since the strategy $\sigma^{(n)}$ is just the strategy $\bar{\sigma}_n$.

The difference $\hat{\pi}_n^{\emptyset}(\bar{s}, \sigma_n^{(m)}) - \hat{\pi}_n^{\emptyset}(\bar{s}, \sigma_n^{(m+1)})$ is given by

$$\begin{aligned}
&\pi_0 \frac{\sum_{i=1}^{n} \mathbf{1}_{(\sigma_{n,i}^{(m)}=\bar{s})} \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_h(\sigma_{n,j}^{(m)})\right)}{\sum_{i=1}^{n} \mathbf{1}_{(\sigma_{n,i}^{(m)}=\bar{s})} \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_\ell(\sigma_{n,j}^{(m)})\right)} - \pi_0 \frac{\sum_{i=1}^{n} \mathbf{1}_{(\sigma_{n,i}^{(m+1)}=\bar{s})} \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_h(\sigma_{n,j}^{(m+1)})\right)}{\sum_{i=1}^{n} \mathbf{1}_{(\sigma_{n,i}^{(m+1)}=\bar{s})} \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_\ell(\sigma_{n,j}^{(m+1)})\right)} \\
=&\pi_0 \frac{\sum_{i=1}^{m} \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_h(\sigma_{n,j}^{(m)})\right)}{\sum_{i=1}^{m} \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_\ell(\sigma_{n,j}^{(m)})\right)} - \pi_0 \frac{\sum_{i=1}^{m+1} \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_h(\sigma_{n,j}^{(m+1)})\right)}{\sum_{i=1}^{m+1} \prod_{j=1}^{i-1} \left(1 - \lambda \bar{F}_\ell(\sigma_{n,j}^{(m+1)})\right)} \\
=&\pi_0 \frac{\sum_{i=1}^{m} (1 - \lambda f_h(\bar{s}))^i}{\sum_{i=1}^{m} (1 - \lambda f_h(\bar{s}))^i} - \pi_0 \frac{\sum_{i=1}^{m+1} (1 - \lambda f_h(\bar{s}))^i}{\sum_{i=1}^{m+1} (1 - \lambda f_h(\bar{s}))^i} \\
=&\pi_0 \frac{\sum_{i=1}^{m} \bar{x}_h^i}{\sum_{i=1}^{m} \bar{x}_\ell^i} - \pi_0 \frac{\sum_{i=1}^{m+1} \bar{x}_h^i}{\sum_{i=1}^{m+1} \bar{x}_\ell^i}.
\end{aligned}$$

The sign of this difference is thus the sign of

$$\begin{aligned}
&\left(\sum_{i=1}^{m} \bar{x}_h^i\right) \left(\bar{x}_\ell^{m+1} + \sum_{i=1}^{m} \bar{x}_\ell^i\right) - \left(\sum_{i=1}^{m} \bar{x}_\ell^i\right) \left(\bar{x}_h^{m+1} + \sum_{i=1}^{m} \bar{x}_h^i\right) \\
&= \sum_{i=1}^{m} \left(\bar{x}_h^i \bar{x}_\ell^{m+1} - \bar{x}_\ell^i \bar{x}_h^{m+1}\right).
\end{aligned}$$

The claim now follows from the fact that $\bar{x}_\ell = 1 - \lambda f_\ell(\bar{s}) > 1 - \lambda f_h(\bar{s}) = \bar{x}_h$ holds. $\square$

Claim B.10. *Let $m \in \{1, \ldots, n\}$. Let $\sigma \in S^n$. If σ_n is a permutation of $\sigma_n^{(m)}$, then $\hat{\pi}_n^{\emptyset}(\bar{s}, \sigma_n) \geq \hat{\pi}_n^{\emptyset}(\bar{s}, \sigma_n^{(m)})$.*

Proof Claim B.10. For $k \in \{1, \ldots m\}$, let $\iota_k(\sigma_n)$ denote the label of the round in which σ_n plays $\bar{s}$ for the k'th time.[18] Defining $\iota_k(\sigma_n^{(m)})$ analogously, notice that we have $\iota_k(\sigma_n^{(m)}) = k$.

[18] That is, $\iota_1(\sigma_n) = \min\{i \in \{1, \ldots, n\} \colon \sigma_{n,i} = \bar{s}\}$. The remaining indices are defined inductively via $\iota_k(\sigma_n) = \min\{i \in \{\iota_{k-1}(\sigma_n) + 1, \ldots, n\} \colon \sigma_{n,i} = \bar{s}\}$.

Given a state ω, consider the sum

$$\sum_{i=1}^{n} \mathbf{1}_{(\sigma_{n,i}=\bar{s})} \prod_{j=1}^{i-1} (1 - \lambda(1 - \underline{F}_\omega(\sigma_{n,i}))) \,. \tag{B.18}$$

The i'th summand is non-zero only if there is some k such that $i = \iota_k(\sigma_n)$ holds. In that case, the definition of $\iota_k(\sigma_n)$ implies the following: Over the course of rounds $\{1, \ldots, \iota_k(\sigma_n) - 1\}$, the strategy σ_n plays $\bar{s}$ exactly $k-1$ times, and $\underline{s}$ otherwise. Hence we have

$$\begin{aligned}
&\prod_{j=1}^{i-1} (1 - \lambda(1 - \underline{F}_\omega(\sigma_{n,i}))) \\
=&(1 - \lambda(1 - \underline{F}_\omega(\underline{s})))^{\iota_k(\sigma_n)-1-(k-1)}(1 - \lambda(1 - \underline{F}_\omega(\bar{s})))^{k-1} \\
&(1-\lambda)^{\iota_k(\sigma_n)-1-(k-1)}(1 - \lambda f_\omega(\bar{s}))^{k-1} \\
=&(1-\lambda)^{\iota_k(\sigma_n)-1-(k-1)}\bar{x}_\omega^{k-1}.
\end{aligned}$$

The sum in (B.18) thus equals

$$\sum_{k=1}^{m} (1-\lambda)^{\iota_k(\sigma_n)-k}\bar{x}_\omega^{k-1}.$$

A similar expression can be derived for $\sigma_n^{(m)}$, with the only change being that we have $\iota_k(\sigma_n^{(m)}) = k$ for all k. The difference $\hat{\pi}_n^\emptyset(\bar{s}, \sigma_n) - \hat{\pi}_n^\emptyset(\bar{s}, \sigma_n^{(m)})$ thus reads

$$\begin{aligned}
&\pi_0 \frac{\sum_{k=1}^{m} (1-\lambda)^{\iota_k(\sigma_n)-k}\bar{x}_h^{k-1}}{\sum_{k=1}^{m} (1-\lambda)^{\iota_k(\sigma_n)-k}\bar{x}_\ell^{k-1}} - \pi_0 \frac{\sum_{k=1}^{m} (1-\lambda)^{\iota_k(\sigma_n^{(m)})-k}\bar{x}_h^{k-1}}{\sum_{k=1}^{m} (1-\lambda)^{\iota_k(\sigma_n^{(m)})-k}\bar{x}_\ell^{k-1}} \\
=&\pi_0 \frac{\sum_{k=1}^{m} (1-\lambda)^{\iota_k(\sigma_n)-k}\bar{x}_h^{k-1}}{\sum_{k=1}^{m} (1-\lambda)^{\iota_k(\sigma_n)-k}\bar{x}_\ell^{k-1}} - \pi_0 \frac{\sum_{k=1}^{m} \bar{x}_h^{k-1}}{\sum_{k=1}^{m} \bar{x}_\ell^{k-1}}.
\end{aligned}$$

The sign of this difference is the sign of

$$
\begin{aligned}
&\sum_{k=1}^{m}\sum_{k'=1}^{m}\Big((1-\lambda)^{\iota_k(\sigma_n)-k}\bar{x}_h^{k-1}\bar{x}_\ell^{k'-1}-(1-\lambda)^{\iota_k(\sigma_n)-k}\bar{x}_\ell^{k-1}\bar{x}_h^{k'-1}\Big)\\
&=\sum_{k,k'\,:\,k>k'}\Big((1-\lambda)^{\iota_k(\sigma_n)-k}\bar{x}_h^{k-1}\bar{x}_\ell^{k'-1}-(1-\lambda)^{\iota_k(\sigma_n)-k}\bar{x}_\ell^{k-1}\bar{x}_h^{k'-1}\\
&\qquad\qquad+(1-\lambda)^{\iota_{k'}(\sigma_n)-k'}\bar{x}_h^{k'-1}\bar{x}_\ell^{k-1}-(1-\lambda)^{\iota_{k'}(\sigma_n)-k'}\bar{x}_\ell^{k'-1}\bar{x}_h^{k-1}\Big)\\
&=\sum_{k,k'\,:\,k>k'}\Big((1-\lambda)^{\iota_k(\sigma_n)-k}-(1-\lambda)^{\iota_{k'}(\sigma_n)-k'}\Big)\Big(\bar{x}_h^{k-1}\bar{x}_\ell^{k'-1}-\bar{x}_\ell^{k-1}\bar{x}_h^{k'-1}\Big)
\end{aligned}
$$

To complete the proof, we argue that each of the summands in the last expression is weakly positive.

First, recall that $\iota_k(\sigma_n)$ denotes the label of the round in which σ_n plays $\bar{s}$ for k'th time, whereas $\iota_{k'}(\sigma_n)$ denotes label of the round with the (k')'th occurence. This means that at least $k-k'$ rounds must pass between the two rounds. Formally, we have $\iota_k(\sigma_n)-\iota_{k'}(\sigma_n)\geq k-k'$. This inequality implies that $(1-\lambda)^{\iota_k(\sigma_n)-k}-(1-\lambda)^{\iota_{k'}(\sigma_n)-k'}$ is negative.

Second, notice that $1-\lambda f_\ell(\bar{s})=\bar{x}_\ell>\bar{x}_h=1-\lambda f_h(\bar{s})$ holds. Given that the summands consider k and k' such that $k>k'$ holds, we conclude that $\bar{x}_h^{k-1}\bar{x}_\ell^{k'-1}-\bar{x}_\ell^{k-1}\bar{x}_h^{k'-1}$ is negative.

The previous two paragraphs imply that

$$
\Big((1-\lambda)^{\iota_k(\sigma_n)-k}-(1-\lambda)^{\iota_{k'}(\sigma_n)-k'}\Big)\Big(\bar{x}_h^{k-1}\bar{x}_\ell^{k'-1}-\bar{x}_\ell^{k-1}\bar{x}_h^{k'-1}\Big)
$$

is weakly positive, which yields the desired conclusion. □

Claim B.11. *If $\mu_n'\in\Delta(S^n)$ is a mixed strategy that plays $\bar{s}$ with non-zero probability, then $\hat{\pi}_n^{\emptyset}(\bar{s},\mu_n')$ is well-defined and we have $\hat{\pi}_n^{\emptyset}(\bar{s},\mu_n')\geq\hat{\pi}_n^{\emptyset}(\bar{s},\bar{\sigma}_n)$.*

Proof of Claim B.11. For a pure strategy σ_n, a state ω and an integer i, let us abbreviate $\delta_\omega(i,\sigma_n)=\prod_{j=1}^{i-1}\big(1-\lambda\bar{F}_\omega(\sigma_{n,j})\big)$. In this notation, the sign of the difference

$\hat{\pi}_n^{\emptyset}(\bar{s}, \mu_n') - \hat{\pi}_n^{\emptyset}(\bar{s}, \bar{\sigma}_n)$ is

$$\operatorname{sgn}\left(\frac{\sum_{\sigma\in S^n}\sum_{i=1}^{n}\mathbf{1}_{(\sigma_{n,i}=\bar{s})}\mu_n'(\sigma_n)\delta_h(i,\sigma_n)}{\sum_{\sigma\in S^n}\sum_{i=1}^{n}\mathbf{1}_{(\sigma_{n,i}=\bar{s})}\mu_n'(\sigma_n)\delta_\ell(i,\sigma_n)} - \frac{\sum_{i=1}^{n}\delta_h(i,\bar{\sigma}_n)}{\sum_{i=1}^{n}\delta_\ell(i,\bar{\sigma}_n)}\right)$$
$$= \operatorname{sgn}\left(\sum_{\sigma\in S^n}\mu_n'(\sigma_n)\left(\left(\sum_{i=1}^{n}\mathbf{1}_{(\sigma_{n,i}=\bar{s})}\delta_h(i,\sigma_n)\right)\left(\sum_{i=1}^{n}\delta_\ell(i,\bar{\sigma}_n)\right) - \left(\sum_{i=1}^{n}\mathbf{1}_{(\sigma_{n,i}=\bar{s})}\delta_\ell(i,\sigma_n)\right)\left(\sum_{i=1}^{n}\delta_h(i,\bar{\sigma}_n)\right)\right)\right).$$

Hence it suffices to show that, for arbitrary σ_n, the difference

$$\left(\sum_{i=1}^{n}\mathbf{1}_{(\sigma_{n,i}=\bar{s})}\delta_h(i,\sigma_n)\right)\left(\sum_{i=1}^{n}\delta_\ell(i,\bar{\sigma}_n)\right) - \left(\sum_{i=1}^{n}\mathbf{1}_{(\sigma_{n,i}=\bar{s})}\delta_\ell(i,\sigma_n)\right)\left(\sum_{i=1}^{n}\delta_h(i,\bar{\sigma}_n)\right)$$

is weakly positive. There is nothing to prove if σ_n never plays $\bar{s}$. If σ_n plays $\bar{s}$ at least once, then the sign of this difference is precisely the sign of

$$\hat{\pi}_n^{\emptyset}(\bar{s}, \sigma_n) - \hat{\pi}_n^{\emptyset}(\bar{s}, \bar{\sigma}_n).$$

A strategy σ_n which plays $\bar{s}$ a total of, say, m times is a permutation of the strategy $\sigma_n^{(m)}$. Thus Claims B.9 and B.10 imply $\hat{\pi}_n^{\emptyset}(\bar{s}, \sigma_n) \geq \hat{\pi}_n^{\emptyset}(\bar{s}, \sigma_n^{(m)}) \geq \hat{\pi}_n^{\emptyset}(\bar{s}, \bar{\sigma}_n)$. In particular, $\hat{\pi}_n^{\emptyset}(\bar{s}, \sigma_n) - \hat{\pi}_n^{\emptyset}(\bar{s}, \bar{\sigma}_n)$ is weakly positive, as promised. □

Claim B.12. *The seller's equilibrium expected utility converges to $\hat{v}_0$*

Proof of Claim B.12. Recall that, for all n, the pair $(\mu_n, \pi_n^{\emptyset})$ is a sequential equilibrium. Claim B.11 therefore implies that $\pi_n^{\emptyset}(\bar{s}) \geq \hat{\pi}_n^{\emptyset}(\bar{s}, \bar{\sigma}_n)$ holds for all n. It follows that the deviation to $\bar{\sigma}_n$ yields an expected utility of at least

$$
\begin{aligned}
&\sum_{i=1}^{n} \sum_{\omega\in\{\ell,h\}} \alpha_\omega \left(\prod_{j=1}^{i-1} \left(1-\lambda \bar{F}_\omega(\sigma_{n,j})\right)\right) \left(\lambda f_\omega(\bar{s}) \hat{v}(\bar{s}, \hat{\pi}_n^{\emptyset}(\bar{s}, \bar{\sigma}_n)) - c_n\right) \\
=&\sum_{i=1}^{n} \sum_{\omega\in\{\ell,h\}} \alpha_\omega \left(1-\lambda f_\omega(\bar{s})\right)^{i-1} \left(\lambda f_\omega(\bar{s}) \hat{v}(\bar{s}, \hat{\pi}_n^{\emptyset}(\bar{s}, \bar{\sigma}_n)) - c_n\right) \\
=&\hat{v}(\bar{s}, \hat{\pi}_n^{\emptyset}(\bar{s}, \bar{\sigma}_n)) \sum_{\omega\in\{\ell,h\}} \alpha_\omega \left(1-(1-\lambda f_\omega(\bar{s}))^n\right) - c_n \sum_{\omega\in\{\ell,h\}} \alpha_\omega \frac{1-(1-\lambda f_\omega(\bar{s}))^n}{f_\omega(\bar{s})}
\end{aligned}
$$

We know from Lemma B.3 that $\hat{\pi}_n^{\emptyset}(\bar{s}, \bar{\sigma}_n)$ converges to $f_\ell(\bar{s})/f_h(\bar{s})$. Hence the expression in the previous line converges to $\hat{v}(\bar{s}, \pi_0 f_\ell(\bar{s})/f_h(\bar{s}))$ as $n \to \infty$. This expectation equals $\hat{v}_0$. Thus we have shown that equilibrium utility admits a lower bound which converges to $\hat{v}_0$. But we also know from Lemma B.1 that equilibrium expected utility is bounded above by $\hat{v}_0$, and hence we arrive at the desired conclusion. $\square$

$\square$

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