Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk[*]

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Abstract

Households face large income uncertainty that varies substantially over the business cycle. We examine the macroeconomic consequences of these variations in a model with incomplete markets, liquid and illiquid assets, and a nominal rigidity. Heightened uncertainty depresses aggregate demand as households respond by hoarding liquid "paper" assets for precautionary motives, thereby reducing both illiquid physical investment and consumption demand. We document the empirical response of portfolio liquidity and aggregate activity to surprise changes in idiosyncratic income uncertainty and find both to be quantitatively in line with our model. The welfare consequences of uncertainty shocks and of the policy response thereto depend crucially on a household's asset position.

Keywords: Incomplete Markets, Nominal Rigidities, Uncertainty Shocks.

JEL-Codes: E22, E12, E32

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1 Introduction

The Great Recession has brought about a reconsideration of the role of uncertainty in business cycles. Increased uncertainty has been documented and studied in various markets. However, uncertainty with respect to household income stands out in its size and importance. Shocks to household income are large and exhibit systematic changes over the business cycle. Storesletten et al. (2001) estimate that for the U.S. the variance of persistent shocks to disposable household income almost doubles in recessions.[1]

The starting point of the present paper is that households use precautionary savings and structure their portfolios to smooth consumption if asset markets are incomplete and assets differ in their liquidity. Therefore, in such a setting, swings in the riskiness of household income lead not only to systematic variations in the propensity to consume but also to a rebalancing of household portfolios.

We quantify the aggregate consequences of precautionary savings and portfolio adjustments in response to shocks to household income risk by means of a dynamic stochastic general equilibrium model. In our model, households have access to two types of assets to smooth consumption. They can either hold liquid (low return) nominal bonds or invest in illiquid, high-dividend-paying physical capital. Illiquidity is modeled by a transaction cost. As a result households trade capital only from time to time.[2] This two-asset structure allows us to disentangle savings and physical investment and thus obtain strong fluctuations in aggregate demand in response to household income risk.[3] To generate aggregate output effects from demand fluctuations, we augment this incomplete markets framework in the tradition of Bewley (1980), Huggett (1993), and Aiyagari (1994) by sticky prices à la Rotemberg (1982).

In this economy, an increase in income risk makes households consume less and save more. In addition, and importantly, they rebalance their portfolios toward the liquid asset because it provides better consumption smoothing. They take into account that they will have to adjust the illiquid asset more often to keep consumption smooth and this drives down the effective return of the illiquid asset because transactions are costly. Thus, higher income risk leads to a flight to liquidity.

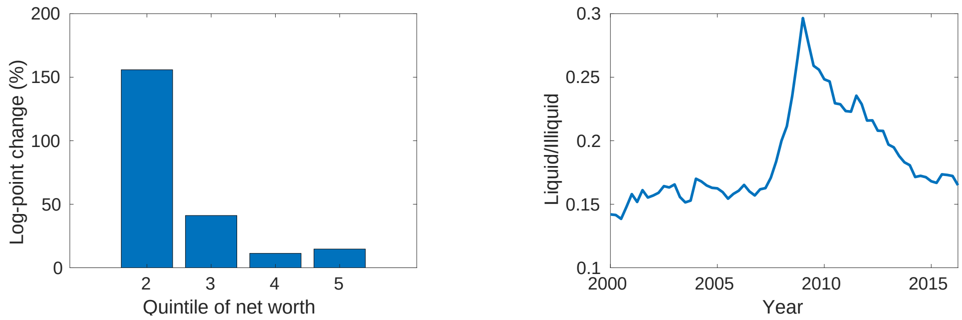
This flight to liquidity is reminiscent of the observed patterns of the share of liquid assets in the portfolios of U.S. households during the Great Recession; see Figure 1.

[1] Work by Guvenen et al. (2014b) documents that changes in individual labor income become left skewed in recessions.

[2] This setup is similar to Kaplan and Violante (2014) and Kaplan et al. (2017) following the tradition of Baumol (1952) and Tobin (1956) in modeling the portfolio choice between liquid and illiquid assets.

[3] In a standard Aiyagari (1994) economy, where all savings are in physical capital, an increase in savings does not lead to a fall in total demand (investment plus consumption) because savings increase investments one-to-one.

Figure 1: Liquid assets relative to illiquid assets on household balance sheets



(a) Cross-section: 2010 vs. 2007 (b) Aggregate: 2000 - 2016

Notes: Net liquid assets are composed of money market, checking, savings and call accounts, as well as corporate and government bonds and T-bills net of credit card debt. All other assets net of all other debt make up net illiquid wealth.
(a) Change between 2007 and 2010 by quintile of the wealth distribution. Based on the Survey of Consumer Finances. Change in the average ratio of liquid to illiquid assets within the four last quintiles of net worth; see Section 6 for details. The first quintile does not consistently hold positive amounts of liquid and/or illiquid assets.
(b) Ratio of liquid to illiquid assets on household balance sheets based on quarterly data from the Flow of Funds; for details, see Appendix G.1.

According to the 2010 Survey of Consumer Finances, the share of liquid assets in household portfolios increased relative to 2007 across all wealth quintiles, with the strongest relative increase for the lower middle class; see panel (a). Also in the aggregate, we see a substantial increase in portfolio liquidity around the crisis; see panel (b). We find this increase in portfolio liquidity to be a general response to estimated shocks to idiosyncratic income uncertainty in the data. In our model, this portfolio rebalancing toward liquid paper reinforces, through a reduction in physical investment, the decline in consumption demand caused by higher uncertainty. Consequently, aggregate demand declines even more strongly than consumption, and investment and consumption co-move.

More generally, we find the following: We estimate the time-series behavior of household income risk from the Survey of Income and Program Participation. We then use these estimates to understand the consequences of a rise in income risk. In the data, a one standard deviation increase in household income risk decreases aggregate activity by 0.2% and investment by 1% over the first year after the shock, and our model matches these numbers. At the zero lower bound, when neither monetary nor fiscal policy stabilizes the economy, our model suggests an output loss of almost 2%.

In addition to the aggregate consequences, an uncertainty shock has rich distributional consequences, as the price of and return on capital fall more than the return on liquid assets when uncertainty increases. We use our model to estimate the welfare consequences of these distributional effects. Our welfare calculations imply that households rich in illiquid physical capital lose the most as capital returns fall strongly in times of high income risk. At the same time, their large but illiquid wealth helps little to smooth consumption. Households rich in liquid assets, by contrast, even though they might hold less total wealth, are much better insured and do not suffer as much from lower capital returns; hence, their welfare losses are smaller.

Our model allows us to assess the importance of systematic monetary and fiscal policy for the stabilization of the economy in response to uncertainty shocks. Aggressive monetary policy can stabilize the economy by cutting interest rates on liquid assets and pushing household portfolios back toward illiquid investments. Expansionary fiscal policy instead supplies the economy with the additional liquid assets demanded by the private sector. Thus, both policies can be used effectively for aggregate stabilization.

Yet, they have different welfare consequences. To understand the consequences of various systematic policy responses, we compare three regimes: first, a regime that corresponds to our baseline calibration of fiscal and monetary policy; second, a regime with perfect stabilization through monetary policy; and third, a regime in which fiscal policy perfectly stabilizes. We find that a one standard deviation increase in household income risk depresses welfare equivalent to 27 basis points of lifetime consumption on average. However, there is a large heterogeneity. Well-insured, wealthy households suffer substantially less from the increase in uncertainty. For them, the equilibrium changes in prices are more important. Therefore, households rich in nominal assets suffer from stabilizing monetary policy as it drives down their asset returns. For the same reason, households rich in real assets like stabilization through fiscal policy. It crowds out investment and keeps capital returns high.

In exploring portfolio adjustment as a new channel through which uncertainty affects real activity, our paper adds to the recent literature that explores the aggregate effects of time-varying uncertainty. In particular, Bloom's (2009) paper on the effects of time-varying (idiosyncratic) productivity uncertainty on firms' factor demand through the real option value of irreversible investment has triggered a stream of research on the aggregate effects of variations in firm-level productivity risk.[4]

A more recent branch of this literature investigates the aggregate implications of uncertainty shocks beyond their transmission through investment and has also broadened the sources of uncertainty studied. The first papers in this vein highlight non-

[4]To name a few: Arellano et al. (2012), Bachmann and Bayer (2013), Christiano et al. (2014), Chugh (2016), Di Tella (2018), Gilchrist et al. (2014), Narita (2011), Panousi and Papanikolaou (2012), Schaal (2012), and Vavra (2014) have studied the business cycle implications of a time-varying dispersion of firm-specific variables, often interpreted as and used to calibrate shocks to firm risk, propagated through various frictions: wait-and-see effects from capital adjustment frictions, financial frictions, search frictions in the labor market, nominal rigidities, balance sheets, and agency problems.

linearities in the New-Keynesian model, in particular the role of precautionary price setting.[5] Fernández-Villaverde et al. (2015), for example, look at a medium-scale DSGE model à la Smets and Wouters (2007). They find that at the zero lower bound output drops by more than 1.5% after a two standard deviation shock to the volatility of taxes. Off the zero lower bound, the drop reduces to 0.2%.[6] In a similar framework, Basu and Bundick (2017) highlight how price stickiness can generate comovement of consumption and investment after a decline in consumption demand driven by a shock to demand uncertainty. Overall, they find aggregate effects similar to those in Fernández-Villaverde et al. (2015).

We focus on idiosyncratic instead of aggregate income uncertainty and abstract from the labor supply effects of income risk by assuming Greenwood et al. (1988) preferences. This paper thereby isolates the precautionary saving and, in particular, the portfolio channel of income uncertainty. The focus on idiosyncratic income risk and the response of precautionary savings links our paper to the burgeoning literature on heterogeneous agent New-Keynesian models, in particular to Ravn and Sterk (2017) and Den Haan et al. (2017). Both papers highlight the importance of uninsurable idiosyncratic unemployment risk in amplifying *first moment shocks* in labor search models. In contrast, we look at second moment shocks. More important, the two papers differ from ours in their asset market setup, assuming that all assets are perfectly liquid such that the portfolio reallocation we highlight is absent by definition.

With respect to the broader literature on New-Keynesian incomplete markets models we share with Gornemann et al. (2012) a focus on the distributional consequences of systematic monetary and fiscal policy (here in response to uncertainty shocks). While they highlight labor market effects, we focus on portfolios. We share this focus with Kaplan et al. (2017), who discuss the transmission of monetary policy,[7] and with Guerrieri and Lorenzoni (2017), who model the effect of a credit crunch.[8]

The remainder of the paper is organized as follows. Section 2 estimates changes in household income risk and their effects on aggregates and household portfolios. Section 3 provides an intuition for these findings. Section 4 develops our quantitative model, and Section 5 discusses the solution method. Section 6 explains the calibration of the model. Section 7 presents the numerical results. Section 8 concludes. An appendix provides details on the properties of the value and policy functions, the numerics, the

[5]With sticky prices, firms target a higher markup the more uncertain the future aggregate price level.

[6]Born and Pfeifer (2014) report an output drop of 0.025% for a similar model and policy risk shock under a slightly different calibration. Regarding TFP risk, they find hardly any aggregate effect.

[7]Luetticke (2017) builds on the framework of our paper to discuss the transmission of monetary policy.

[8]Further examples of the New-Keynesian incomplete markets literature are Auclert (2015); Challe and Ragot (2016); McKay et al. (2016); McKay and Reis (2016); Werning (2015), all of which, however, build on a single-asset framework.

estimation of the uncertainty process from income data, and further robustness checks.

2 Empirical Evidence

To analyze the aggregate effects of shocks to household income risk, we first need to identify these shocks. For this purpose, we employ data from the Survey of Income and Program Participants (SIPP), covering the time period 1984-2013, and estimate a process for household income and its shock distribution similar to Storesletten et al. (2001, 2004), who estimate differences in income risk between recessions and booms. Different to their approach, we do not restrict ourselves to a generic business cycle relationship but, instead, want to estimate a sequence of shocks to income risk first and then study their effect on household portfolios and a manifold of aggregate variables.

The central idea of Storesletten et al.'s approach is to identify differences in the variance of persistent income shocks over time by comparing different cohorts of households at a given age. Like differences in the growth rings of a tree, the variance of income within a cohort memorizes the variances of shocks the cohort faced in the past to the extent that income is persistent. Since households of different cohorts accumulated income shocks at different times, differences across cohorts in terms of their within-cohort variance can identify the evolution of income-shock variances over time. Take for example a pure unit-root income process. In this example, the income variance of a cohort at a given point in time is simply the sum of all the variances of income shocks that cohort went through. Averaging the increase in income variances over all working-age cohorts between two adjacent quarters gives an estimate of the average income risk in that period. Below unit-root persistence in income, transitory income shocks, sampling uncertainty, and the persistence of income risk complicate the estimation, but the procedure we lay out below follows this basic intuition.

2.1 Estimating Income Risk Over Time

2.1.1 Income Process

Since the focus of this paper is on private self-insurance, our income measure is household labor income after taxes and transfers. The SIPP data are originally available at monthly frequency and represent individual-level income data. We aggregate these data to the household level and to quarterly frequency, restricting the data to households whose head is at least 30 and below 56 years of age. We generate household labor income by summing over household head and spouse and impute taxes and transfers using TAXSIM.

We assume that the labor income of a household after taxes and transfers is composed of a transitory, a persistent, a household-fixed and a deterministic component, i.e., income y of household i in quarter t is given by:

$$\begin{aligned}
\log y_{it} &= f(o_{it}) + \tau_{it} + h_{it} + \mu_i, \qquad (1) \\
h_{it} &= \sum_{s=c}^{t} \rho_h^{t-s} \epsilon_{is}^h, \\
\tau_{it} &= \epsilon_{it}^\tau + \rho_\tau \epsilon_{it-1}^\tau, \\
\epsilon_{it}^\tau \sim \mathcal{N}(0, \sigma_\tau^2), &\qquad \epsilon_{it}^h \sim \mathcal{N}(0, \sigma_{\epsilon,t}^2), \qquad \mu_i \sim \mathcal{N}(0, \sigma_\mu^2),
\end{aligned}$$

where c defines a cohort by the quarter when a household head turns 30, $f(o_{it})$ measures the effect of observable household characteristics o_{it}, τ_{it} is a MA(1) transitory shock or measurement error, μ_i is a household fixed effect, and h_{it} is a persistent component.

2.1.2 Risk Process

We assume transitory shocks and fixed effects to be homoscedastic, while the variance $\sigma_{\epsilon,t}^2$ of the shocks ϵ_{it}^h to the persistent component, h_{it}, evolves slowly according to a log-AR(1) process around a quadratic time trend:

$$\begin{aligned}
\sigma_{\epsilon,t}^2 &= \bar{\sigma}_\epsilon^2 \exp\left(s_t + t\theta_1 + t^2\theta_2\right), \qquad (2) \\
s_{t+1} &= \rho_s s_t + \epsilon_t^s, \\
\epsilon_t^s &\sim \mathcal{N}\left(-\frac{\sigma_s^2}{2(1+\rho_s)}, \sigma_s^2\right).
\end{aligned}$$

2.1.3 Autocovariance Structure of Residual Income

In a first step, we estimate $f(o_{it})$ by OLS to remove this non-stochastic part and work with residual income, $\tau_{it} + h_{it} + \mu_i$. We then generate short panels of residual income and its first two lags for each household in a sample quarter $t = 1 \ldots T$. From these data sets, we calculate $\mathbf{ac}_{0,j}(c,t), j = 0,1,2$, i.e., the sample variance ($j = 0$) and first two auto-covariances ($j = 1,2$) of residual income for cells defined by survey quarter, t, and c, the quarter when a household head turned 30.

These empirical auto-covariances equal their theoretical counterparts $\omega_{0,j}(c,t), j = 0,1,2$, up to sampling error. Substituting in the variances for the various terms in equation (2) yields for the theoretical auto-covariances

$$\begin{aligned}
\omega_{0,0}^2(c,t) &= (1+\rho_\tau)\sigma_\tau^2 + \sigma_{\mu,c}^2 + \sigma_h^2(c,t), \\
\omega_{0,1}^2(c,t) &= \rho_\tau\sigma_\tau^2 + \sigma_{\mu,c}^2 + \rho_h\sigma_h^2(c,t-1), \\
\omega_{0,2}^2(c,t) &= \sigma_{\mu,c}^2 + \rho_h^2\sigma_h^2(c,t-2),
\end{aligned} \tag{3}$$

where $\sigma_h^2(c,t)$ is the variance of the persistent income component of cohort c at quarter t. This itself evolves slowly and accumulates income-risk shocks according to:

$$\begin{aligned}
\sigma_h^2(c,t) &= \sum_{j=c}^{t} \rho_h^{2(t-j)}\sigma_{\epsilon,j}^2 = \bar{\sigma}_\epsilon^2 \sum_{j=c}^{t} \rho_h^{2(t-j)} \exp\left(s_j + j\theta_1 + j^2\theta_2\right), \\
s_t &= \rho_s s_{t-1} + \epsilon_t^s.
\end{aligned} \tag{4}$$

2.1.4 Estimator

Equations (3) and (4) allow us to formulate a quasi-maximum likelihood estimator for the parameters of interest $(\rho_h, \rho_s, \rho_\tau, \bar{\sigma}_\epsilon^2, \sigma_\tau^2, \sigma_\mu^2, \sigma_s)$ along with the *sequence of shocks to income risk* ϵ_t^s. Approximating the sampling noise by Gaussian error terms, the (quasi) log-likelihood function of our model is given by

$$-2\log L = \sum_{(c,t)\in S} \psi(c,t)'\Sigma(c,\tau)^{-1}\psi(c,t) + \sum_{j\in T}(\epsilon_j^s)^2/\sigma_s^2 + \#T\log\sigma_s^2, \tag{5}$$

where

$$\psi(c,t) = \begin{pmatrix} \mathbf{ac}_{0,0}(c,t) - \omega_{0,0}(c,t) \\ \mathbf{ac}_{0,1}(c,t) - \omega_{0,1}(c,t) \\ \mathbf{ac}_{0,2}(c,t) - \omega_{0,2}(c,t) \end{pmatrix} \tag{6}$$

is the difference between theoretical and empirical auto-covariances and, as such, is a function of the model parameters and the sequence of income-risk shocks. The matrix $\Sigma(c,t)$ is the corresponding variance-covariance matrix of $\psi(c,t)$ resulting from sampling uncertainty. Note that Σ is cell specific because differences in the income risk between cells lead to differences in sampling uncertainty regarding these income risks. We estimate the matrices $\Sigma(c,t)$ by block-bootstrapping the micro data clustered at the (c,t)-cell level, i.e., preserving the cell and autocorrelation structure of the data. The set S captures all cohort-quarter pairs we observe, i.e., the cohorts 1959Q1 - 2013Q1 (denoted by the quarter they turn 30) between 1983Q4 and 2013Q1 and T is the set of quarters for which we estimate shocks, i.e., 1976Q1-2013Q1. Note that since we estimate the (auto-co-)variances within a cohort-age cell, we control for anything that is common

Table 1: Parameter estimates

ρ_h	ρ_s	$\rho_\mathcal{T}$	$\bar{\sigma}_\epsilon$	$\sigma_\mathcal{T}$	σ_μ	σ_s
0.98 (0.06)	0.84 (0.06)	0.34 (0.01)	0.06 (0.03)	0.12 (0.00)	0.27 (0.01)	0.54 (0.10)

Notes: All parameters correspond to quarterly frequency of the data. Bootstrapped standard errors using a wild bootstrap in parenthesis; see Appendix F.3 for details. A quadratic time trend in the variance of persistent income shocks is included (not reported). The estimate for the average uncertainty $\bar{\sigma}_\epsilon$ includes the average time-trend effect for 1983-2013.

across households in this cell such as average wages, average hours, or average taxes.[9]

Our methodology extends Storesletten et al.'s (2001,2004) method of moments estimator for income risk, reformulating it as a quasi-maximum likelihood one.[10] More fundamentally, it follows the idea of pseudo-panels as pioneered by Deaton (1985), i.e., we treat each short panel of residual income and its two lags as an independent data set and aggregate the data in terms of second moments to the cohort level. These aggregated data are the unit of observation on which we estimate the non-linear model for the laws of motion for income and shock variances. Further details can be found in Appendix F.

2.1.5 Estimation Results

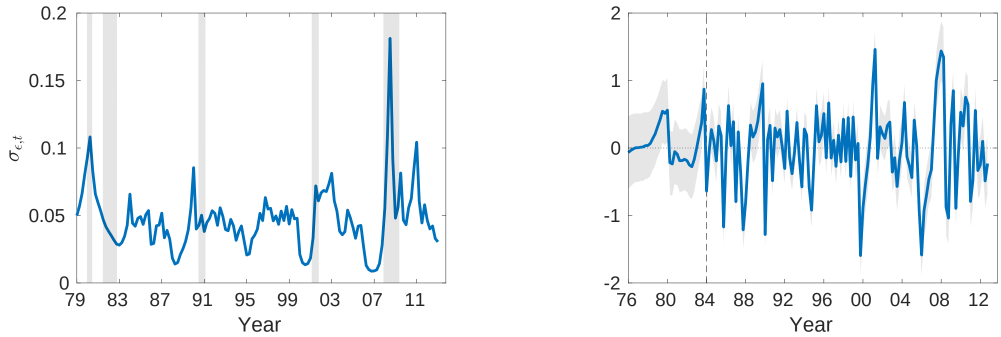
Table 1 presents the estimation results. Income is persistent with a quarterly autocorrelation of ($\rho_h = 0.98$), which is slightly below the corresponding annual autocorrelation that Storesletten et al. (2004) report. The baseline persistent income risk is with $\bar{\sigma}_\epsilon = 0.06$ comparable to their numbers.

The estimated variability of income risk is large. The standard deviation of s, $\sigma_s = 0.54$, implies that on average roughly every 10 years there is an income risk shock that triples income risk, which is equivalent to a two standard deviation shock. The persistence of income risk ($\rho_s = 0.84$) is in the range of typical business cycle fluctuations.

[9] We force $\sum_{j \in T} \epsilon_j^s = 0$. For 1959Q1-1975Q4 we set the shocks ϵ_t^s to zero because, as the persistence of h is below one, income has limited memory. In turn, risk shocks occurring long before the first observation of income, i.e., long before 1983Q4, have very little impact on any empirical variance and are hence weakly identified and the estimate of σ_s becomes biased.

[10] The parameters could also be estimated using such a method of moments approach. For example, one could use a two step procedure. In a first step, one minimizes the first sum in (5) searching over the parameters of the income process. The second step then uses the residuals from this first step and fits the autoregressive process for income risk $\sigma_h^2(c, t)$.

Figure 2: Estimated level of household income risk over time



(a) Level (b) Shocks

Notes: Quarterly data come from the SIPP files 1984-2013 for after tax household level income. Only households with at least two married adults, the oldest male being age 30-55, are admitted. Household income is the sum of the incomes of the oldest male and female in a household. Left panel: Estimated standard deviation of persistent income shocks for the period 1979 to 2013. NBER recession dates in gray. Right panel: Shocks to income risk, bootstrapped one standard deviation confidence bounds shaded in gray. The dotted vertical line shows the first quarter for which we have observations from the SIPP.

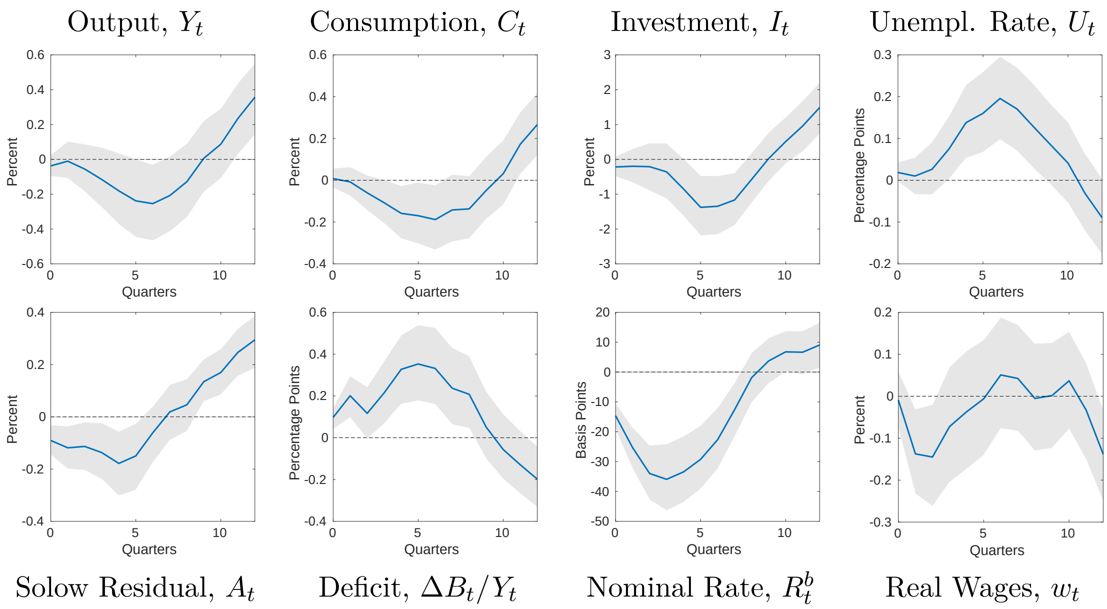
Figure 2 (a) displays the estimated series of persistent income risk and panel (b) displays the estimated sequence of shocks, ϵ_t^s, to income risk together with their confidence bounds. As one can see, income risk is low late in a boom and typically increased at the onset of a recession. The Great Recession stands out in size of income risk.

2.2 Responses to Shocks to Income Risk

We use the estimated sequence of shocks to household income risk, $\{\epsilon_t^s\}_{t=1976Q1...2013Q1}$, to estimate their aggregate repercussions and the effects they have on the portfolios households hold. We focus on the post-Volcker disinflation era and discard all estimated shocks before 1983Q1 as structural breaks in monetary policy may impact results. Another reason is that the shocks before the start of the SIPP sample are not well identified because ρ_h is below 1.

We first estimate the effect of household income risk on aggregate economic activity and average household portfolios. As we will see, upon an increase in income risk, aggregate output falls. Investment declines particularly strongly, as households rebalance their portfolios toward liquid assets, while the (equilibrium) return premium on illiquid assets goes up. Combining the income risk series with cross-sectional information on household portfolios from the Survey of Consumer Finances (SCF), we find that the increase in the liquidity of household portfolios is particularly concentrated among the

Figure 3: Empirical response to household income risk shock



Notes: Estimated response of $\mathbf{X}_{t+j}, j = 0 \ldots 12$, where $\mathbf{X}_t = [Y_t, C_t, I_t, A_t, \Delta B_t/Y_t, w_t, R_t^b, U_t]$, to the estimated shocks to household income risk, ϵ_t^s. The regressions control for the lagged state of the economy $\mathbf{X}_{t-1}$ and lagged levels of income risk s_{t-1}. The nominal rate is the 3-month T-bill rate. Bootstrapped 66% confidence bounds in gray (block bootstrap).

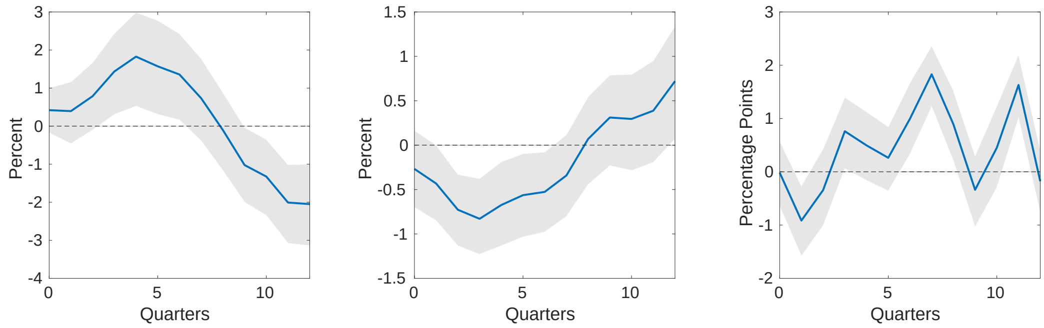
relatively poor.

2.2.1 Aggregate Response

Figure 3 shows the response of aggregate variables to an increase in household income risk. We estimate the response by local projections using the shock series we identified from the SIPP data while controlling for lagged aggregate variables, lagged income risk and a time trend. In line with the specification of our model, we assume that the realized uncertainty shock in time $t+1$ is observed at time t. As a robustness check, we have estimated impulse responses using a different ordering of variables controlling also for the contemporaneous response of aggregates. Results are similar and can be found in Appendix H.2.

Upon a one standard deviation increase in income risk, output falls by roughly 0.2% on average over the first year. The trough is reached six quarters after the shock with a 0.3% decrease in output. Consumption has very similar dynamics but goes down slightly less. Investment drops too, but its reaction is roughly five times as strong as the output reaction. The measured Solow residual from Fernald's TFP series (Fernald, 2012) falls as well. One explanation could be that upon a decrease in aggregate demand,

Figure 4: Response of household portfolios, house prices and the liquidity premium to household income risk shock



Portfolio Liquidity House Prices Liquidity Premium

Notes: Estimated response of the liquidity of household portfolios, the price of houses (Case-Shiller S&P Index), and the difference between the return on housing and the nominal rate (Liquidity Premium) to income risk using local projections. The set of control variables is as in Figure 3. Bootstrapped 66% confidence bounds in gray (block bootstrap).

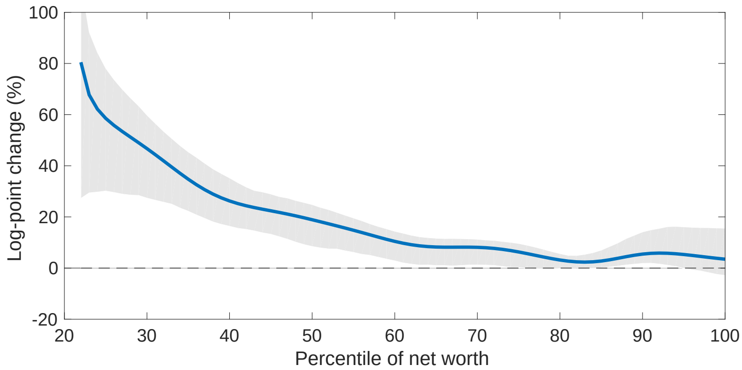
markups go up as they do in New-Keynesian models and this is captured as a decrease in measured TFP (see Hall, 1989). Real wages fall slightly, the unemployment rate goes up by 0.2 percentage points. The government seems to react systematically by making use of stabilizing monetary and fiscal policy – government deficits go up by 0.25 percentage point of GDP over the first year and the nominal return on 3-month Treasury bills goes down on average by 35 basis points (annualized) over the first four quarters after the shock. After roughly ten quarters the recessionary effect of the income risk shock becomes expansionary and output, consumption, and investment overshoot their trends.

The decline in investment – despite a decrease in interest rates – finds its repercussions in household balance sheets; see Figure 4. The ratio of liquid-to-illiquid assets goes up after an increase in household income risk. We calculate this ratio from the Flow of Funds (Table Z1-B.101) by subsuming as liquid assets all deposits, cash, debt securities (including government bonds), and loans held directly, while we treat all other real and financial assets as illiquid.[11]

A part of the increase in the liquidity of household portfolios is driven by real house prices as houses make up the lion's share of the illiquid assets of households (close to 50% on average; see Kaplan and Violante, 2014). Hence any change in house prices directly

[11] Kaplan et al. (2017) use a very similar taxonomy to split assets into liquid and illiquid. The reason for treating equities as illiquid is that most equities are held in the form of pension funds. Equity shares held directly play a role only above the 85th wealth percentile, but even these are often closely held equities such as S-corporations or other illiquid forms. Publicly traded equities, which a single household can sell without price impact, play a significant role in household portfolios only for a relatively small fraction of households and a small fraction of the aggregate capital stock.

Figure 5: Response of portfolio liquidity to household income risk by wealth percentile



Notes: Estimated log difference of liquid-to-illiquid ratio of household portfolios across the wealth distribution in response to a one standard deviation shock to household income risk. Income risk shocks are identified from the SIPP. Portfolio composition is estimated from the SCF years 1983-2013, only households with at least two adults and the household head is between 30 and 55 years of age. Bootstrapped 66% confidence bands in gray.

affects portfolio liquidity. However, as house prices, measured by the Case-Shiller index, fall only by 1% after an increase in uncertainty (see Figure 4), they can make up only for about a quarter of the increase in liquidity. The largest part of the increase in portfolio liquidity must therefore come from outright different reactions in the demand for liquid and illiquid assets. In fact, the return premium of houses over liquid assets, measured as the rent plus price increase of houses relative to the three month T-bill return, increases relatively quickly after the shock to household income risk; see again Figure 4.[12]

2.2.2 Response by Wealth Group

As our theoretical explanation focuses on heterogeneity among households as a result of uninsurable risk, it will have rich cross-sectional implications for households' responses to income risk that go beyond average household portfolios and the differential changes in the return on liquid and illiquid assets.

To provide evidence along this dimension, we use the waves 1983-2013 of the Survey of Consumer Finances (SCF). The data sets contain detailed information on household balance sheets. In line with our treatment of the SIPP data, which we use to estimate income risk, we restrict the sample to households whose head is between 30 and 55 years

[12]We proxy the liquidity premium by the realized return on housing (rent-price ratio in t plus realized growth rate of house prices in $t+1$) relative to the nominal rate. The house price we use is the Case-Shiller S&P national house price index. Rents are imputed on the basis of the CPI for rents of primary residences, fixing the rent-price ratio in 1981Q1 to 4%.

of age and married, and to households with at least two adults. This sample selection not only makes the wealth and income data comparable, but also limits the compositional effects of demographic change.

For each wave, t, of the SCF we estimate a function that maps the percentile rank, prc, of a household in the total wealth distribution into liquid, $\lambda^{LI}(prc, t)$, and illiquid asset holdings, $\lambda^{IL}(prc, t)$, by a local-linear regression; see Appendix G for details. Using this function, we then calculate the average ratio of liquid to illiquid assets of a household at a given percentile in the wealth distribution $\lambda(prc, t) = \frac{\lambda^{LI}(prc,t)}{\lambda^{IL}(prc,t)}$.

We calculate the average shock in the year preceding an SCF wave, $\bar{\epsilon}^s_{t-1}$, and regress the liquidity ratio of all percentiles (above the 20th) on the shock, an intercept and a linear time trend, following the idea of a local projection here, too:

$$\lambda(prc, t) = \gamma_0(prc) + \gamma_1(prc)t + \gamma_2(prc)\bar{\epsilon}^s_{t-1} + \zeta.$$

Figure 5 shows the coefficients, $\gamma_2(prc)$, of the uncertainty shock for this regression. Again, we bootstrap the confidence bands. The poorer a household, the stronger its increase in liquidity holdings.

3 A Simple Expository Partial Equilibrium Model

To provide intuition on why households want to increase the liquidity of their portfolios upon an increase in income uncertainty, we commence with a stylized 3-period model of income uncertainty and portfolio decisions. We present our quantitative general equilibrium model in Section 4. The 3-period model is meant for illustrative purposes and focuses on the effect of uncertainty on asset demand without discussing aggregate effects.

Households hold an endowment $y > 0$ in period 1, which they can either consume, c_1, or invest. When investing they have to decide between illiquid assets, k_1, and liquid assets, b_1. The liquid asset pays a zero net return in period 2 (a storage technology). The illiquid asset pays a positive net return, but only in period 3. In period 2, half of the households obtain high income, $y^H = y + \sigma, \sigma > 0$, half obtain low income $y^L = y - \sigma \geq 0$. They can again invest in a liquid zero-net-return asset in positive amounts, but they can neither borrow against the illiquid asset nor sell it. There is no endowment in period 3.

With their consumption and savings decisions, households maximize the sum over period felicities, $u(c_t) = \frac{c_t^{1-\xi}}{1-\xi}$, i.e., we abstract from discounting. Since in period 2 all uncertainty is resolved, consumption in period 3 will only depend on income in period 2. Let $c_t^{H,L}$ denote consumption in period t after income is H(igh) or L(ow) in period 2, respectively. Since all uncertainty is revealed by then, the household splits resources evenly between period 2 and 3 if no borrowing constraint binds. When the constraint binds, the household does not save in period 2 and hence consumes all income and liquid assets in period 2, being left with only illiquid asset income in period 3. If gross returns on the illiquid asset, R^k, are not too high, the household will not be constrained in the high income state. Moreover, households will only hold liquid assets despite the higher return on the illiquid asset if they expect to be constrained in the low income case. Therefore, we focus on this case (see Appendix A for details), such that:

$$\begin{aligned} c_2^H &= \tfrac{1}{2}(b_1 + y + \sigma + R^k k_1) \quad & c_3^H &= c_2^H \\ c_2^L &= b_1 + y - \sigma & c_3^L &= R^k k_1. \end{aligned}$$

This means that, keeping investment and savings decisions in period 1 fixed, an increase in income risk, σ, leaves consumption c_3^L unchanged because the borrowing constraint binds in period 2, but it increases consumption c_3^H because the household saves some of its income from period 2 in the high income state. Therefore average consumption in period 3 rises with income risk σ. Conversely, consumption in period 2 falls more in the bad state in σ than it rises in the good one and average consumption in period 2 falls in σ. In other words, an increase in risk shifts consumption *ceteris paribus* from period 2, when capital is illiquid, to period 3, when all assets become liquid. Anticipating this shift, the household has an incentive to undo it by rebalancing its portfolio in period 1, increasing the share of liquid assets. Importantly, this comes on top of the precautionary motives that lead to an increased demand for liquid assets also in a setup without any illiquid investment option.

This implies that the demand for *liquid* assets increases more strongly in uncertainty, when households can invest in an illiquid asset, than it does when they cannot:

Proposition 1. *Define $b_1^*(\sigma), k_1^*(\sigma)$ the optimal liquid and illiquid asset holdings. Define $\tilde{b}_1(\sigma)$ the liquid asset holdings of a household that does not have the option to invest in an illiquid asset. Now, suppose income uncertainty is large enough, such that $b_1^*(\sigma) > 0$ and the returns on the illiquid investment are positive but not too large, i.e. $1 < R^k < \left(1 + R^{k\,\frac{\xi-1}{\xi}}\right)^{\xi}$. Then $\frac{\partial b_1^*}{\partial \sigma} > \frac{\partial \tilde{b}_1}{\partial \sigma} > 0 > \frac{\partial k_1^*}{\partial \sigma}$, i.e., liquid asset holdings increase in σ and they increase more than in a model where all assets are liquid, while illiquid asset holdings decrease.*

Proof. See Appendix A. □

From a macroeconomic perspective, this shift in portfolios becomes important when changes in the demand for liquid and illiquid assets impact the demand for final goods differently. In our general equilibrium model in the next section, this is the case. A decrease in the demand for illiquid assets leads to an immediate decline in physical investment. An increase in the demand for liquid assets leads to an increase in goods demand only when the suppliers of these assets, households and the government, decide to use the extra funds. In turn, when output is demand determined, because prices are sticky, this decrease in demand due to higher income uncertainty leads to a decrease in output. Portfolio rebalancing exacerbates this demand-driven downturn, all of which is in line with our empirical findings in the previous section.

4 Quantitative Model

To understand the quantitative importance of portfolio rebalancing in response to changes in income uncertainty, we build a dynamic model of heterogeneous households with incomplete markets, time variations in income risks and sticky prices. The economy is composed of a firm sector, a household sector and a government sector. Firms are either perfectly competitive intermediate goods producers or final goods producers that face monopolistic competition, producing differentiated final goods out of homogeneous intermediate inputs. Price setting for these goods is subject to a pricing friction à la Rotemberg (1982). Households supply labor and capital and own all final goods producers, absorbing their rents. The government sector runs both a fiscal authority and a monetary authority. The fiscal authority levies a time-constant labor income tax, issues government bonds, and adjusts expenditures both to business cycle conditions and to stabilize debt in the long run. The monetary authority sets the nominal interest rate on government bonds according to a Taylor rule.

4.1 Households

The household sector is subdivided into two types of agents: workers and entrepreneurs. The transition between both types is stochastic. Both rent out physical capital, but only workers supply labor. The efficiency of a worker's labor evolves randomly exposing worker-households to labor-income risk. Entrepreneurs do not work, but earn all pure rents in our economy. All households self-insure against the income risks they face by saving in a liquid nominal asset (bonds) and a less liquid physical asset (capital). Trading illiquid capital is costly as in Kaplan and Violante (2014).

To be specific, there is a continuum of ex-ante identical households of measure one, indexed by i. Households are infinitely lived, have time-separable preferences with time-discount factor β, and derive felicity from consumption c_{it} and leisure. They obtain income from supplying labor, n_{it}, from renting out capital, k_{it}, and from interest on bonds, b_{it}. Whenever a household adjusts its holdings of capital, it needs to pay some felicity cost χ_{it} that is an i.i.d. draw from a logistic distribution.[13] Holdings of bonds have to be above an exogenous debt limit $\underline{B}$, and holdings of capital have to be non-negative.

A household's labor income $w_t h_{it} n_{it}$ is composed of the aggregate wage rate, w_t, the household's hours worked, n_{it}, and its idiosyncratic labor productivity, h_{it}. In line with our empirical specification, we assume that productivity evolves according to a log-AR(1) process with time-varying volatility and a fixed probability of transition between the worker and the entrepreneur state:

$$\tilde{h}_{it} = \begin{cases} \exp\left(\rho_h \log \tilde{h}_{it-1} + \epsilon^h_{it}\right) & \text{with probability } 1-\zeta \text{ if } h_{it-1} \neq 0, \\ 1 & \text{with probability } \iota \text{ if } h_{it-1} = 0, \\ 0 & \text{else,} \end{cases} \tag{7}$$

with individual productivity $h_{it} = \frac{\tilde{h}_{it}}{\int \tilde{h}_{it} di}$ such that $\tilde{h}_{it}$ is scaled by its cross-sectional average, $\int \tilde{h}_{it} di$ to make sure that average worker productivity is constant. The shocks ϵ^h_{it} to productivity are normally distributed with time-varying variance as given by

$$\begin{aligned} \sigma^2_{h,t} &= \bar{\sigma}^2_h \exp s_t, \\ s_{t+1} &= \rho_s s_t + \epsilon^s_t, \\ \epsilon^s_t &\sim \mathcal{N}\left(-\frac{\sigma^2_s}{2(1+\rho_s)}, \sigma^2_s\right), \end{aligned} \tag{8}$$

i.e., at time t households observe a change in the variance of shocks that drive the next period's productivity. In words, we assume that idiosyncratic productivity normally evolves according to a log AR(1) process with time-varying variance.[14] With probability ζ households become entrepreneurs ($h = 0$). With probability ι an entrepreneur returns to the labor force with median productivity. An entrepreneurial household obtains a fixed share of the pure rents, Π_t, in the economy (from monopolistic competition and

[13] Kaplan and Violante (2014) find that physical transaction costs and utility costs yield similar results for the portfolio problem. Assuming a logistic distribution of adjustment costs yields closed-form solutions for expected adjustment costs given the value of adjustment.

[14] For simplicity, we abstract from transitory income shocks and permanent income differences in the model. We assume that uncertainty fluctuations are exogenous partly for analytical clarity. Likely some of the fluctuations in uncertainty in the data reflect endogenous responses through, say, unemployment as in Ravn and Sterk (2017) or, beyond the labor income focus of our paper, a change in the insurance offered by financial markets as in Brunnermeier and Sannikov (2016).

creation of capital).[15] We assume that the claim to the pure rent cannot be traded as an asset.

This modeling strategy, the introduction of an exogenous entrepreneur state, serves two purposes. First and foremost, it solves the problem of the allocation of pure rents without distorting factor returns and without introducing another tradable asset – an issue in any heterogeneous agent New-Keynesian model.[16] Second, we use the entrepreneur state – as a transitory state in which incomes are typically extremely high – to match the wealth distribution following the idea by Castaneda et al. (1998). The entrepreneur state does not change the asset returns or investment opportunities available to households.

With respect to leisure and consumption, households have Greenwood-Hercowitz-Huffman (GHH) preferences and maximize the discounted sum of felicity:

$$E_0 \max_{\{c_{it}, n_{it}, \Delta k_{it}\}} \sum_{t=0}^{\infty} \beta^t u\left[c_{it} - G(h_{it}, n_{it})\right] - \mathbb{I}_{\Delta k_{it} \neq 0} \chi_{it}, \tag{9}$$

where χ_{it} is the utility cost of adjustment and $\mathbb{I}_{\Delta k_{it} \neq 0}$ is an indicator function that takes value one if a household adjusts its holdings of physical capital and zero otherwise. The assumption of GHH preferences simplifies the numerical analysis substantially and allows us to abstract from the labor supply effects of uncertainty.[17] The maximization is subject to the budget constraints described further below. The felicity function u

[15]Note that the notation with $h = 0$ for the entrepreneur state is somewhat counterintuitive: the entrepreneur state will be a high income state. The assumption of stochastic transitions to the entrepreneurial state, $h = 0$, can be thought of as a household inventing a new version of a differentiated product that replaces an older existing version of that product (keeping the mass of products constant). We assume that the innovation is drastic such that the old product disappears and plays no role in price setting. The innovating household then focuses exclusively on the production of this product and can no longer supply any additional labor. The incumbent household returns to the labor force.

[16]The assumption of how to allocate pure rents is borrowed from Romei (2015). Attaching the rents in the economy to an exogenously determined group of households instead of distributing it with the factor incomes for capital or labor has the advantage that the factor prices and thus factor supply decisions remain the same as in any standard New-Keynesian framework. Allocating pure rents exogenously is not the only way to allocate them without distorting factor returns, but it is the only way to avoid the introduction of a third asset, which then would need to be priced. If one is willing to assume that pure and capital rents come in illiquid form, as in Kaplan et al. (2017), pure rents can be priced using the rate of return on capital. However, this approach requires not only assets (claims on physical capital and claims on future pure rents) to be illiquid, but also their corresponding asset income.

[17]Basu and Bundick (2017) (their appendix Figure D.6) show that the assumption of Greenwood et al. (1988) preferences leads to a smaller recessionary response to uncertainty than under King et al. (1988) preferences. On the other hand, Auclert and Rognlie (2017) show that under Greenwood et al. (1988) preferences, monetary and fiscal multipliers tend to be larger than under King et al. (1988) preferences. This ought to be taken into account when interpreting our results.

exhibits a constant relative risk aversion (CRRA) with risk aversion parameter $\xi > 0$,

$$u(x_{it}) = \frac{1}{1-\xi} x_{it}^{1-\xi},$$

where $x_{it} = c_{it} - G(h_{it}, n_{it})$ is household i's composite demand for goods consumption c_{it} and leisure and G measures the disutility from work. Goods consumption bundles varieties j of differentiated goods according to a Dixit-Stiglitz aggregator:

$$c_{it} = \left(\int c_{ijt}^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}.$$

Each of these differentiated goods is offered at price p_{jt}, so that for the aggregate price level, $P_t = \left(\int p_{jt}^{1-\eta} dj \right)^{\frac{1}{1-\eta}}$, the demand for each of the varieties is given by

$$c_{ijt} = \left(\frac{p_{jt}}{P_t} \right)^{-\eta} c_{it}.$$

The disutility of work, $G(h_{it}, n_{it})$, determines a household's labor supply given the aggregate wage rate, w_t, and a labor income tax, τ, through the first-order condition:

$$\frac{\partial G(h_{it}, n_{it})}{\partial n_{it}} = (1-\tau) w_t h_{it}. \tag{10}$$

Assuming that G has a constant elasticity w.r.t. n, $\frac{\partial G(h_{it}, n_{it})}{\partial n_{it}} = (1+\gamma) \frac{G(h_{it}, n_{it})}{n_{it}}$ with $\gamma > 0$, we can simplify the expression for the composite consumption good x_{it} making use of the first-order condition (10):

$$x_{it} = c_{it} - G(h_{it}, n_{it}) = c_{it} - \frac{(1-\tau) w_t h_{it} n_{it}}{1+\gamma}. \tag{11}$$

When the Frisch elasticity of labor supply is constant, the disutility of labor is always a constant fraction of labor income. Therefore, in both the budget constraint of the household and its felicity function only after-tax income enters and neither hours worked nor productivity appears separately.

This implies that we can assume $G(h_{it}, n_{it}) = h_{it} \frac{n_{it}^{1+\gamma}}{1+\gamma}$ without further loss of generality as long as we treat the empirical distribution of income as a calibration target. This functional form simplifies the household problem as h_{it} drops out from the first-order condition and all households supply the same number of hours $n_{it} = N(w_t)$. Total effective labor input, $\int n_{it} h_{it} di$, is hence also equal to $N(w_t)$ because $\int h_{it} di = 1$. This means that we can read off productivity risk directly from the estimated income risk and treat both interchangeably. Correspondingly, we will – as a shorthand notation – call the risk households face regarding their productivity "income risk" and the shocks to h "income shocks," accordingly.

The households optimize subject to their budget constraint:

$$c_{it} + b_{it+1} + q_t k_{it+1} = b_{it}\frac{R(b_{it}, R_t^b)}{\pi_t} + (q_t + r_t)k_{it} + (1-\tau)(w_t h_{it} N_t + \mathbb{I}_{h_{it}=0}\Pi_t),$$
$$k_{it+1} \geq 0, b_{it+1} \geq \underline{B},$$

where b_{it} is real bond holdings, $\underline{B}$ is an exogenous borrowing constraint, k_{it} is the amount of illiquid assets, q_t is the price of these assets, r_t is their dividend, $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ is realized inflation, and R is the nominal interest rate on bonds, which depends on the portfolio position of the household and the central bank's interest rate R_t^b, which is set one period before. All households that decide not to participate in the capital market ($k_{it+1} = k_{it}$) still obtain dividends and can adjust their bond holdings. Depreciated capital has to be replaced for maintenance, such that the dividend, r_t, is the net return on capital.

We assume that there is a wasted intermediation cost, $\overline{R}$, when households resort to unsecured borrowing and specify:

$$R(b_{it}, R_t^b) = \begin{cases} R_t^b & \text{if } b_{it} \geq 0 \\ R_t^b + \overline{R} & \text{if } b_{it} < 0. \end{cases}$$

This assumption creates a mass of households with zero unsecured credit but with the possibility to borrow, though at a penalty rate.

Substituting the expression $c_{it} = x_{it} + \frac{(1-\tau)w_t h_{it} N_t}{1+\gamma}$ for consumption, we obtain:

$$x_{it} + b_{it+1} + q_t k_{it+1} = b_{it}\frac{R(b_{it}, R_t^b)}{\pi_t} + (q_t + r_t)k_{it} + (1-\tau)\left(\frac{\gamma}{1+\gamma} w_t h_{it} N_t + \mathbb{I}_{h_{it}=0}\Pi_t\right),$$
$$k_{it+1} \geq 0, \quad b_{it+1} \geq \underline{B}. \tag{12}$$

Since a household's saving decision will be some non-linear function of that household's wealth and productivity, inflation, π_t, and accordingly aggregate real bond holdings, B_{t+1}, will be functions of the joint distribution, Θ_t, of (b, k, h) in t. This makes Θ_t a state variable of the household's planning problem. This distribution evolves as a result of the economy's reaction to shocks to uncertainty that we model as in (2).

Three functions thus characterize the household's problem: The value function V_a for the case where the household adjusts its capital holdings, the value function V_n for the case in which it does not adjust, and the expected envelope value, EV, over both:

$$
\begin{aligned}
V_a(b,k,h;\Theta,R^b,s) &= \max_{k',b'_a} u[x(b,b'_a,k,k',h)] + \beta EV(b'_a,k',h,\Theta',R^{b'},s') \\
V_n(b,k,h;\Theta,R^b,s) &= \max_{b'_n} u[x(b,b'_n,k,k,h)] + \beta EV(b'_n,k,h,\Theta',R^{b'},s') \qquad (13) \\
EV(b',k',h;\Theta,R^b,s) &= E_{\chi',h',s'}\left\{\max\left[V_a(b',k',h';\Theta',R^{b'},s') - \chi', V_n(b',k',h';\Theta',R^{b'},s')\right]\right\}
\end{aligned}
$$

Expectations about the continuation value are taken with respect to all stochastic processes (productivity, adjustment costs, and uncertainty) conditional on the current states.

Conditional on paying the adjustment cost, the household will choose a portfolio that trades off the higher liquidity of bonds against the higher return that illiquid assets pay (in equilibrium). The value of liquidity stems from smoother consumption. We denote the optimal consumption policies for the adjustment and non-adjustment cases as x_a^* and x_n^*, the bond holding policies as b_a^* and b_n^*, and the capital investment policy as k^*.

The household will pay the fixed cost to adjust its portfolio if and only if

$$
V_a(b',k',h';\Theta',R^{b'},s') - \chi' \geq V_n(b',k',h';\Theta',R^{b'},s'),
$$

such that the probability to adjust is given by

$$
\nu^*(b',k',h';\Theta',s') := F_\chi\left[V_a(b',k',h';\Theta',R^{b'},s') - V_n(b',k',h';\Theta',R^{b'},s')\right], \qquad (14)
$$

where F_χ is the cumulative distribution function of χ. We assume this distribution to be logistic, so that the EV term has a closed-form expression given $V_{a,n}$. Details on the properties of the value functions and policy functions (differentiable and increasing in total resources), the first-order conditions, and the algorithm we employ to calculate the policy functions can be found in Appendices B and C.

4.2 Intermediate Goods Producers

Intermediate goods are produced with a constant returns to scale production function:

$$
Y_t = N_t^\alpha K_t^{(1-\alpha)}.
$$

Let MC_t be the relative price at which the intermediate good is sold to entrepreneurs.

The intermediate-good producer maximizes profits,

$$MC_tY_t - w_tN_t - (r_t+\delta)K_t = MC_tN_t^{\alpha}K_t^{(1-\alpha)} - w_tN_t - (r_t+\delta)K_t,$$

but it operates in perfectly competitive markets, such that the real wage and the user costs of capital are given by the marginal products of labor and capital:

$$w_t = \alpha MC_t\left(K_t/N_t\right)^{1-\alpha}, \qquad r_t+\delta = (1-\alpha)MC_t\left(N_t/K_t\right)^{\alpha}.$$

4.3 Price Setting

Final-goods producers differentiate the intermediate good and set prices. We assume price adjustment costs à la Rotemberg (1982). For tractability, we assume that price setting is delegated to a mass-zero group of households (managers) that are risk neutral and compensated by a share in profits. They do not participate in any asset market. Under this assumption, managers maximize the present value of real profits given the demand for good j,

$$y_{jt} = \left(p_{jt}/P_t\right)^{-\eta} Y_t,$$

and quadratic costs of price adjustment, i.e., they maximize:

$$E_0\sum_{t=0}^{\infty}\beta^tY_t\left\{\left(\frac{p_{jt}}{P_t}-MC_t\right)\left(\frac{p_{jt}}{P_t}\right)^{-\eta} - \frac{\eta}{2\kappa}\left(\log\frac{p_{jt}}{p_{jt-1}}\right)^2\right\}, \tag{15}$$

with a time constant discount factor.[18] From the corresponding first-order condition for price setting, it is straightforward to derive the Phillips curve:

$$\log(\pi_t) = \beta E_t\left[\log(\pi_{t+1})\tfrac{Y_{t+1}}{Y_t}\right] + \kappa\left(MC_t - \tfrac{\eta-1}{\eta}\right), \tag{16}$$

where π_t is the gross inflation rate, $\pi_t := \frac{P_t}{P_{t-1}}$, and MC_t is the real marginal costs. The price adjustment then creates real costs $\frac{\eta}{2\kappa}Y_t\log(\pi_t)^2$.

Since managers are a mass-zero group in the economy, their consumption does not show up in any resource constraint and all profits – net of price adjustment costs – go to the entrepreneur households (whose $h=0$). In addition, these households also obtain

[18]The choice of the discount factor has relatively little impact on results. Given that we calibrate to a zero inflation steady state, and approximate the aggregate dynamics linearly, only the steady-state value of the discount factor matters in the manager's problem. Our baseline sets the discount factor of managers and households to be equal. We tried setting the managers' time preference rate to the interest rate on bonds and to the median discount factor of entrepreneurs as alternatives. The results are robust.

profit income from adjusting the aggregate capital stock. They can transform I_t consumption goods into ΔK_{t+1} capital goods (and back) according to the transformation function:

$$I_t = \frac{\phi}{2} \left(\Delta K_{t+1}/K_t\right)^2 K_t + \Delta K_{t+1}.$$

Since they are facing perfect competition in this market, entrepreneurs will adjust the stock of capital until the following first-order condition holds:

$$q_t = 1 + \phi \Delta K_{t+1}/K_t.$$

4.4 Government

The government operates a monetary and a fiscal authority. The monetary authority controls the nominal interest rate on liquid assets, while the fiscal authority issues government bonds to finance deficits and adjusts expenditures to stabilize debt in the long run and output in the short run.

We assume that monetary policy sets the nominal interest rate on bonds following a Taylor (1993)-type rule with interest rate smoothing:

$$\frac{R^b_{t+1}}{\bar{R}^b} = \left(\frac{R^b_t}{\bar{R}^b}\right)^{\rho_R} \left(\frac{\pi_t}{\bar{\pi}}\right)^{(1-\rho_R)\theta_\pi}. \tag{17}$$

The coefficient $\bar{R}^b \geq 0$ determines the nominal interest rate in the steady state. The coefficient $\theta_\pi \geq 0$ governs the extent to which the central bank attempts to stabilize inflation around its steady-state value: the larger θ_π the stronger is the reaction of the central bank to deviations from the inflation target. When $\theta_\pi \to \infty$, inflation is perfectly stabilized at its steady-state value. $\rho_R \geq 0$ captures interest rate smoothing.

We assume that the government issues bonds according to the rule (c.f. Woodford, 1995):

$$\frac{B_{t+1}}{\bar{B}} = \left(\frac{B_t R^b_t/\pi_t}{\bar{B}\bar{R}^b/\bar{\pi}}\right)^{\rho_B} \left(\frac{\pi_t}{\bar{\pi}}\right)^{-\gamma_\pi} \left(\frac{\mathcal{T}_t}{\bar{\mathcal{T}}}\right)^{-\gamma_\mathcal{T}}, \tag{18}$$

using tax revenues $\mathcal{T}_t = \tau(w_t N_t + \Pi_t)$ to finance government consumption, G_t, and interest on debt. The coefficient ρ_B captures whether and how fast the government seeks to repay its outstanding obligations $B_t R^b_t/\pi_t$. For $\rho_B < 1$ the government actively stabilizes real government debt, and for $\rho_B = 1$ the government rolls over all outstanding debt including interest. The coefficients $\gamma_\pi, \gamma_\mathcal{T}$ capture the cyclicality of debt issuance: for $\gamma_\pi = \gamma_\mathcal{T} = 0$, new debt does not actively react to tax revenues and inflation, but only to the value of outstanding debt. For $\gamma_\pi > 0 > \gamma_\mathcal{T}$, debt is countercyclical, for $\gamma_\pi < 0 < \gamma_\mathcal{T}$ it is procyclical.

4.5 Goods, Bonds, Capital, and Labor Market Clearing

The labor market clears at the competitive wage given in (**??**). The bond market clears whenever the following equation holds:

$$B_{t+1} = B^d(\Theta_t; R_t^b, s_t; q_t, \pi_t) := E\left[\nu^* b_a^* + (1-\nu^*) b_n^*\right], \tag{19}$$

where ν^*, b_a^*, b_n^* are functions of the states $(b, k, h; R_t^b, s_t)$, of current prices q_t, π_t, and of expectations of future prices. Expectations in the right-hand-side expression are taken w.r.t. the distribution $\Theta_t(b, k, h)$. Equilibrium requires the total *net* amount of bonds the household sector demands, B^d, to equal the supply of government bonds. In gross terms there are more liquid assets in circulation as some households borrow up to $\underline{B}$.

Last, the market for capital has to clear:

$$\begin{aligned} q_t &= 1 + \phi \frac{K_{t+1} - K_t}{K_t}, \\ K_{t+1} &= K^d(\Theta_t; R_t^b, s_t; q_t, \pi_t) := E[\nu^* k^* + (1-\nu^*) k], \end{aligned} \tag{20}$$

where the first equation stems from competition in the production of capital goods, and the second equation defines the aggregate supply of funds from households – both those that trade capital, $\nu^* k^*$, and those that do not, $(1-\nu^*)k$. Again ν^* and k^* are functions of the state variables $(\Theta_t; R_t^b, s_t)$, and current and expected future prices. The goods market then clears due to Walras' law, whenever labor, bonds, and capital markets clear.

4.6 Recursive Equilibrium

A *recursive equilibrium* in our model is a set of policy functions $\{x_a^*, x_n^*, b_a^*, b_n^*, k^*, \nu^*\}$, value functions $\{V_a, V_n, EV\}$, pricing functions $\{r, w, \pi, q, R^b\}$, aggregate capital and labor supply functions $\{K, N\}$, distributions Θ_t over individual asset holdings and productivity, and a perceived law of motion Γ, such that

1. Given $\{V_a, V_n\}$, Γ, prices, and distributions, the policy functions $\{x_a^*, x_n^*, b_a^*, b_n^*, k^*, \nu^*\}$ solve the households' planning problem, and given the policy functions $\{x_a^*, x_n^*, b_a^*, b_n^*, k^*, \nu^*\}$, prices and distributions, the value functions $\{V_a, V_n\}$ are a solution to the Bellman equations (13).
2. The labor, the final goods, the bond, the capital and the intermediate good markets clear, and interest rates on bonds are set according to the central bank's Taylor rule, i.e., (**??**), (16), (19), and (20) hold.
3. The actual and the perceived law of motion Γ coincide, i.e., $\Theta' = \Gamma(\Theta, s')$.

4.7 Price-Level Determinacy

Since our economy is non-Ricardian, price-level determinacy depends not only on monetary policy alone but also on fiscal policy (see Bénassy, 2005; Leith and von Thadden, 2008, for a treatment in an OLG framework, and Linnemann, 2006, for the case of distortionary taxation), because the demand for bonds does not increase one-for-one in outstanding real government debt, even abstracting from the real effects of inflation. Thereby, the demand for government bonds creates a "nominal anchor."

Given expected future inflation, a nominal interest rate and a wealth distribution, households demand a real amount B^d of bonds. Any change in the price level scales the real liquid wealth holdings of all households as well as real government debt. Yet, the change in real liquid wealth does not lead to a proportional increase in the demand for liquid wealth because households want to hold a certain portfolio structure and precautionary savings. Conversely, there is a "Pigou effect" on the demand for goods (Pigou, 1943) and only one price level clears the bond market. This is important as it constitutes the key difference from a representative agent model. Hagedorn (2016) provides a discussion of the special case of an interest rate peg and a government debt rule that allows the government to stabilize nominal government debt and shows that the price level is determinate then. We sketch the idea in Appendix D, where we also show that indeterminacy may still arise if the government overly aims at returning to the steady-state level of real debt. Conversely, if the fiscal policy does not stabilize real debt at all ($\rho_B \geq 1$), the monetary authority needs to violate the Taylor principle ($\theta_\pi < 1$) such that higher inflation reduces the real rate on bonds even in the long run in order to keep real debt stable.

5 Numerical Implementation

The dynamic program (13) and hence the recursive equilibrium is not computable, because it involves the infinite dimensional object Θ_t. We discretize the distribution Θ and represent it by its histogram, a finite dimensional object.

5.1 Solving the Household's Planning Problem

We approximate the idiosyncratic productivity process by a discrete Markov chain with 26 states. We obtain the time-varying transition probabilities for this Markov chain using the method proposed by Tauchen (1986).[19]

In solving for the household's policy functions, we apply an endogenous gridpoint method as originally developed in Carroll (2006) and extended by Hintermaier and Koeniger (2010), iterating over the first-order conditions. We start with a guess for the adjustment probabilities and use (14) to update the adjustment probabilities until convergence. In each iteration we check for concavity of the value functions and find that the value functions are concave on the entire domain on which we solve them, i.e., we operate a special case of the algorithm suggested by Fella (2014). Details on the algorithm can be found in Appendices B.4 and C.

5.2 Aggregate Fluctuations

Even though the histogram is a finitely dimensional object, it is still highly dimensional, which makes it difficult to apply standard techniques to solve for a competitive equilibrium with aggregate risk.

Our baseline approach builds on and extends Reiter (2009) and solves for aggregate dynamics by first-order perturbation around the stationary equilibrium without aggregate shocks. What we add to Reiter's method is to approximate the three-dimensional distribution Θ by a distribution that has a fixed copula and time-varying marginals. To reduce the dimensionality of the value function and its derivatives, we approximate them by a sparse polynomial around their stationary equilibrium solutions. Alternatively, we solve for a Krusell-Smith equilibrium, with very similar results. Details on both methods can be found in Appendix E.

6 Calibration

We calibrate the model to the U.S. economy. The aggregate data used for calibration spans 1983 to 2015 (post-Volcker disinflation). One period in the model refers to a quarter of a year. The choice of parameters as summarized in Tables 2 to 4 is explained next. We present the parameters as if they were individually chosen in order to match a specific data moment, but all calibrated parameters are determined jointly of course.

[19] We solve the household policies for 80 points on the grid for bonds and on the grid for capital using log-scaled grids. We experimented with changing the number of grid points without a noticeable impact on results.

Table 2: Calibrated parameters: Firms and households

Parameter	Value	Description	Target
Households			
β	0.9795	Discount factor	see Table 2
ξ	4	Relative risk aversion	Standard value
γ	1	Inverse of Frisch elasticity	Chetty et al. (2011)
μ_χ	63563	Participation utility costs	see Table 2
σ_χ	22500	Participation utility costs	see Table 2
$\overline{R}$	11%	Borrowing penalty	see Table 2
Intermediate Goods			
α	0.70	Share of labor	Income share of labor of 2/3
δ	1.35%	Depreciation rate	NIPA: Fixed assets
Final Goods			
κ	0.09	Price stickiness	Mean price duration of 4 quarters
η	20	Elasticity of substitution	5% markup
Capital Goods			
ϕ	11.4	Capital adjustment costs	Relative investment volatility of 4.5

6.1 Technology and Preferences

While we can estimate the income process directly from the data, all other parameters are calibrated within the model. Table 2 summarizes our calibration with respect to non-government parameters. In detail, we choose the parameter values as follows.

6.1.1 Intermediate, Final, and Capital Goods Producers

We parameterize the production function of the intermediate good producer according to the U.S. National Income and Product Accounts (NIPA). In the U.S. economy the income share of labor is about 2/3. Accounting for profits we hence set $\alpha = 0.73$.

To calibrate the parameters for the monopolistic competition, we use standard values for markup and price stickiness that are widely employed in the New-Keynesian literature. The Phillips curve parameter κ implies an average price duration of 4 quarters (in the equivalent Calvo setting), assuming flexible capital at the firm level. The steady-state marginal costs, $\frac{\eta-1}{\eta} = 0.95$, imply a markup of 5%.

We calibrate the adjustment cost of capital, $\phi = 11.4$, to match an investment to output volatility of 4.5 conditional on a TFP shock (see Appendix I).

6.1.2 Households

For the felicity function, $u = \frac{1}{1-\xi}x^{1-\xi}$, we set the coefficient of relative risk aversion, $\xi = 4$. The chosen value for the inverse Frisch elasticity of labor supply, $\gamma = 1$, reflects the fact that estimates for the aggregate inverse elasticity typically range between 0.5 and 1 (Chetty et al., 2011).

For the labor income process, we use the estimated coefficients for the persistent component of after-tax household income from Section 2; see Table 1. Because taxes are linear in our model, pre-tax and after-tax incomes are proportional, and our estimator takes out average tax rate changes by controlling for year-effects. We calibrate the probability of leaving the entrepreneurial state to 1/16 per quarter following the numbers that Guvenen et al. (2014a) report on the probability of dropping out of the top 1% income group in the U.S. (see their Table 2, roughly 25% p.a.). The fraction of households in the entrepreneurial state, and hence the probability of entering this state, is calibrated to match the average Gini coefficient of total net worth in our SCF sample (78%).

The time-discount factor, β, and the distribution of costs (pinned down by its mean and variance) of asset market participation, F_{χ}, are jointly calibrated to match the *average ratios* of liquid and illiquid assets to output and the *portfolio liquidity of the poor*. In particular, we target the average portfolio liquidity of the second wealth quintile.

We equate illiquid assets to all capital goods at current replacement values in the NIPA tables (1983-2015) because all illiquid assets in our model are both productive and produced. Because they are not productive assets in the NIPA sense, we disregard non-housing durable consumption goods. This implies for the total value of illiquid assets relative to nominal GDP a capital-to-output ratio of 286% and an annual real return for illiquid assets of 4.5%.[20] We use the Survey of Consumer Finances (SCF) to estimate the liquidity of household portfolios as described in Section 2. We fix the aggregate supply of government bonds, B_t, so as to match the average ratio of aggregate net liquid to net illiquid assets (average 1983-2013: 9%).[21] We consider all deposits, money market accounts, and bonds net of credit card debt as liquid assets. All other assets in the SCF

[20]We calibrate to the capital to GDP ratio in NIPA instead of using the illiquid assets to GDP ratio from the Flow of Funds (roughly 3.3) because the latter contain land as a non-produced asset, debt-assets turned illiquid when held in pension funds, and foreign illiquid assets, but they lack foreign owned capital used in production. We view it as important that our model replicates the production structure of the economy.

[21]This number is relatively close to the ratio of average U.S. federal debt held by domestic private agents relative to capital of 10.5%.

Table 3: Moments targeted in calibration

Targets	Model	Data	Source	Parameter
Mean illiquid assets (K/Y)	2.86	2.86	NIPA	Discount factor
Mean liquidity (B/K)	0.09	0.09	SCF	Mean adj. costs
2nd quintile liquidity (b/k)	0.33	0.33	SCF	Variance adj. costs
Gini total wealth	0.78	0.78	SCF	Fraction of entrepreneurs
Fraction borrowers	0.16	0.16	SCF	Borrowing penalty

and all non-credit-card debts are considered illiquid as in Kaplan et al. (2017). Since we abstracted from consumer durables, we also disregard car wealth and auto loans here.

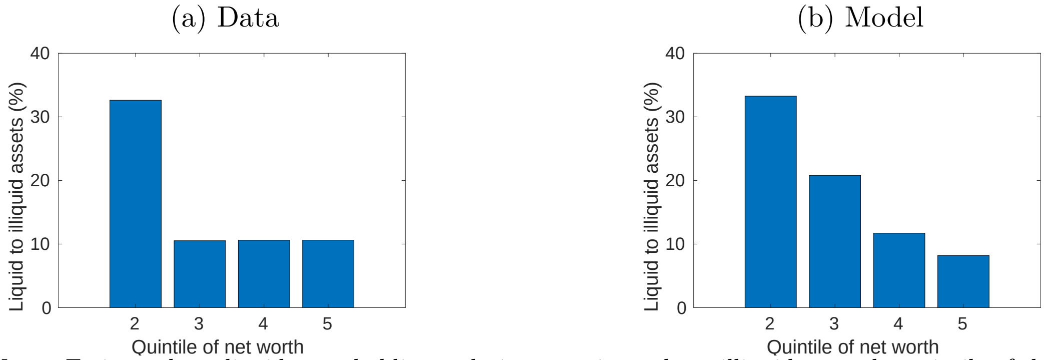
The empirical distribution of portfolio liquidity sheds light on how state-dependent liquidation decisions are, i.e., whether the logistic distribution for adjustment costs has a high or a low variance. Figure 6 shows the average holding of liquid relative to illiquid assets over the period 1983-2013 in the SCF and implied by the model. Portfolio liquidity is estimated using a local linear regression as described in Section 2. Note that only households above the 20th percentile have typically non-negligible amounts of positive illiquid asset holdings net of illiquid mortgage debt in every year of the sample. In the data, richer households hold a smaller fraction of their wealth in liquid form.[22]

Our model produces this downward sloping curve, too. The intuition is that households hold bonds because they provide better short-term consumption smoothing than capital and that this value of liquidity decreases in the amount of bonds a household holds. Furthermore, for richer households a larger share of income comes from capital and is hence not subject to labor income risk. Therefore, richer households, which typically hold both more bonds and more capital (even relative to their income), hold less liquid portfolios. Table 3 summarizes how we match our targets from the wealth distribution. The calibrated adjustment costs imply an average adjustment frequency of 5.6% per quarter that increases for households with unbalanced portfolios to up to 14% probability of adjustment.[23] The average adjustment frequency is close to the implied adjustment probability in Kaplan and Violante (2014). The maximum adjustment cost a household is willing to pay is equivalent to 6.3% of one quarter's consumption on average. The top 10% households in terms of willingness to pay for adjusting their portfolios

[22]Despite this fact, the Gini coefficient of liquid wealth is larger than the Gini of illiquid wealth.

[23]We provide robustness checks on the distribution of adjustment costs in Appendix L and find our results to be robust.

Figure 6: Average holdings of liquid assets relative to illiquid assets by wealth quintile



Notes: Estimated net liquid asset holdings relative to estimated net illiquid assets by quintile of the net wealth distribution. Average over the estimates from the SCFs 1983-2013. We select only households composed of at least two adults whose head is between 30 and 55 years of age. Estimation by a local linear estimator with a Gaussian kernel and a bandwidth of 0.05. Relative holdings below the 21st wealth percentile are not reported, because the net illiquid asset holdings can be zero and net liquid holdings negative.

are willing to pay a felicity cost equivalent to 21% of a quarter's consumption.[24]

Of course, it is a highly stylized treatment of the financial sector to assume that no physical investment is directly financed by the issuance of liquid assets. This stylized view, however, is motivated by the data. The Flow of Funds show (Z1-Table L.213) that roughly 73% of all corporate equities held or issued in the U.S. are either not publicly traded (11%) or held by agents other than households or depository institutions (in total 62% of all equities; out of these 24% are held by mutual funds, 16 % by the rest of the world, 12% by pension funds, and the remaining 12% by all other sectors). Importantly, the non-debt assets issued by these non-household and non-bank sectors are typically less liquid.[25] On top of that, some of the corporate equities held by households will be held by large holders (e.g., company founders) who impact the asset price when transacting. Even more extreme is the distribution of corporate bond holdings, of which more than 80% are held outside households and depository institutions (Flow of Funds, Z1-Table L.223).

6.1.3 Monetary Policy

We set the coefficients of the Taylor rule to standard values commonly used for New-Keynesian models. The coefficient θ_π describes the reaction of the nominal interest

[24]The consumption equivalents ce are calculated by solving $u(x_a^*) - u[(1-\mathsf{ce})x_a^*] = V_a - V_n$, where the right-hand side is the maximal adjustment costs a household is willing to pay.

[25]Roughly 50% of the mutual funds are directly held by households; the rest are mostly held by pension funds.

Table 4: Calibrated parameters: Government

Parameter	Value	Description	Target
Monetary Policy			
$\bar{R}^b$	1.0062	Nominal rate	2.5% p.a.
$\bar{\pi}$	1.00	Inflation	0% p.a.
θ_π	1.25	Reaction to inflation	Standard value
ρ_R	0.80	Inertia in Taylor rule	Standard value
Fiscal Policy			
ρ_B	0.86	Reaction to debt	Autocorrelation of government debt
γ_π	1.5	Reaction to inflation	Deficit response to uncertainty
$\gamma_\mathcal{T}$	0.5075	Reaction to tax rev.	Standard deviation of deficits
τ	0.30	Labor tax rate	$G/Y = 20\%$

rate to deviations of inflation from the steady state and ρ_R captures persistence in the nominal interest rate. We set $\theta_\pi = 1.25$ and $\rho_R = 0.8$. We set steady-state inflation to zero. The steady-state nominal interest rate is therefore equal to the real rate, which we set to 2.5% (annual).[26] In order to match the fraction of indebted households, we add a wedge between the lending and the borrowing rate of $\overline{R} = 11\%$ (annual).

6.1.4 Fiscal Policy

Government spending evolves according to a fiscal rule similar to Woodford (1995) or Bi et al. (2013). We choose the tax rate and government expenditures such that they account for 20% of output in the steady state, implying a tax rate of 30%. We estimate the persistence of government debt by the autocorrelation of government debt in the U.S. $\rho_B = 0.86$. We calibrate γ_π by matching the estimated peak response in primary surpluses after an uncertainty shock after 4 quarters and $\gamma_\mathcal{T}$ such that the model with TFP shocks replicates the volatility of primary surpluses relative to GDP in the data.

[26]The zero steady-state inflation assumption is equivalent to assuming any non-zero steady-state inflation rate together with perfect indexation to this rate (see e.g. Basu and Bundick, 2017). Under indexation, the steady-state inflation rate drops out of the price-adjustment costs and hence the Phillips curve.

7 Quantitative Results

Having determined the parameters of the model, we can quantify the aggregate effects of shocks to household income risk in our model. They turn out to be very similar in size to what we found empirically in Section 2.

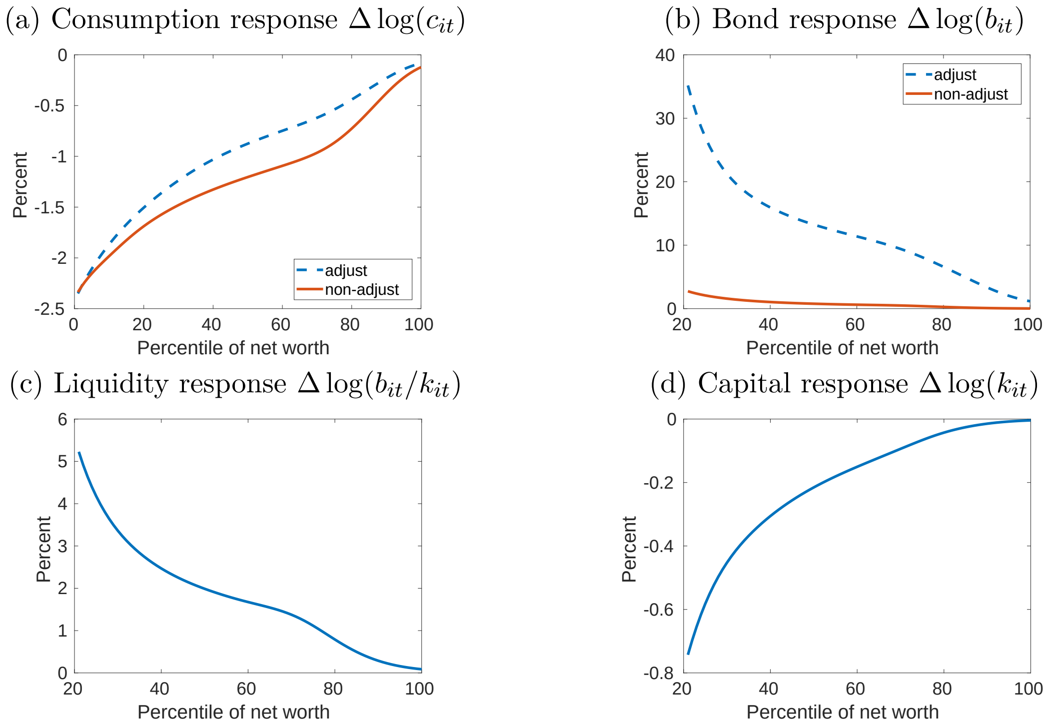
7.1 Household Portfolios and the Individual Response to Income Risk

We first describe the individual household response in partial equilibrium to clarify the mechanics of a shock to household income risk. For that purpose, we fix prices and expectations at their steady-state level and solve for the household decisions by discretizing the uncertainty process. We also solve the model without risk shocks and use this variant to obtain the stationary cross-sectional distribution of households in liquid and illiquid assets and income. We then order households along the net worth dimension, and estimate from the policy functions of the model with risk shocks average consumption as well as liquid and illiquid asset holdings by net worth using a local linear regression technique. We compare a situation when uncertainty is at its average value to an increase of income risk by one standard deviation (an increase in the variance of income shocks of 54%). Given that the so-specified planning problem of households uses the actual uncertainty process but fixed prices, this identifies the average partial equilibrium effect of uncertainty.

Figure 7 presents the results. For all households, consumption declines. As income risk goes up, households want to save more and they want to do this in liquid form. In fact, those households that decide to adjust their portfolios sell illiquid capital in exchange for liquid assets. Therefore, the liquidity of portfolios increases across all wealth groups.

Figure 8 shows the general equilibrium response of portfolio liquidity and consumption across the wealth distribution, where we allow prices to adjust and expectations to be consistent with equilibrium. In equilibrium, wage incomes fall and pure profits increase. Therefore, poorer households use some of their liquidity to smooth consumption, and compared to the partial equilibrium response, their liquidity increase is muted. On the other hand, some rich entrepreneur households see a temporary increase in income, and invest in liquid assets. This picture resembles what we found in Section 2. The increase in the liquidity of the portfolios is strongest for the lower middle class.

Figure 7: Partial equilibrium response –
Change in individual policy at constant prices and expectations



Response of individual consumption and asset demand policies at constant prices and price expectations to a one standard deviation increase in income risk. Policies by wealth percentile are estimated using a local linear regression technique with a Gaussian kernel and a bandwidth of 0.05. The figures compare the estimated function at average risk and at a one standard deviation increase, which is equal to a 54% increase in the variance of income shocks.
Top Panels: Conditional on adjustment decisions.
Bottom Panels: Average response over both adjusters and non-adjusters.

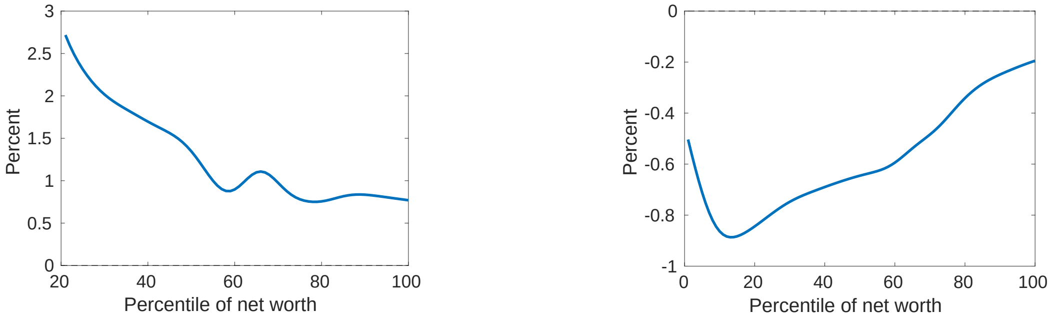
7.2 Aggregate Consequences of Shocks to Household Income Risk

7.2.1 Main Findings

As Figure 7 shows, upon an increase in income risk, the demand for consumption and capital simultaneously falls. Given that output is partly demand determined, output, wages and employment, and dividends need to fall in equilibrium. Figure 9 displays the impulse responses of aggregate output and its components, real bond holdings and the capital stock as well as asset prices and returns for our baseline calibration. The assumed monetary policy reacts to the uncertainty-induced deflation by cutting rates. Fiscal policy expands government expenditure. After a one standard deviation increase in the variance of idiosyncratic income shocks, output drops on impact by 0.2% and

Figure 8: General equilibrium response –
Change in the liquidity of household portfolios and consumption

(a) Liquidity response $\Delta \log \left(\frac{b_{it}}{q_t k_{it}} \right)$ (b) Consumption response $\Delta \log(c_{it})$



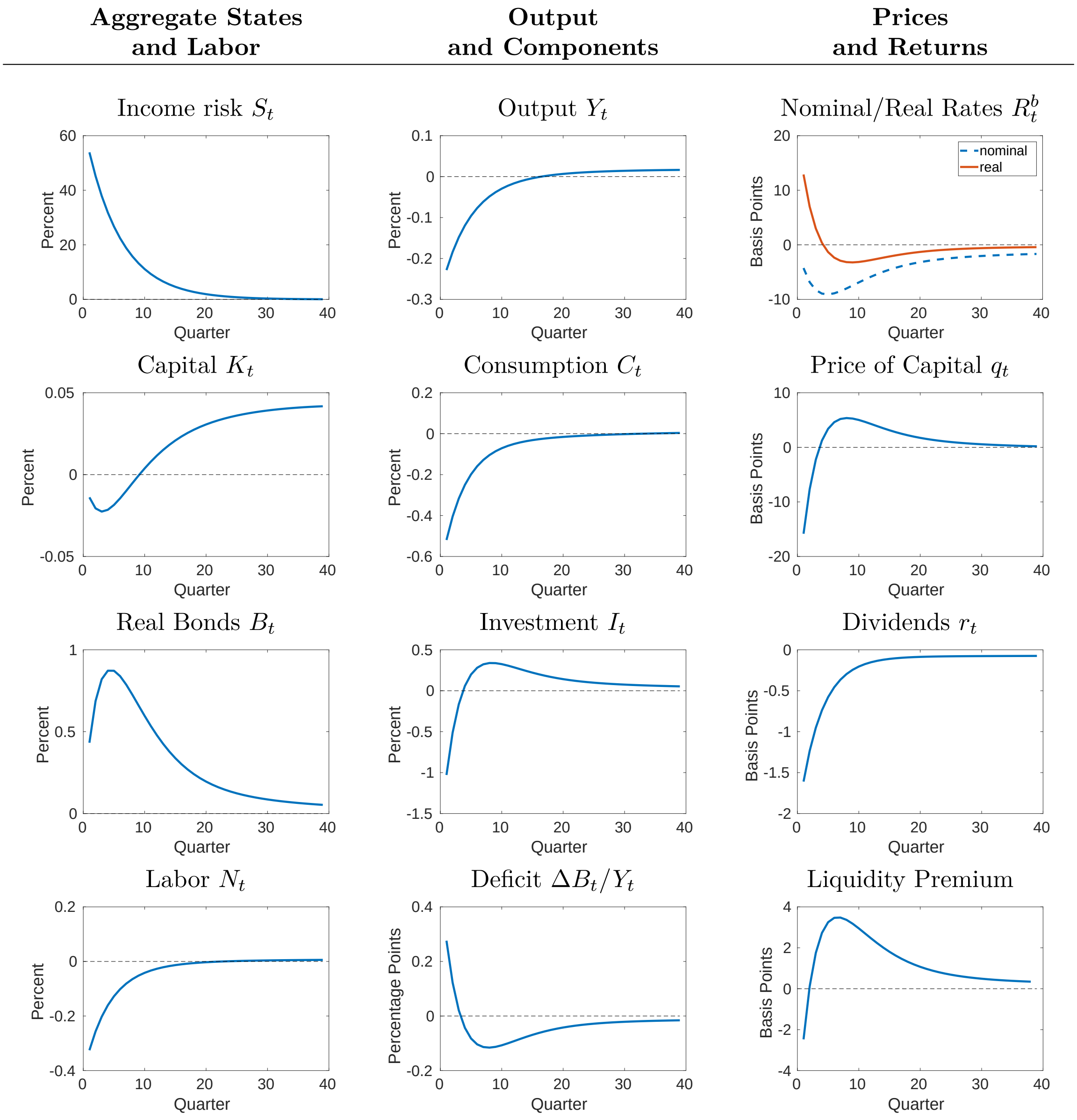
Notes: Change in the distribution of liquidity and consumption at all percentiles of the wealth distribution after 2 quarters at equilibrium prices and price expectations after a one standard deviation shock to income risk. The liquidity of the portfolios is averaged using frequency weights from the simulated wealth distribution and reported conditional on a household falling into the x-th wealth percentile. The left-hand panel shows the change in portfolio liquidity; the right-hand panel shows the consumption response. As with the data, we use a Gaussian kernel-weighted local linear smoother with bandwidth 0.05.

only recovers after 12 quarters. Consumption falls even more and remains subdued for roughly 20 quarters. Investment on impact sees the sharpest decline of all aggregates - almost five times stronger than output.

Overall, we find very similar responses to uncertainty shocks in terms of the size of the peak response in the model and the data. The data typically show hump-shaped responses, which our model cannot generate because both government expenditures and investment can adjust on impact in the model, while they do so slowly in reality.

The output drop in our model results from households increasing their precautionary savings in conjunction with a portfolio adjustment toward the liquid asset. In times of high uncertainty, households dislike illiquid assets because of their limited use for short-run consumption smoothing. Consequently, the price of capital decreases on impact. Since the demand for the liquid asset is a demand for paper and not for (investment) goods, demand for both consumption and investment goods falls. At the same time the central bank cuts interest rates on bonds, which stabilizes the demand for illiquid assets. Despite an increase in the quantity of bonds, the liquidity premium, i.e., the return difference between illiquid and liquid assets, increases. Quantitatively, we find that fluctuations in household income risk explain a significant fraction (21%) of the business cycle in terms of standard deviations; see Appendix I.

Figure 9: Aggregate response to household income risk shock



Notes: Liquidity Premium: $\frac{E_t q_{t+1}+r_t}{q_t} - \frac{R_t^b}{E_t \pi_{t+1}}$.
Impulse responses to a one standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized.

Appendix J presents details on four model extensions that help to understand the importance of certain aspects of the model. First, to understand the importance of portfolio rebalancing, we show that in an economy where capital is also liquid the effects of income-risk shocks are substantially muted, and while output still falls, investment increases. Second, to show the importance of price stickiness, we solve the model for flexible prices. In that case, income risk has a small expansionary effect through investment, very similar to the monetary stabilization case we discuss in the next section. Inflation falls until the demand for goods equals supply, which itself does not depend on inflation. Hence, we see larger movements in inflation. These are large enough to let the interest rate, which is still given by the Taylor rule, fall sufficiently to undo the thirst for liquidity. Instead, households then save more in illiquid assets and a small investment boom follows. Third, to understand the importance of stabilization policies, we look at a case where the interest rate is pegged and fiscal policy also does not stabilize. In this "crisis" case, the output loss is ten times larger, roughly 2%. Fourth, to understand the potential role of endogenous liquidity, we study a version of the model where a banking sector generates liquid assets out of the illiquid investments of households, mortgaging a fixed fraction of the investment. Results are all very similar to our baseline. Further robustness checks regarding the calibration can be found in Appendix L, where we keep the model structure the same but vary the risk aversion, the Frisch elasticity, the manager's discount factor, and the degree of state dependence in portfolio adjustment decisions.

7.2.2 Stabilization Policy

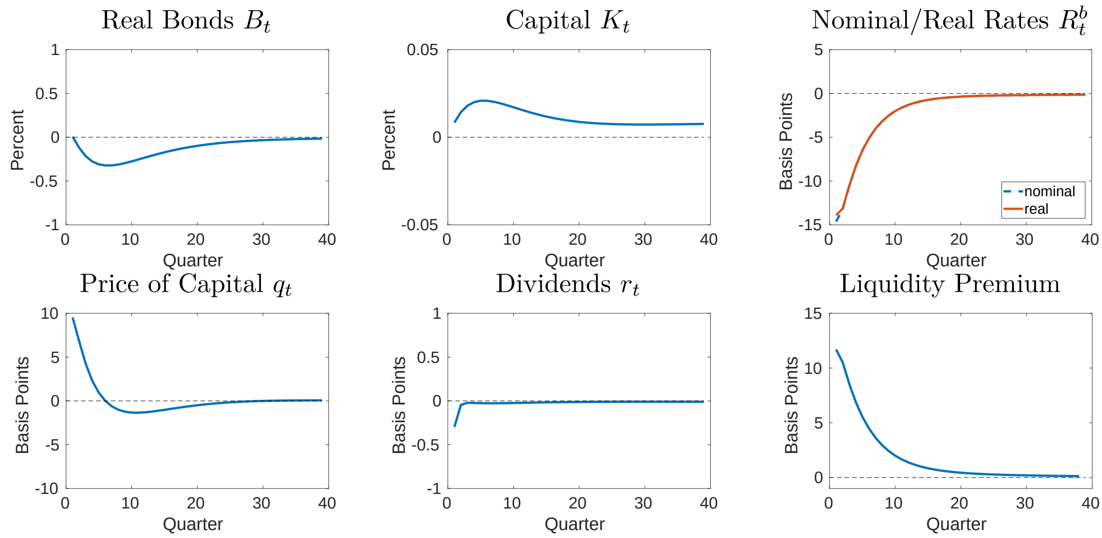
There are two ways the government can stabilize the economy in our setup: by cutting rates on bonds to shift asset demand from liquid to illiquid assets, i.e., by monetary policy, or by increasing the supply of government bonds, i.e., through fiscal policy. Our baseline calibration is a mix of the two following the empirical results in Section 2.

To obtain a better understanding of the differences between the two policy options, we next consider two extreme scenarios: One where monetary policy reacts very strongly to inflation, $\theta_\pi = 100$, but fiscal policy does not at all, $\gamma_\pi = 0$, and an alternative scenario, where monetary policy keeps a nominal interest rate peg, $\theta_\pi = 0$, and fiscal policy reacts strongly to inflation, $\gamma_\pi = 100$.

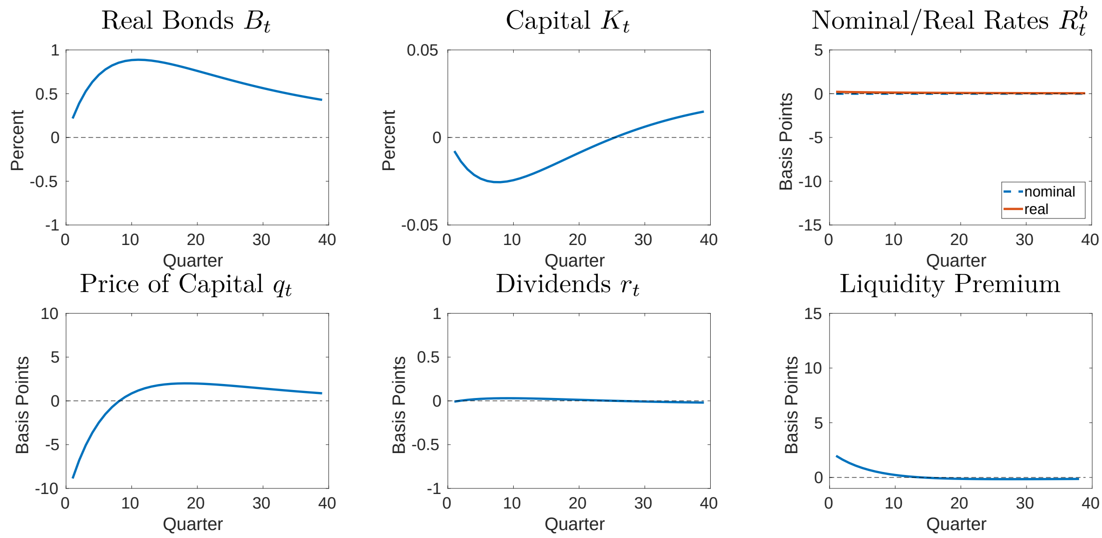
Both regimes successfully stabilize inflation and output at their steady-state levels. Yet, they still see a drop in consumption, as households want to increase their precautionary savings. The results for the key variables that differ across the regimes are depicted in Figure 10. Under monetary stabilization, the central bank increases the liq-

Figure 10: Aggregate response to household income risk shock with stabilization policy

(a) **Monetary Stabilization**



(b) **Fiscal Stabilization**



Notes: Liquidity Premium: $\frac{E_t q_{t+1}+r_t}{q_t} - \frac{R_t^b}{E_t \pi_{t+1}}$.
Impulse responses to a one standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized.
Top Panels: Taylor-rule coefficient on inflation is set to $\theta_\pi = 100$, and fiscal policy does not respond to inflation $\gamma_\pi = 0$ and taxes $\gamma_\mathcal{T} = 0$.
Bottom Panels: Fiscal policy strongly responds to inflation $\gamma_\pi = 100$ ($\gamma_\mathcal{T} = 0.5075$), and the Taylor-rule coefficient on inflation is zero, $\theta_\pi = 0$.

uidity premium by lowering the interest rate on bonds until the excess supply of goods at steady-state inflation is eliminated. The lower interest rate spurs investment such that the capital stock increases. In the fiscal policy case, the government instead supplies the liquid assets that households demand until the increase in the liquidity premium is eliminated as the return on liquid assets rises when they are more abundant. The increased savings of households are thus held in the form of government bonds used to finance government expenditures in the fiscal stabilization case. The welfare consequences for the different groups of households vary in the two regimes due to their different implications for the price of and return on liquid and illiquid assets. Monetary stabilization drives down the return on liquid assets and increases the price of capital. Fiscal stabilization, by contrast, increases the return on liquid assets but lowers the price of capital.

7.3 Redistributive and Welfare Effects

Since the aggregate consequences of uncertainty shocks affect asset prices, dividends, wages, and entrepreneurial incomes differently, our model predicts that not all agents (equally) lose from the uncertainty shock. For example, if capital prices fall, those agents who have high productivity and hence are rich in human capital, but hold little physical capital, could actually gain from the uncertainty shock. These agents are net savers. They increase their holdings of physical capital and can now do so more cheaply.

To quantify and understand the relative welfare consequences of the uncertainty shock and of systematic policy responses, we calculate the difference in expected value EV after a one standard deviation increase in uncertainty relative to its steady-state value for all (b, k, h) triples.[27] To put this number into perspective we normalize by the expected discounted felicity stream from consumption and leisure given (b, k, h) that a household expects. This way, we can calculate how much larger lifetime consumption would need to be to compensate a household for the effect of the uncertainty shock. This consumption equivalent takes the form:

$$\begin{aligned} \texttt{CE}(b,k,h) &= \left[\frac{EV(b,k,h;\Theta^{ss},\sigma_s) - EV(b,k,h;\Theta^{ss},0)}{EU(b,k,h)} + 1\right]^{1/(1-\xi)} - 1, \qquad (21) \\ EU(b,k,h) &= \sum_{t=0}^{\infty} \beta^t u(x_t^*), \end{aligned}$$

where Θ^{ss} is the steady-state distribution and the sequence x_t^* results from optimal

[27] For this purpose, we calculate the value functions iterating backward given the equilibrium price and uncertainty paths after an uncertainty shock, which we obtain by linearization using Reiter's procedure. We check with the Krusell-Smith variant for our baseline and find virtually the same results.

decisions of households using stationary equilibrium policies.

Table 5 provides the consumption equivalents for both baseline and stabilization policies. The average welfare loss (one-sided) from the uncertainty shock is 0.27% of lifetime consumption. Table 5 shows how much larger or smaller the losses are across population groups. What confounds results somewhat is that households with low labor income mechanically gain from an increase in the variance of shocks to productivity h, because expected productivity growth is positively related to uncertainty for low-productivity and negatively related for high-productivity households.[28] Also, entrepreneurs profit from the uncertainty shock as markups and thus profits go up. Therefore, it is particularly useful to look at the differences in welfare effects across groups, keeping the other characteristics constant; see the "Median" rows in Table 5.

In general, welfare losses are substantially more pronounced for those households with few asset holdings. In fact, comparing the average welfare loss across the policy regimes, we find that the numbers are very similar. It follows that the main source of welfare losses is the lack of idiosyncratic insurance. The (one-sided) welfare costs of the aggregate downturn itself is less important and on the order of 0.04% of lifetime consumption (baseline minus fiscal stabilization).

Notwithstanding, stabilization policies have sizable distributional consequences. Fiscal stabilization benefits particularly those who hold a lot of illiquid wealth as it stabilizes the dividend payments from these assets. Across liquid asset holdings the welfare benefits are relatively evenly distributed; see Figure 11 (b). Stabilization through monetary policy, by contrast, redistributes from households with particularly liquid portfolios to households with very little total wealth. Focusing on households with close to no illiquid wealth (1st quintile), we observe that the relative welfare gains amount to as much as 10 basis points of lifetime consumption for indebted households (1st quintile) to minus 10 basis points for households in the top quintile of liquid wealth; see Figure 11 (a). Households that hold large amounts of illiquid wealth also benefit under monetary stabilization from stable markups and hence relatively stable dividends. Yet, even households with more balanced portfolios, who are rich in liquid assets and illiquid assets, lose from monetary stabilization relative to the baseline because in equilibrium their asset returns fall.

[28] The conditional expected productivity growth is $g(\sigma_{h,t}, \log \tilde{h}_{it}) := E_t \exp(\Delta \log \tilde{h}_{it+1}) \frac{\int \tilde{h}_{it}}{\int \tilde{h}_{it+1}} = \exp[(\rho_h - 1) \log \tilde{h}_{it} + 0.5\sigma^2_{h,t}] \frac{\int \tilde{h}_{it}}{\int \tilde{h}_{it+1}}$. Across all households expected productivity growth is zero. The cross derivative is negative $\frac{\partial^2 g}{\partial \sigma_{h,t} \partial \tilde{h}_{it}} = g(\sigma_{h,t}, \log \tilde{h}_{it})(\sigma_{h,t}[1 - (\int \tilde{h}_{it})^{-\rho_h^2})](\rho_h - 1) < 0$.

Table 5: Welfare effects of household income risk shock

(a) Policy regime: Baseline

	Quintiles of bond holdings					Percentiles of human capital			
	1.	2.	3.	4.	5.	0-33	33-66	66-99	Entr.
Conditional	0.04	-0.11	-0.06	-0.00	0.13	0.14	-0.04	-0.10	1.03
Median	-0.08	-0.04	-0.01	0.02	0.08	0.20	-0.03	-0.15	1.88
	Quintiles of capital holdings								
	1.	2.	3.	4.	5.	Average CE: -0.27			
Conditional	-0.06	-0.07	-0.04	0.02	0.15				
Median	-0.18	-0.10	-0.02	0.06	0.15				

(b) Policy regime: Monetary stabilization

	Quintiles of bond holdings					Percentiles of human capital			
	1.	2.	3.	4.	5.	0-33	33-66	66-99	Entr.
Conditional	0.11	-0.07	-0.07	-0.05	0.09	0.17	-0.04	-0.10	0.07
Median	-0.11	-0.05	-0.05	-0.05	-0.02	0.20	-0.05	-0.15	0.21
	Quintiles of capital holdings								
	1.	2.	3.	4.	5.	Average CE: -0.24			
Conditional	-0.02	-0.09	-0.06	-0.01	0.17				
Median	-0.20	-0.14	-0.04	0.04	0.19				

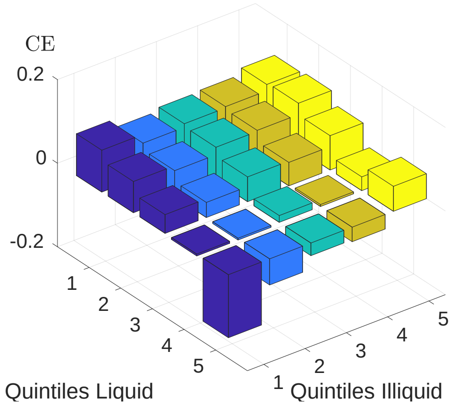
(c) Policy regime: Fiscal stabilization

	Quintiles of bond holdings					Percentiles of human capital			
	1.	2.	3.	4.	5.	0-33	33-66	66-99	Entr.
Conditional	0.07	-0.11	-0.08	-0.02	0.13	0.16	-0.04	-0.09	0.23
Median	-0.13	-0.08	-0.06	-0.03	0.05	0.19	-0.06	-0.18	0.18
	Quintiles of capital holdings								
	1.	2.	3.	4.	5.	Average CE: -0.23			
Conditional	-0.06	-0.09	-0.06	0.01	0.18				
Median	-0.22	-0.14	-0.06	0.04	0.18				

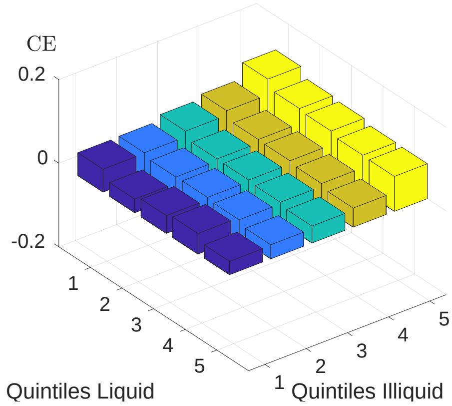
Notes: Welfare costs of a one standard deviation increase in income risk in terms of consumption equivalents (CE) as defined in (21) in % minus the population average for each regime. "Conditional" refers to integrating out the other individual states, whereas "Median" refers to median asset holdings/productivity in the other states.
(b) Policy coefficients are: $\theta_\pi = 100$, $\gamma_\pi = 0$, $\gamma_\mathcal{T} = 0$.
(c) Policy coefficients are: $\theta_\pi = 0$, $\gamma_\pi = 100$, $\gamma_\mathcal{T} = 0.5075$.

Figure 11: Welfare effects of household income risk shock with stabilization

(a) Monetary Stabilization



(b) Fiscal Stabilization



Notes: Welfare costs of a one standard deviation increase in income risk relative to the baseline policy specification in terms of consumption equivalents (CE) as defined in (21) integrating out h_{it} (excluding entrepreneurs). The graphs refer to the conditional expectations of CE with respect to the joint distribution of liquid and illiquid assets for workers.
Left Panel: Taylor-rule coefficient on inflation is set to $\theta_\pi = 100$, and fiscal policy does not respond to inflation $\gamma_\pi = 0$ and $\gamma_\mathcal{T} = 0$.
Right Panel: Fiscal policy strongly responds to inflation $\gamma_\pi = 100$ ($\gamma_\mathcal{T} = 0.5075$), and the Taylor-rule coefficient on inflation is zero, $\theta_\pi = 0$.

8 Conclusion

This paper examines how variations in the riskiness of household income affect the macroeconomy through precautionary savings. For this purpose, we develop a tractable framework that combines nominal rigidities and incomplete markets in which households choose portfolios of liquid paper and illiquid physical assets. In this model, higher income risk triggers a flight to liquidity because the liquid asset is better suited for short-run consumption smoothing. This reduces not only consumption but also investment, and hence depresses economic activity.

Calibrating the model to match the evolution of uncertainty about household income in the U.S., we find that, in line with the empirical evidence, a spike in income risk leads to losses in output, consumption, and investment. The decline in aggregate activity predicted by our model becomes sizable at the zero lower bound. This may help us to understand the severity of the Great Recession, for which we document a shift toward liquid assets across all percentiles of the U.S. wealth distribution.

The welfare effects of uncertainty shocks crucially depend on a household's asset position and the stance of monetary and fiscal policy. Monetary policy that lowers the return on liquid assets in times of increased uncertainty limits the negative aggregate consequences of uncertainty but redistributes resources from liquid to illiquid asset holders – both of which are typically wealthy. Fiscal policy can similarly ameliorate the uncertainty shock by providing the liquid asset that households demand. This keeps the return on all assets high at the expense of lower future capital and wages.

This highlights the importance of the supply of liquid funds in the economy. We have extended our baseline model to incorporate inside money created by banks in exchange for mortgages. This extension sketches the effect it has when households in times of high uncertainty demand less illiquid assets and hence banks write fewer mortgages: When uncertainty increases, the supply of liquid assets is depressed, while demand for liquid assets is high. However, it would require going beyond our current model with two assets to obtain a detailed account of these effects.

Relative to the existing literature on uncertainty, our model introduces portfolio rebalancing of households as a new channel to explain how an increase in uncertainty can lead to a recession. In modeling this new channel, we abstract from a richer model of firms and therefore from at least two important channels through which uncertainty impacts economic activity: first, from a "wait-and-see" channel that leads to a freeze in investment activity (see Bloom, 2009; Bloom et al., 2012; Bachmann and Bayer, 2013); second, from a cost-of-finance channel (see, e.g., Gilchrist et al., 2014), in which increased uncertainty magnifies the financial frictions firms face. In terms of our model, both channels imply that the effective return on illiquid funds declines, a situation that should amplify the household sector's flight to liquidity. In addition, a richer model of the firm sector would allow for introducing liquidity concerns for the firm sector, too (see, e.g., Del Negro et al., 2017). Changes in aggregate uncertainty come on top of the idiosyncratic uncertainty changes we highlight and likely go hand in hand with them, affecting aggregate outcomes through similar precautionary savings channels (Basu and Bundick, 2017) or other more complementary ones (Fernández-Villaverde et al., 2015). Similarly, in future work it might be interesting to investigate the effect of changes in higher moments of income shocks along the lines of Guvenen et al. (2014b).

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A Proof of Proposition 1

In the following appendix, we prove Proposition 1. For this purpose, we first establish the following two lemmas.

Lemma 1. *In the high income state in period 2, the borrowing constraint never binds if* $R^k < \left(1 + R^{k\frac{\xi-1}{\xi}}\right)^{\xi}$

Proof. Suppose the opposite. Then also in the low income state the constraint must bind, thus $c_3^{H,L} = R^k k_1$ and $c_2^H = y + b_1 + \sigma$. Optimality of $b_2 = 0$ requires $c_2^H < R^k k_1$. At the same time optimality in period 1 requires $u'(c_1) = R^k u'(c_3)$ (there is no uncertainty regarding period 3 consumption) and thus $c_3 = R^{k1/\xi} c_1$, implying $c_1 = \frac{y-b_1}{1+R^{k\frac{1-\xi}{\xi}}}$ and $c_3 = (y - b_1)\frac{R^{k1/\xi}}{1+R^{k\frac{1-\xi}{\xi}}}$. However, then $c_2^H > c_3$, as $R^{k1/\xi} < 1 + (R^k)^{\frac{\xi-1}{\xi}}$ by assumption. This contradicts optimality in period 2. □

Lemma 2. *If the household holds positive amounts of both assets in period 1 and* $R^k \neq 1$*, then the borrowing constraint binds in period 2 in the low income state.*

Proof. As the household holds positive amounts of both assets, it is not borrowing constrained in period 1 and hence $u'(c_1) = Eu'(c_2)$ and $u'(c_1) = R^k Eu'(c_3)$ from the optimality in period 1. Optimality in period 2 in the absence of binding borrowing constraints implies $c_2^{H,L} = c_3^{H,L}$, such that $Eu'(c_3) = R^k Eu'(c_3)$, which contradicts $R^k \neq 1$. □

We can use these Lemmas to prove the actual proposition:

Proposition 1. *Define* $b_1^*(\sigma), k_1^*(\sigma)$*, the optimal liquid and illiquid asset holdings. Define* $\tilde{b}_1(\sigma)$*, the liquid asset holdings of a household that does not have the option to invest in an illiquid asset. Now, suppose income uncertainty is large enough, such that* $b_1^*(\sigma) > 0$ *and the returns on the illiquid investment are larger than one* $R^k > 1$*, but not too large, i.e.* $1 < R^k < \left(1 + R^{k\frac{\xi-1}{\xi}}\right)^{\xi}$*. Then* $\frac{\partial b_1^*}{\partial \sigma} > \frac{\partial \tilde{b}_1}{\partial \sigma} > 0 > \frac{\partial k_1^*}{\partial \sigma}$*, i.e. liquid asset holdings increase in* σ *and they increase more than in a model where all assets are liquid, while illiquid asset holdings decrease.*

Proof. The optimal liquid and illiquid asset holdings are determined by the two Euler equations in period 1. From Lemma 1 and 2, we know that in period 2 the household will not be borrowing constrained in the high income state but it will be constrained in the low income state. Moreover, we also know that then $k_1 > 0$ follows as marginal utility diverges to infinity for $c \to 0$. Therefore, the two Euler equations for b_1 and k_1 read:

$$u'(y - b_1^* - k_1^*) - \frac{1}{2}\left[u'\left(\frac{y + b_1^* + \sigma + R^k k_1^*}{2}\right) + u'\left(y + b_1^* - \sigma\right)\right] \quad = \quad 0. \qquad (22)$$

$$u'(y - b_1^* - k_1^*) - R^k \frac{1}{2}\left[u'\left(\frac{y + b_1^* + \sigma + R^k k_1^*}{2}\right) + u'\left(R^k k_1^*\right)\right] \quad = \quad 0. \qquad (23)$$

These Euler equations define $k_1^*(\sigma), b_1^*(\sigma)$ as implicit functions of σ.

Removing the option to invest in the illiquid asset, the household is never borrowing constrained in period 2 and the demand for liquid assets $\tilde{b}_1(\sigma)$ is given by the following Euler equation:

$$u'(y - \tilde{b}_1) - \frac{1}{2}\left[u'\left(\frac{y + \tilde{b}_1 + \sigma}{2}\right) + u'\left(\frac{y + \tilde{b}_1 - \sigma}{2}\right)\right] \quad = \quad 0. \qquad (24)$$

We can now use the implicit function theorem to calculate how asset demand changes in σ. This yields

$$\begin{pmatrix} \frac{\partial b_1^*}{\partial \sigma} \\ \frac{\partial k_1^*}{\partial \sigma} \end{pmatrix} = \frac{1}{4}\begin{pmatrix} A_1 & A_2 \\ A_2 & A_3 \end{pmatrix}^{-1} \begin{pmatrix} 2u''_{2L} - u''_{2H} \\ -R^k u''_{2H} \end{pmatrix}$$

with

$$\begin{aligned}
A_1 &:= u''_1 + 1/4\left(u''_{2H} + 2u''_{2L}\right) < 0 \\
A_2 &:= u''_1 + 1/4 R^k u''_{2H} < 0 \\
A_3 &:= u''_1 + 1/4 R^{k2}\left(u''_{2H} + u''_{3L}\right) < 0
\end{aligned}$$

and

$$u''_{2L} := u''(c_2^L) < u''_{3L} := u''(c_3^L) < u''_1 := u''(c_1), < u''_{2H} := u''(c_2^H) < 0,$$

and thus:

$$\begin{aligned}
\begin{pmatrix} \frac{\partial b_1^*}{\partial \sigma} \\ \frac{\partial k_1^*}{\partial \sigma} \end{pmatrix} &= \frac{1}{4(A_1 A_3 - A_2^2)}\begin{pmatrix} A_3 & -A_2 \\ -A_2 & A_1 \end{pmatrix}\begin{pmatrix} 2u''_{2L} - u''_{2H} \\ -R^k u''_{2H} \end{pmatrix} \\
&= \frac{1}{4(A_1 A_3 - A_2^2)}\begin{pmatrix} A_3(2u''_{2L} - u''_{2H}) + R^k A_2 u''_{2H} \\ -A_2(2u''_{2L} - u''_{2H}) - R^k A_1 u''_{2H} \end{pmatrix}
\end{aligned}$$

In particular, making use of $u''_{2H} < u''_{3L} < u''_{2L} < 0$, we obtain that the numerator

$$\begin{aligned} A_1A_3 - A_2^2 &> \left(u''_1 + 3/4u''_{2H}\right)\left(u''_1 + 1/2R_k^2u''_{2H}\right) - \left(u''_1 + 1/4R_ku''_{2H}\right) \\ &= \frac{5}{16}R_k^2(u''_{2H})^2 + \frac{3+2R_k^2}{4}u''_1(u''_{2H}) - \frac{2}{4}R_ku''_1(u''_{2H}) > 0 \end{aligned}$$

is positive. This implies, as $A_{1,2,3}$, $2u''_{2L} - u''_{2H}$, and u''_{2H} are all negative:

$$\frac{\partial b_1^*}{\partial\sigma} > 0 > \frac{\partial k_1^*}{\partial\sigma}.$$

Moreover, we can estimate a lower bound on $\frac{\partial b_1^*}{\partial\sigma}$ as

$$\begin{aligned} \frac{\partial b_1^*}{\partial\sigma} &= \frac{1}{4}\frac{A_3}{A_1A_3 - A_2^2}\left(2u''_{2L} - u''_{2H}\right) + \frac{1}{4}\frac{A_2}{A_1A_3 - A_2^2}R^ku''_{2H} \\ &> \frac{1}{4}\frac{A_3}{A_1A_3}\left(2u''_{2L} - u''_{2H}\right) + \frac{1}{4}\frac{A_2}{A_1A_3 - A_2^2}R^ku''_{2H} \\ &> \frac{\frac{1}{4}(2u''_{2L} - u''_{2H})}{u''_1 + \frac{1}{4}(u''_{2H} + 2u''_{2L})} \end{aligned}$$

This term, the lower bound on $\frac{\partial b_1^*}{\partial\sigma}$, has a form similar to the derivative of liquid asset demand to income risk when the household can only hold liquid assets. In that case, we obtain:

$$\frac{\partial\tilde{b}_1}{\partial\sigma} = \frac{\frac{1}{4}(\tilde{u}''_{2L} - \tilde{u}''_{2H})}{\tilde{u}''_1 + \frac{1}{4}(\tilde{u}''_{2H} + \tilde{u}''_{2L})},$$

Now define a function $U(u'(c)) = u''(c)$. This function $U = -\xi u'^{\frac{\xi+1}{\xi}}$ is negative, decreasing, and convex in marginal utility. Therefore, the Euler equation implies $0 > 1/2(u''_{2H} + u''_{2L}) > u''_1$ due to convexity and $u''_1 > u''_{2L}$ because U is decreasing. Analogous formulas apply for the case without illiquid assets. We can use these estimates to obtain an upper bound on $\frac{\partial \tilde{b}_1}{\partial \sigma}$ and to simplify the lower bound on $\frac{\partial b_1^*}{\partial \sigma}$:

$$\begin{aligned}
\frac{\partial \tilde{b}_1}{\partial \sigma} &= \frac{\tilde{u}''_{2L} - \tilde{u}''_{2H}}{4\tilde{u}''_1 + (\tilde{u}''_{2H} + \tilde{u}''_{2L})} \leq \frac{\tilde{u}''_{2L} - \tilde{u}''_{2H}}{3(\tilde{u}''_{2H} + \tilde{u}''_{2L})} = \frac{\tilde{u}''_{2L} - \tilde{u}''_{2H}}{6\tilde{u}''_{2H} + 3(\tilde{u}''_{2L} - \tilde{u}''_{2H})} \\
&= \frac{1}{3 + 6\frac{\tilde{u}''_{2H}}{\tilde{u}''_{2L} - \tilde{u}''_{2H}}}, \\
\frac{\partial b_1^*}{\partial \sigma} &> \frac{2u''_{2L} - u''_{2H}}{4u''_1 + (u''_{2H} + 2u''_{2L})} \geq \frac{2u''_{2L} - u''_{2H}}{6u''_{2L} + u''_{2H}} = \frac{2u''_{2L} - u''_{2H}}{3(2u''_{2L} - u''_{2H}) + 4u''_{2H}} \\
&= \frac{1}{3 + 4\frac{u''_{2H}}{2u''_{2L} - u''_{2H}}}
\end{aligned}$$

which implies $\frac{\partial \tilde{b}_1}{\partial \sigma} < \frac{\partial b_1^*}{\partial \sigma}$ because $4\frac{u''_{2H}}{2u''_{2L} - u''_{2H}} < 6\frac{\tilde{u}''_{2H}}{\tilde{u}''_{2L} - \tilde{u}''_{2H}}$ as $0 > \tilde{u}''_{2L} > u''_{2L}$ and $0 > u''_{2H} > \tilde{u}''_{2H}$. This completes the proof. □

B Dynamic Planning Problem with Two Assets and Fixed Adjustment Probabilities

The dynamic planning problem of a household in the model is characterized by two Bellman equations, V_a in the case where the household can adjust its capital holdings and V_n otherwise. We will first go through the problem with exogenous adjustment probabilities, as the first-order conditions of the model with adjustment decisions that describe portfolio and consumption choices turn out to be of the same structure as under given adjustment probabilities.

With fixed adjustment probabilities, the value functions are given by

$$\begin{aligned}
V_a(b,k,h;\Theta,R^b,s) &= \max_{k',b'_a\in\Gamma_a} u[x(b,b'_a,k,k',h)] \\
&\quad + \beta\left[\nu EV_a(b'_a,k',h';\Theta',R'_b,s') + (1-\nu)EV_n(b'_a,k',h';\Theta',R'_b,s')\right] \\
V_n(b,k,h;\Theta,R^b,s) &= \max_{b'_n\in\Gamma_n} u[x(b,b'_n,k,k,h)] \\
&\quad + \beta\left[\nu EV_a(b'_n,k,h';\Theta',R'_b,s') + (1-\nu)EV_n(b'_n,k,h';\Theta',R'_b,s')\right]
\end{aligned} \tag{25}$$

where the budget sets are given by

$$\begin{aligned}
\Gamma_a(b,k,h;\Theta,R^b,s) &= \left\{k'\geq 0, b'\geq \underline{B} \mid q(k'-k)+b'\leq rk+\frac{R^b}{\pi}b\right. \\
&\qquad \left. +(1-\tau)\left(\frac{\gamma}{1+\gamma}whN+\mathbb{I}_{h=0}\Pi\right)\right\} \\
\Gamma_n(b,k,h;\Theta,R^b,s) &= \left\{b'\geq \underline{B} \mid b'\leq rk+\frac{R^b}{\pi}b\right. \\
&\qquad \left. +(1-\tau)\left(\frac{\gamma}{1+\gamma}whN+\mathbb{I}_{h=0}\Pi\right)\right\} \\
x(b,b',k,k',h;\Theta,R^b,s) &= \frac{\gamma}{1+\gamma}whN+rk+\frac{R^b}{\pi}b-q(k'-k)-b',
\end{aligned}$$

where q,π, and Π are functions of (Θ,R^b,s).

To save on notation, let Ω be the set of idiosyncratic state variables controlled by the household, let Z be the set of states outside the household's control, let $\Gamma_i:\Omega\to\Omega$ be the correspondence describing the feasibility constraints, and let $A_i(z)=\{(\omega,y)\in\Omega\times\Omega: y\in\Gamma_i(\omega,z)\}$ be the graph of Γ_i. Hence the states and controls of the household problem can be defined as

$$\begin{aligned}\Omega =&\{\omega = (b,k) \in \mathbb{R}^2 : \underline{B} \leq b < \infty, 0 \leq k < \infty\}\\ z =&\{h, \Theta, R^b, s\}\end{aligned}$$

and the return function $F : A \rightarrow R$ reads:

$$F(\Gamma_i(\omega,z),\omega;z) = \frac{x_i^{1-\gamma}}{1-\gamma}$$

Define the value before the adjustment/non-adjustment shock is realized as

$$v(\omega,z) := \nu V_a(\omega,z) + (1-\nu)V_n(\omega,z).$$

Now we can rewrite the optimization problem of the household in terms of the definitions above in a compact form:

$$V_a(\omega,z) = \max_{y\in\Gamma_a(\omega,z)}[F(\omega,y;z) + \beta Ev(y,z')] \tag{26}$$

$$V_n(\omega,z) = \max_{y\in\Gamma_n(\omega,z)}[F(\omega,y;z) + \beta Ev(y,z')]. \tag{27}$$

Finally we define the mapping $T : C(\Omega) \rightarrow C(\Omega)$, where $C(\Omega)$ is the space of bounded, continuous and weakly concave functions.

$$\begin{aligned}(Tv)(\omega,z) &= \nu V_a(\omega,z) + (1-\nu)V_n(\omega,z)\\ V_a(\omega,z) &= \max_{y\in\Gamma_a(\omega,z)}[F(\omega,y;z) + \beta Ev(y,z')]\\ V_n(\omega,z) &= \max_{y\in\Gamma_n(\omega,z)}[F(\omega,y;z) + \beta Ev(y,z')].\end{aligned}$$

B.1 Properties of Primitives

Abstracting from the non-continuity in R at $b = 0$, the following properties of the primitives of the problem obviously hold:

P 1. *Properties of sets* $\Omega, \Gamma_a(\omega,z), \Gamma_n(\omega,z)$

1. Ω *is a convex subset of* $\mathbb{R}^3$.
2. $\Gamma_i(\cdot,z) : \Omega \rightarrow \Omega$ *is non-empty, compact-valued, continuous, monotone and convex for all z.*

P 2. *Properties of return function F*

F is bounded, continuous, strongly concave, $\mathbb{C}^2$ differentiable on the interior of A, and strictly increasing in each of its first two arguments.

B.2 Properties of the Value and Policy Functions

Lemma 3. *The mapping T defined by the Bellman equation for v fulfills Blackwell's sufficient conditions for a contraction on the set of bounded, continuous and weakly concave functions $\mathbb{C}(\Omega)$.*

a) It satisfies discounting.

b) It is monotonic.

c) It preserves boundedness (assuming an arbitrary maximum consumption level).

d) It preserves strict concavity.

Hence, the solution to the Bellman equation is strictly concave. The policy is a single-valued function in (b, k), and so is optimal consumption.

Proof. The proof proceeds item by item and closely follows Stokey and Lucas (1989) taking into account that the household problem in the extended model consists of two Bellman equations.

a) Discounting
Let $a \in R_+$ and the rest be defined as above. Then it holds that:

$$\begin{aligned}(T(v+a))(\omega, z) =& \nu \max_{y \in \Gamma_a(\omega,z)} [F(\omega, y, z) + \beta E v(y, z') + a] \\ &+ (1-\nu) \max_{y \in \Gamma_n(\omega,z)} [F(\omega, y, z) + \beta E v(y, z') + a] \\ =& (Tv)(\omega, z) + \beta a\end{aligned}$$

Accordingly, T fulfills discounting.

b) Monotonicity
Let $g : \Omega \times Z \to R^2$, $f : \Omega \times Z \to R^2$ and $g(\omega, z) \geq f(\omega, z)$ $\forall \omega, z \in \Omega \times Z$, then it

follows that:

$$\begin{aligned}(Tg)(\omega, z) =&\nu \max_{y\in\Gamma_a(\omega,z)}[F(\omega, y, z) + \beta E g(y, z')]\\ &+ (1-\nu) \max_{y\in\Gamma_n(\omega,z)}[F(\omega, y, z) + \beta E g(y, z')]\\ \geq&\nu \max_{y\in\Gamma_a(\omega,z)}[F(\omega, y, z) + \beta E f(y, z')]\\ &+ (1-\nu) \max_{y\in\Gamma_n(\omega,z)}[F(\omega, y, z) + \beta E f(y, z')]\\ =&Tf(\omega, z)\end{aligned}$$

The objective function for which Tg is the maximized value is uniformly higher than the function for which Tf is the maximized value. Therefore, T preserves monotonicity.

c) Boundedness
From properties **P1** it follows that the mapping T defines a maximization problem over the continuous and bounded function $[F(\omega, y)+\beta Ev(y, z'))]$ over the compact sets $\Gamma_i(\omega, z)$ for $i = \{a, n\}$. Hence the maximum is attained. Since F and v are bounded, Tv is also bounded.

d) Strict Concavity
Let $f \in C''(\Omega)$, where $C^{''}$ is the set of bounded, continuous, strictly concave functions on Ω. Since the convex combination of two strictly concave functions is strictly concave, it is sufficient to show that $T_i[C''(\Omega)] \subseteq C''(\Omega)$, where T_i is defined by

$$T_i v = \max_{y\in\Gamma_i(\omega,z)}[F(\omega, y, z) + \beta E v(y, z')], i \in \{a, n\}$$

Let $\omega_0 \neq \omega_1, \theta \in (0, 1), \omega_\theta = \theta\omega_0 + (1-\theta)\omega_1$.
Let $y_j \in \Gamma_i(\omega_j, z)$ be the maximizer of $(T_i f)(\omega_j)$ for $j = \{0, 1\}$ and $i = \{a, n\}$, $y_\theta = \theta y_0 + (1-\theta)y_1$.

$$\begin{aligned}(T_i f)(\omega_\theta, z) \geq&[F(\omega_\theta, y_\theta, z) + \beta E f(y_\theta, z')]\\ >&\theta[F(\omega_0, y_0, z) + \beta E f(y_0, z'))] + (1-\theta)[F(\omega_1, y_1, z) + \beta E f(y_1, z')]\\ =&\theta(Tf)(\omega_0, z) + (1-\theta)(Tf)(\omega_1, z)\end{aligned}$$

The first inequality follows from y_θ being feasible because of convex budget sets.

The second inequality follows from the strict concavity of f. Since ω_0 and ω_1 are arbitrary, it follows that $T_i f$ is strictly concave, and since f is arbitrary that $T[C''(\Omega)] \subseteq C''(\Omega)$.

□

Lemma 4. *The value function is $\mathbb{C}^2$ and the policy function $\mathbb{C}^1$ differentiable.*

Proof. The properties of the choice set **P1**, of the return function **P2**, and the properties of the value function proven in (3) fulfill the assumptions of Santos's (1991) theorem on the differentiability of the policy function. According to the theorem, the value function is C^2 and the policy function C^1 differentiable.
Note that strong concavity of the return function holds for CRRA utility, because of the arbitrary maximum we set for consumption. □

Lemma 5. *The total savings $S_i^* := b_i^*(\omega, z) + q(z)k_i^*(\omega, z)$ and consumption $c_i^*, i \in \{a, n\}$ are increasing in ω if $r(z)$ is positive. In the adjustment case, total savings and consumption are increasing in total resources $\mathcal{R}_a(z) = [q(z) + r(z)]k + b\frac{R(b,z)}{\pi(z)}$ for any r(z).*

Proof. Define $\tilde{v}(S, z) := \max_{\{b,k|b+q(z)k\leq S\}} Ev(b, k; z')$ and resources in the case of no adjustment $\mathcal{R}_n = r(z)k + b\frac{R(b,z)}{\pi(z)}$. Since v is strictly concave and increasing, so is $\tilde{v}$ by the line of the proof of Lemma 3.d). Denote $\varphi(z) = (1 - \tau)\left(\frac{\gamma}{1+\gamma}w(z)hN + \mathbb{I}_{h=0}\Pi(z)\right)$. Now we can (re)write the planning problem as

$$
\begin{aligned}
V_a(b, k; z) &= \max_{S\leq\varphi(z)+\mathcal{R}_a} \left[u\left(\varphi(z) + [q(z) + r(z)]k + b\frac{R(b, z)}{\pi(z)} - S\right) + \beta\tilde{v}(S, z)\right] \\
V_n(b, k; z) &= \max_{b'\leq\varphi(z)+\mathcal{R}_n} \left[u\left(\varphi(z) + r(z)k + b\frac{R(b, z)}{\pi(z)} - b'\right) + \beta Ev(b', k; z')\right].
\end{aligned}
$$

Due to differentiability we obtain the following (sufficient) first-order conditions:

$$
\begin{aligned}
\frac{\partial u\left(\varphi(z) + \mathcal{R}_a - S\right)}{\partial c} &= \beta\frac{\partial\tilde{v}(S, z)}{\partial S} \\
\frac{\partial u\left(\varphi(z) + \mathcal{R}_n - b'\right)}{\partial c} &= \beta\frac{\partial v(b', k; z)}{\partial b'}.
\end{aligned} \tag{28}
$$

Since the left-hand sides are decreasing in $\omega = (b, k)$, and increasing in S (respectively b'), and the right-hand side is decreasing in S (respectively b'), $S_i^* = \begin{cases} qk' + b' & \text{if } i = a \\ qk + b' & \text{if } i = n \end{cases}$ must be increasing in ω.

Since the right-hand side of (28) is hence decreasing in ω, so must be the left-hand side of (28). Hence consumption must be increasing in ω.
The last statement follows directly from the same proof. $\square$

B.3 Euler Equations

Denote the optimal policies for consumption, for bond holdings and capital as $x_i^*, b_i^*, k^*, i \in \{a, n\}$ respectively. The first-order conditions for an inner solution in the (non-)adjustment case read:

$$k^* : \frac{\partial u(x_a^*)}{\partial x} q = \beta E \left[\nu \frac{\partial V_a(b_a^*, k^*; z')}{\partial k} + (1-\nu) \frac{\partial V_n(b_a^*, k^*; z')}{\partial k} \right] \tag{29}$$

$$b_a^* : \frac{\partial u(x_a^*)}{\partial x} = \beta E \left[\nu \frac{\partial V_a(b_a^*, k^*; z')}{\partial b} + (1-\nu) \frac{\partial V_n(b_a^*, k^*; z')}{\partial b} \right] \tag{30}$$

$$b_n^* : \frac{\partial u(x_n^*)}{\partial x} = \beta E \left[\nu \frac{\partial V_a(b_n^*, k; z')}{\partial b} + (1-\nu) \frac{\partial V_n(b_n^*, k; z')}{\partial b} \right] \tag{31}$$

Note the subtle difference between (30) and (31), which lies in the different capital stocks k' vs. k in the right-hand side expressions.

Differentiating the value functions with respect to k and m, we obtain:

$$\begin{aligned}
\frac{\partial V_a(b, k; z)}{\partial k} &= \frac{\partial u[x_a^*(b, k; z)]}{\partial x}(q(z) + r(z)) \\
\frac{\partial V_a(b, k; z)}{\partial b} &= \frac{\partial u[x_a^*(b, k; z)]}{\partial x}\frac{R(b, z)}{\pi(z)} \\
\frac{\partial V_n(b, k; z)}{\partial b} &= \frac{\partial u[x_n^*(b, k; z)]}{\partial x}\frac{R(b, z)}{\pi(z)} \\
\frac{\partial V_n(b, k; z)}{\partial k} &= r(z)\frac{\partial u[x_n^*(b, k; z)]}{\partial x} \\
&\quad + \beta E \left[\nu \frac{\partial V_a[b_n^*(b, k; z), k; z']}{\partial k} + (1-\nu) \frac{\partial V_n[b_n^*(b, k; z), k; z']}{\partial k} \right] \\
&= r(z)\frac{\partial u[x_n^*(b, k; z)]}{\partial x} + \beta \nu E \frac{\partial u\{x_a^*[b_n^*(b, k; z), k; z], k; z'\}}{\partial x}(q(z') + r(z')) \\
&\quad + \beta(1-\nu) E \frac{\partial V_n\{[b_n^*(b, k; z), k; z], k; z'\}}{\partial k}
\end{aligned} \tag{32}$$

such that the marginal value of capital in non-adjustment is defined recursively.

Now we can plug the second set of equations into the first set of equations and obtain the following Euler equations (in slightly shortened notation):

$$\frac{\partial u[x_a^*(b,k;z)]}{\partial x}q(z) = \beta E\left[\nu\frac{\partial u[x_a^*(b_a^*,k^*;z')]}{\partial x}[q(z')+r(z')] + (1-\nu)\frac{\partial V_n(b_a^*,k^*;z')}{\partial k}\right] \tag{33}$$

$$\frac{\partial u[x_a^*(b,k;z)]}{\partial x} = \beta E\frac{R(b^*,z')}{\pi(z')}\left[\nu\frac{\partial u[x_a^*(b_a^*,k^*;z')]}{\partial x} + (1-\nu)\frac{\partial u[x_n^*(b_a^*,k^*;z')]}{\partial x}\right] \tag{34}$$

$$\frac{\partial u[x_n^*(b,k;z)]}{\partial x} = \beta E\frac{R(b^*,z')}{\pi(z')}\left[\nu\frac{\partial u[x_a^*(b_n^*,k;z')]}{\partial x} + (1-\nu)\frac{\partial u[x_n^*(b_n^*,k;z')]}{\partial x}\right] \tag{35}$$

In words, when deciding between the liquid and the illiquid asset, the household compares the one-period return difference between the two assets $E\frac{R(b^*,z')}{\pi(z')} - E\frac{r(z')+q(z')}{q(z)}$ weighted with the marginal utility under adjustment and the probability of adjustment and the difference between the return in the no adjustment case, $E\frac{R(b^*,z')}{\pi(z')}\frac{\partial u[x_n^*(b_a^*,k^*;z')]}{\partial x}$, and the marginal value of illiquid assets when not adjusting $\frac{\partial V_n(b_a^*,k^*;z')}{\partial k}$. The latter reflects both the utility derived from the dividend stream and the utility from occasionally selling the asset. (We abstract from the non-differentiability at $b=0$ in this.)

B.4 Algorithm

The algorithm we use to solve for optimal policies is a version of Hintermaier and Koeniger's (2010) extension of the endogenous grid method, originally developed by Carroll (2006).

It works iteratively until convergence of policies as follows: Start with some guess for the policy functions x_a^* and x_n^* on a given grid $(b,k) \in B \times K$. Define the shadow value of capital

$$\begin{aligned}
\beta^{-1}\psi(b,k;z) :=& \nu E\left\{\frac{\partial u\{x_a^*[b_n^*(b,k,z),k;z']\}}{\partial x}[q(z')+r(z')]\right\} \\
&+ (1-\nu)E\frac{\partial V_n[b_n^*(b,k,z),k;z']}{\partial k} \\
&= \nu E\left\{\frac{\partial u\{x_a^*[b_n^*(b,k,z),k;z']\}}{\partial x}[q(z')+r(z')]\right\} \\
&+ (1-\nu)E\left\{\frac{\partial u\{x_n^*(b_n^*(b,k,z),k;z')]}{\partial x}r(z')\right\} \\
&+ (1-\nu)E\left\{\psi[b_n^*(b,k,z),k;z']\right\}.
\end{aligned}$$

Guess initially $\psi = 0$. Then

1. Solve for an update of x_n^* by standard endogenous grid methods using equation

(35), and denote $b_n^*(b,k;z)$ as the optimal bond holdings without capital adjustment.

2. Find for every k' on-grid some (off-grid) value of $\tilde{b}_a^*(k';z)$ such that combining (34) and (33) yields:

$$0 = \nu E\left\{\frac{\partial u[x_a^*(\tilde{b}_a^*(k',z),k';z')]}{\partial x}\left[\frac{q(z')+r(z')}{q(z)} - \frac{R(b',z')}{\pi(z')}\right]\right\}$$
$$+ (1-\nu)E\left\{\frac{\partial u[x_n^*(\tilde{b}_a^*(k',z),k';z')]}{\partial x}\left[\frac{r(z')}{q(z)} - \frac{R(b',z')}{\pi(z')}\right]\right\} + (1-\nu)E\left[\frac{\psi(\tilde{b}_a^*(k',z),k';z')}{q(z)}\right]$$

N.B. that $E\psi$ takes the stochastic transitions in h' into account and does not replace the expectations operator in the definition of ψ. If no solution exists, set $\tilde{b}_a^* = \underline{B}$. Uniqueness (conditional on existence) of $\tilde{b}_a^*$ follows from the strict concavity of v.

3. Solve for total initial resources, by solving the Euler equation (34) for $\tilde{x}^*(k',z)$, such that:

$$\tilde{x}^*(k',z)$$
$$= \frac{\partial u}{\partial x}^{-1}\left\{\beta E\frac{R(b^*,z')}{\pi(z')}\left[\nu\frac{\partial u\{x_a^*[b_a^*(k',z),k';z']\}}{\partial x} + (1-\nu)\frac{\partial u\{x_n^*[b_a^*(k',z),k';z']}{\partial x}\right]\right\}$$

where the right-hand side expressions are obtained by interpolating $x_a^*(b_a^*(k',z),k',z')$ from the on-grid guesses $x_a^*(b,k;z)$ and taking expected values with respect to z'.

This way we obtain total non-human resources $\tilde{\mathcal{R}}_a(k',z)$ that are compatible with plans $(b^*(k'),k')$ and a consumption policy $\tilde{\tilde{x}}_a^*\left(\tilde{\mathcal{R}}_a(k',z),z\right)$ in total resources.

4. Since (consumption) policies are increasing in resources, we can obtain consumption policy updates as follows: Calculate total resources for each (b,k) pair $\mathcal{R}_a(b,k) = (q+r)k+b\frac{R(b)}{\pi}$ and use the consumption policy obtained before to update $x_a^*(b,k,z)$ by interpolating at $\mathcal{R}_a(b,k)$ from the set $\left\{(\tilde{\tilde{x}}_a^*(\tilde{\mathcal{R}}_a(k',z),z),\mathcal{R}_a(k',z))\middle|k'\in K\right\}$.[29]

[29]If a boundary solution $\tilde{b}^*(\underline{B}) > \underline{B}$ is found, we use the "n" problem to obtain consumption policies for resources below $\tilde{b}^*(\underline{B})$.

5. Update ψ: Calculate a new value of ψ using (32), such that:

$$\begin{aligned}\psi^{new}(b,k,z) =& \beta\nu E\left\{\frac{\partial u\{x_a^*[b_n^*(b,k,z),k;z']\}}{\partial x}[q(z')+r(z')]\right\}\\ &+\beta(1-\nu)E\left\{\frac{\partial u\{x_n^*(b_n^*(b,k,z),k;z')]}{\partial x}r(z')\right\}\\ &+\beta(1-\nu)E\left\{\psi^{old}[b_n^*(b,k,z),k;z']\right\}.\end{aligned}$$

making use of the **updated** consumption policies.

Note that we wrote up the algorithm in a general form that covers both Krusell-Smith equilibria, steady states and first-order perturbations in aggregate dynamics. The difference lies in specifying the prices $r(z), q(z), R(b,z), \pi(z)$. In a Krusell-Smith equilibrium these are given by the forecasting rules, when computing the steady state prices are fixed, and when approximating the aggregate dynamics by first-order perturbation following Reiter (2002, 2009, 2010) current and future prices get perturbed.

C Dynamic Planning Problem with Two Assets and Logistic Distribution of Adjustment Costs

With logistically distributed adjustment costs, concavity of the value function is no longer guaranteed, because ν will depend on $(\omega;z)$. If the function EV in equation (13) is convex, then the policy functions will still be continuously differentiable and the value function twice differentiable because the prerequisites of Lemmas 4 and 5 are still fulfilled.

Let $f(\chi)$ be the density function of the adjustment costs. Since $V_a \geq V_n$ we can write

$$EV(\omega;z) = V_n(\omega;z) + \int_0^{V_a(\omega;z)-V_n(\omega;z)} (V_a(\omega;z) - V_n(\omega;z) - \chi)f(\chi)d\chi$$

In turn if $f(\chi) > 0$ for all $\chi > 0$ (the adjustment cost distribution has unbounded support on $\mathbb{R}_+$), the derivative of EV w.r.t. ω takes the form

$$\frac{\partial EV}{\partial\omega} = \frac{\partial V_n}{\partial\omega} + \nu^*(\omega;z)\left[\frac{\partial V_a}{\partial\omega} - \frac{\partial V_n}{\partial\omega}\right].$$

In words, first-order conditions of a model with fixed adjustment probabilities and a model with state dependent adjustment probabilities are the same. We make use of this fact and simply replace the state independent adjustment probability by a guess for an adjustment probability function in the algorithm described in Appendix B.4. We then update the adjustment probabilities by making use of the closed-form solution to the expected adjustment costs under the logistic distribution assumption for χ when calculating the value functions in iteration (n):

$$\begin{aligned}
V_a^{(n)} &= u(x_a^{*(n)}) + \beta EV_{(n)}(b_a^{*(n)}, k^{*(n)}; z') \\
V_n^{(n)} &= u(x_n^{*(n)}) + \beta EV_{(n)}(b_n^{*(n)}, k; z') \\
\text{where} & \\
EV^{(n)} &= \nu^{*(n)} V_a^{(n)} + (1-\nu^{*(n)}) V_n^{(n)} - AC(\nu^{*(n)}; \mu_\chi, \sigma_\chi)
\end{aligned}$$

where μ_χ and σ_χ are the mean and the scale of the logistic distribution

$$F(\chi) = \frac{1}{1+\exp\{-\frac{\chi-\mu_\chi}{\sigma_\chi}\}}.$$

The adjustment probability can be updated after the two value functions have been calculated for a given $\nu^*(\omega, z)$ as

$$\nu^{*(n+1)}(\omega, z) = F[V_a^{(n)}(\omega, z) - V_n^{(n)}(\omega, z)].$$

Given the new adjustment probabilities, consumption and savings policies can be determined again using the endogenous grid method. The expected conditional adjustment cost is given by

$$\begin{aligned}
AC(\nu; \mu_\chi, \sigma_\chi) &= \int_0^{F^{-1}(\nu)} \chi dF(\chi) = \int_0^{\nu} F^{-1}(p) dp \\
&= \int_0^{\nu} \mu_\chi + \sigma_\chi[\log p - \log(1-p)] dp \\
&= \mu_\chi \nu + \sigma_\chi \left[\nu \log \nu + (1-\nu)\log(1-\nu)\right]
\end{aligned}$$

Given that concavity of the value functions is not guaranteed, we check for monotonicity of the derivatives of the value function and for uniqueness of the optimal portfolio solution in the algorithm, implementing thereby a version of Fella's (2014) algorithm, and find that the solution turns out to be globally concave.

D Price-Level Determinacy

This appendix sketches in a more technical way than the main text the question of price-level determinacy in our non-Ricardian setup. Making the argument in full is beyond the scope of the paper, so that we restrict ourselves to a setup with only bonds, no aggregate shocks, and flexible prices. With flexible prices there is no output-inflation feedback, so that we can drop output as a determinant of government rules.

The equation that then determines the price level is the bond-market clearing condition (see Woodford, 1995), which simplifies to the well-known series of consumption Euler equations under complete markets and a representative agent. Given that total output is fixed under flexible prices, this market clearing condition, when substituted in for the supply of bonds, simplifies to:

$$\left(\frac{B_t R_t^b/\pi_t}{\bar{B}\bar{R}^b/\bar{\pi}}\right)^{\rho_B}\left(\frac{\pi_t}{\bar{\pi}}\right)^{-\gamma_\pi} = \frac{B^d(\Theta_t; R_t^b; \pi_t)}{\bar{B}}. \tag{36}$$

Since in our model tax policy does not adjust, there is no direct feedback from government policy to bond demand through household budgets, but only through goods/bond markets. Of course, the demand for bonds depends on the entire path of future prices and wealth distributions (and insofar our notation is sloppy). We can, however, use the logic of a local approximation in aggregates (c.f. Reiter, 2002, a variant of which we use to solve the model), and write the first-order expansion of (36) as

$$\rho_B(\hat{B}_t + \hat{R}_t^b - \hat{\pi}_t) - \gamma_\pi \hat{\pi}_t = \zeta_B(\hat{B}_t - \hat{\pi}_t) + \zeta_R(\hat{R}_{t+1}^b - E_t \hat{\pi}_{t+1}),$$

where the $\hat{x}$ is a log deviation in a variable x from its steady-state value, and $\zeta_{B,R}$ are the *aggregate* wealth and interest elasticities of savings. We approximate the true dynamics assuming approximate aggregation, i.e., that all changes in aggregate government debt affect debt demand as if they were proportionally distributed according to the steady-state distribution. This implies that any change in end of period $t-1$ government debt, B_t, has the same impact on total demand of liquid assets as has a change in beginning of period real debt, $\hat{B}_t - \hat{\pi}_t$, through inflation and we do not need to model the dynamics of Θ_t explicitly. Note that this is a simplifying assumption that we make only here to obtain analytical results, but not in the actual solution of our model.

Inserting the Taylor rule and plugging in the laws of motion for bonds and nominal rates, we obtain a system of three equations:

$$\begin{aligned}
E_t\hat{\pi}_{t+1} &= \left[(1-\rho_R)\theta_\pi + \frac{\gamma_\pi}{\zeta_R}\right]\hat{\pi}_t + \frac{\zeta_B - \rho_B}{\zeta_R}(\hat{B}_t - \hat{\pi}_t + \hat{R}^b_t) + \frac{\rho_R\zeta_R - \zeta_B}{\zeta_R}\hat{R}^b_t, \\
\hat{B}_{t+1} &= \rho_B(\hat{B}_t - \hat{\pi}_t + \hat{R}^b_t) - \gamma_\pi\hat{\pi}_t, \\
\hat{R}^b_{t+1} &= \rho_R\hat{R}^b_t + (1-\rho_R)\theta_\pi\pi_t,
\end{aligned}$$

which we can write in terms of total real outstanding government obligations $\hat{O}_t = \hat{B}_t - \hat{\pi}_t + \hat{R}^b_t$ as:

$$\begin{aligned}
E_t\hat{\pi}_{t+1} &= \left[(1-\rho_R)\theta_\pi + \frac{\gamma_\pi}{\zeta_R}\right]\hat{\pi}_t + \frac{\zeta_B - \rho_B}{\zeta_R}\hat{O}_t + \frac{\rho_R\zeta_R - \zeta_B}{\zeta_R}\hat{R}^b_t, \\
E_t\hat{O}_{t+1} &= -\left(\frac{(\zeta_R + 1)}{\zeta_R}\gamma_\pi\right)\hat{\pi}_t + \left(\frac{\rho_B - \zeta_B}{\zeta_R} + \rho_B\right)\hat{O}_t + \frac{\zeta_B}{\zeta_R}\hat{R}^b_t, \\
\hat{R}^b_{t+1} &= \rho_R\hat{R}^b_t + (1-\rho_R)\theta_\pi\pi_t.
\end{aligned}$$

For government obligations we can invoke a transversality condition to rule out exploding paths. This means, if $\hat{O}$ explodes for $\hat{O} \neq 0$, $\hat{O} = 0$ is the only solution. This directly implies that $\hat{\pi} = 0$ because real beginning of period bonds and interest rates are predetermined.

For the special case of an interest rate peg, $\theta_\pi = \rho_R = 0$, and no active fiscal stabilization, $\gamma_\pi = 0$, this implies

$$E_t(\hat{O}_{t+1}) = \left(\frac{\rho_B - \zeta_B}{\zeta_R} + \rho_B\right)\hat{O}_t,$$

such that the government obligations are stable, whenever $-1 < \frac{\rho_B - \zeta_B}{\zeta_R} + \rho_B < 1$, which implies local indeterminacy of the price level. If by contrast $\frac{\rho_B - \zeta_B}{\zeta_R} + \rho_B > 1$, then any deviation from steady-state inflation leads to an explosive path of log real debt, and $\hat{B}_t - \hat{\pi}_t = 0$ is the only solution to the system.

In a representative agent model, we have $\zeta_B = 1$ and $\zeta_R = \sigma$, which gives rise to fiscal theories of the price level (see Leeper, 1991), when households assume that primary surpluses do not adjust when the real value of debt changes, $\rho_B = 1$ ("fiscal dominance"). With incomplete markets, the elasticity of savings to wealth $\zeta_B < 1$, such that the critical value for $\rho_B < 1$. Therefore, we have that even when the government does make sure that future surpluses repay any level of government debt, the price level is still determinate, because households are not indifferent about the paths of government debt. Yet, if government debt is "too" stable, in the sense that debt reverts relatively fast to its steady-state level, $\rho_B < \zeta_B$, indeterminacy still arises.

E Solving the Model with Aggregate Shocks

E.1 Local Approximation

Our model has a three-dimensional idiosyncratic state space with two endogenous states. We experimented with the grid size for liquid and illiquid asset holdings as well as for the process of productivity. Given that we focus on second moment changes, we require $n_h = 26$ productivity states and find that with a log-spaced grid for assets, results are no longer affected by grid size beyond $n_b = 80, n_k = 80$ points. This means that a tensor grid contains $n_b \times n_k \times n_h = 166,400$ points. This renders solving the model by perturbing the histogram and the value functions on a tensor grid infeasible such that we cannot apply a perturbation method without state-space reduction, as in Reiter (2002).

Instead, we develop a variant of Reiter's (2009) method to solve heterogeneous agent models with aggregate risk. We represent the dynamic system as a set of non-linear difference equations, for which

$$E_t F\left(X_t, X_{t+1}, Y_t, Y_{t+1}\right) = 0$$

holds, where the set of control variables is $Y_t = (V_t, \frac{\partial V_t}{\partial b}, \frac{\partial V_t}{\partial k}, \tilde{Y}_t)$, i.e., value functions and their marginals with respect to k, b as well as some aggregate controls $\tilde{Y}_t$ such as dividends, wages, etc. The set of state variables $X_t = (\Theta_t, R^b_t, s_t)$ is given by the histogram Θ_t of the distribution over (b, k, h) and the aggregate states R^b_t, s_t. In principle, we can solve this system with Schmitt-Grohé and Uribe's (2004) method as argued in Reiter (2002), but in practice the state space is too rich and the solution becomes numerically infeasible and unstable.

Hence, we need to reduce the dimensionality of the system. We therefore first approximate value functions and their derivatives at all grid points around their value in the stationary equilibrium without aggregate risk, $V^{SS}(b, k, h)$, by a sparse polynomial $P(b, k, h)$ with parameters $\Omega_t = \Omega(\Theta_t, R^b_t, s_t)$. For example, we write the value function as

$$V(b, k, h; \Theta_t, R^b_t, s_t)/V^{SS}(b, k, h) \approx P(b, k, h)\Omega_t.$$

Note the difference from a global approximation of the functions for finding the stationary equilibrium without aggregate risk. Here, we only use the sparse polynomial to capture *deviations* from the stationary equilibrium values, cf. Ahn et al. (2017) and different from Winberry (2016) and Reiter (2009). We define the polynomial basis functions in such a way that the grid points of the tensor grid coincide with the Chebyshev nodes for this basis.

In the system F, we then use the Bellman equation to obtain V_t from V_{t+1} on a tensor grid and then calculate the difference of Ω_t to the regression coefficients for the polynomial that fits $V_t(b,k,h)/V^{SS}(b,k,h)$.

This reduces the number of variables in the difference equation substantially, but leaves us still with too many state variables from the histogram at the tensor grid. Reiter (2010) and Ahn et al. (2017) suggest using state-space reduction techniques to deal with this issue. In continuous time, the state-space reduction can be done based on a Taylor expansion in time derivatives. In discrete time, there is no obvious basis for the state-space reduction and the Jacobi matrices involved are substantially less sparse.

Yet, we use Sklar's theorem and write the distribution function in its copula form such that $\Theta_t = \mathcal{C}_t(F_t^b, F_t^k, F_t^h)$ with the copula $\mathcal{C}_t$ and the marginal distributions for liquid and illiquid assets and productivity $F_t^{b,k,h}$. Now fixing $\mathcal{C}_t = \mathcal{C}$ can break the curse of dimensionality, reducing the number of state variables from $n_b \times n_k \times n_h = 166,400$ to $n_b + n_k + n_h = 186$, as we now only need to perturb the marginal distributions.

Fixing the copula $\mathcal{C}$ to the one from the stationary distribution, the approximation does not impose any restriction on the stationary distribution when aggregate shocks are absent, such that the approximation then becomes exact. Therefore, it is less restrictive for the stationary state then assuming a parametric form for the distribution function. The copula itself is obtained by fitting a cubic spline to the stationary distribution of ranks in b, k, h.

The idea behind this approach is that given the economic structure of the model, prices only depend on aggregate asset demand and supply, as in Krusell and Smith (1998), and not directly on higher moments of the *joint* distributions Θ_t, Θ_{t+1}. Our approach imposes no restriction on how the marginal distributions change, i.e., how many more or less liquid assets the portfolios of the x-th percentile have. It only restricts the change in the likelihood of a household being among the x-percent richest in liquid assets to be among the y-percent richest in illiquid assets. We check whether the time-constant copula assumption creates substantial numerical errors and find none by comparing it to the Krusell and Smith (1998) solution. See Figure 12 for a comparison of the IRFs for our baseline calibration.

In addition, we calculate the $\mathbf{R}^2$ statistics for the estimate $\mathcal{C}(F_{t+1}^b, F_{t+1}^k, F_{t+1}^h)$ of distribution Θ_{t+1}:

$$\mathbf{R}^2 = 1 - \frac{\int \left[d\mathcal{C}(F_{t+1}^b, F_{t+1}^k, F_{t+1}^h) - d\Theta_{t+1}\right]^2}{\int \left[d\Theta_{t+1}\right]^2},$$

Table 6: Den Haan (2010) statistic

	Absolute error (in %) for			
	Price of Capital q_t	Capital K_t	Inflation π_t	Real Bonds B_t
Mean	0.0310	0.1071	0.0359	0.4434
Max	0.1030	0.4543	0.2119	1.6634

Notes: Differences in percent between the simulation of the linearized solution of the model and a simulation in which we solve for the actual intratemporal equilibrium prices in every period for $t = \{1, ..., 1.000\}$; see Den Haan (2010).

plugging in for $F_{t+1}^{b,k,h}$ the linearized solutions $H(F_t^{b,k,h}, R_t^B, s_t)$ and for $d\Theta_{t+1}$ the solution from iterating the histogram forward given the policy functions. This yields a measure of fit for our approximation of the distribution function by a fixed copula. Absent aggregate shocks, the measure is 100% by construction. Given the solution technique, the appropriateness of the fixed copula assumption is captured by the derivative $\frac{\partial \mathbf{R}^2}{\partial x_t}$ of the $\mathbf{R}^2$ statistics with respect to state variable x_t. We find that this derivative is roughly 0.00019% with respect to uncertainty, such that, extrapolating linearly, the $\mathbf{R}^2$ at 99.9999% remains extremely high after a one standard deviation increase in uncertainty (a shock of size 0.54).

Finally, we check the quality of the linearized solution (in aggregate shocks) by solving the household planning problem given the implied expected continuation values from our solution technique but solving for the actual intratemporal equilibrium, as suggested by Den Haan (2010). We simulate the economy over T=1.000 periods and calculate the differences between our linearized solution and the non-linear one. The maximum difference is 0.45% for the capital stock and 1.66% for bonds while the mean absolute errors are substantially smaller; see Table 6.

E.2 Krusell-Smith Equilibrium

As an alternative to the solution method laid out above, we assume that households use forecasting rules to predict future prices on the basis of a restricted set of moments, as in Krusell and Smith (1997, 1998).

Specifically, these rules nowcast inflation, π_t, and capital price, q_t, and forecast the term $\left[\log(1 + \pi_{t+1})\frac{Y_{t+1}}{Y_t}\right]$ in the Phillips curve. These rules are used when calculating the continuation values in the Bellman equation. We assume these functions to be log-linear in government debt, B_t, last period's nominal interest rate, R_t^b, the aggregate stock of capital, K_t, average h_{it} (denoted H_t below), and the uncertainty state, s_t (and s_{t+1} for the forecasting term).

We formulate the problem in terms of relative price nowcasts and inflation forecasts such that we have a description of the conditional distributions of all future prices households expect. Note also that it is sufficient to write the problem in terms of *price* nowcasts and the Phillips curve forecast, because given these, households can back out future state variables describing aggregate *quantities*, $\{K_{t+s}, B_{t+s}\}$, from the government's budget constraint and the capital supply function, and future nominal rates R_{t+s}^b from the Taylor rule.

In detail, this means that when households know $K_t, B_t, R_t^b, s_t, H_t$, they can back out markups from the Phillips curve (16) using the stipulated rules for inflation in t and the conditional inflation forecasts for $t+1$. Given this, they can calculate real wages and total output. In turn, they know future government debt, B_{t+1}, from the government's budget constraint (18). The future nominal interest rate, R_{t+1}^b, is pinned down by the Taylor rule (17). Finally, from the nowcast for capital prices (20) households can determine the next period's capital stock K_{t+1}. Using these model-implied forecasts for $K_{t+1}, B_{t+1}, R_{t+1}^b, H_{t+1}$, households can then forecast the next period's inflation, capital prices, etc., conditional on shock realizations ad infinitum. The law of motion for average productivity is given analytically by

$$\begin{aligned}\log H_{t+1} &:= \log \int h_{it+1} = \frac{1}{2} var(\log h_{it+1}) \\ &= \rho_h^2 \frac{1}{2} var(\log h_{it}) + \frac{1}{2}\bar{\sigma}_h^2 \exp(s_t) = \rho_h^2 \log H_t + \frac{1}{2}\bar{\sigma}_\epsilon^2 \exp(s_t).\end{aligned}$$

Below are the functional forms we use in the nowcasts/ forecasts of prices. We let the coefficients depend on the uncertainty state (a hat denotes deviations from steady state):

$$\begin{aligned}\log \pi_t &= \beta_\pi^1(s_t) + \beta_\pi^2(s_t)\log \hat{B}_t + \beta_\pi^3(s_t)\log \hat{K}_t \\ &+ \beta_\pi^4(s_t)\hat{R}_t^b + \beta_\pi^5 H_t^{-1}, \qquad (37)\\ \log q_t &= \beta_q^1(s_t) + \beta_q^2(s_t)\log \hat{B}_t + \beta_q^3(s_t)\log \hat{K}_t \\ &+ \beta_q^4(s_t)\hat{R}_t^b + \beta_q^5 H_t^{-1}, \qquad (38)\\ \left[\log(1+\pi_{t+1})\frac{Y_{t+1}}{Y_t}\right] &= \beta_{E\pi}^1(s_t) + \beta_{E\pi}^2(s_t)\log \hat{B}_t + \beta_{E\pi}^3(s_t)\log \hat{K}_t \\ &+ \beta_{E\pi}^4(s_t)\hat{R}_t^b + \beta_{E\pi}^5 H_t^{-1} + \beta_{E\pi}^6(s_{t+1}). \qquad (39)\end{aligned}$$

Whether these rules yield good nowcasts of prices depends on the asset-demand functions, $b^*_{a,n}$ and k^*. If these are sufficiently close to linear in human capital, h, and in non-human wealth, b and k, at the mass of Θ_t, B_t and K_t will suffice and we can expect approximate aggregation to hold. For our exercise, the four endogenous aggregate states – R^b_t, H_t, B_t, and K_t – and the aggregate stochastic state s_t are sufficient to describe the evolution of the aggregate economy.

Technically, finding the equilibrium is the same as in Krusell and Smith (1997), as we need to find market clearing prices within each period. Concretely, this means that the posited rules, (37) to (39), are used to solve for households' policy functions. Having solved for the policy functions conditional on the forecasting rules, we then simulate n independent sequences of economies for $t = 1, \ldots, T$ periods, keeping track of the actual distribution Θ_t. In each simulation, the sequence of distributions starts from the stationary distribution implied by our model without aggregate risk. We then calculate in each period t the optimal policies for market clearing inflation rates and capital prices assuming that households resort to the policy functions derived under rules (37) to (39) from period $t + 1$ onward. Having determined the market clearing prices, we obtain the next period's distribution Θ_{t+1}. In doing so, we obtain n sequences of equilibria. The first 250 observations of each simulation are discarded to minimize the impact of the initial distribution. We next re-estimate the parameters of (37) to (39) from the simulated data and update the parameters accordingly. By using $n = 20$ and $T = 750$, it is possible to make use of parallel computing resources and obtain $10,000$ equilibrium observations. Subsequently, we recalculate policy functions and iterate until convergence in the forecasting rules.

The posited rules, (37) to (39), approximate the aggregate behavior of the economy fairly well. The minimal within sample R^2 is above 99%. The forecast performance is not perfect because we need to force households to effectively approximate the process for $\log \int h$ by a three-state Markov chain. This variable moves slowly and leads to small but persistent low frequency errors.

E.3 Comparison of Results

Figure 12 compares the impulse response functions obtained from the Reiter-method solution to the non-linear Krusell-Smith solution. The Krusell-Smith impulse response functions are generated by linearly interpolating the policy functions, setting the uncertainty state to exactly its expected path after a one standard deviation shock, i.e., they are obtained without simulation.

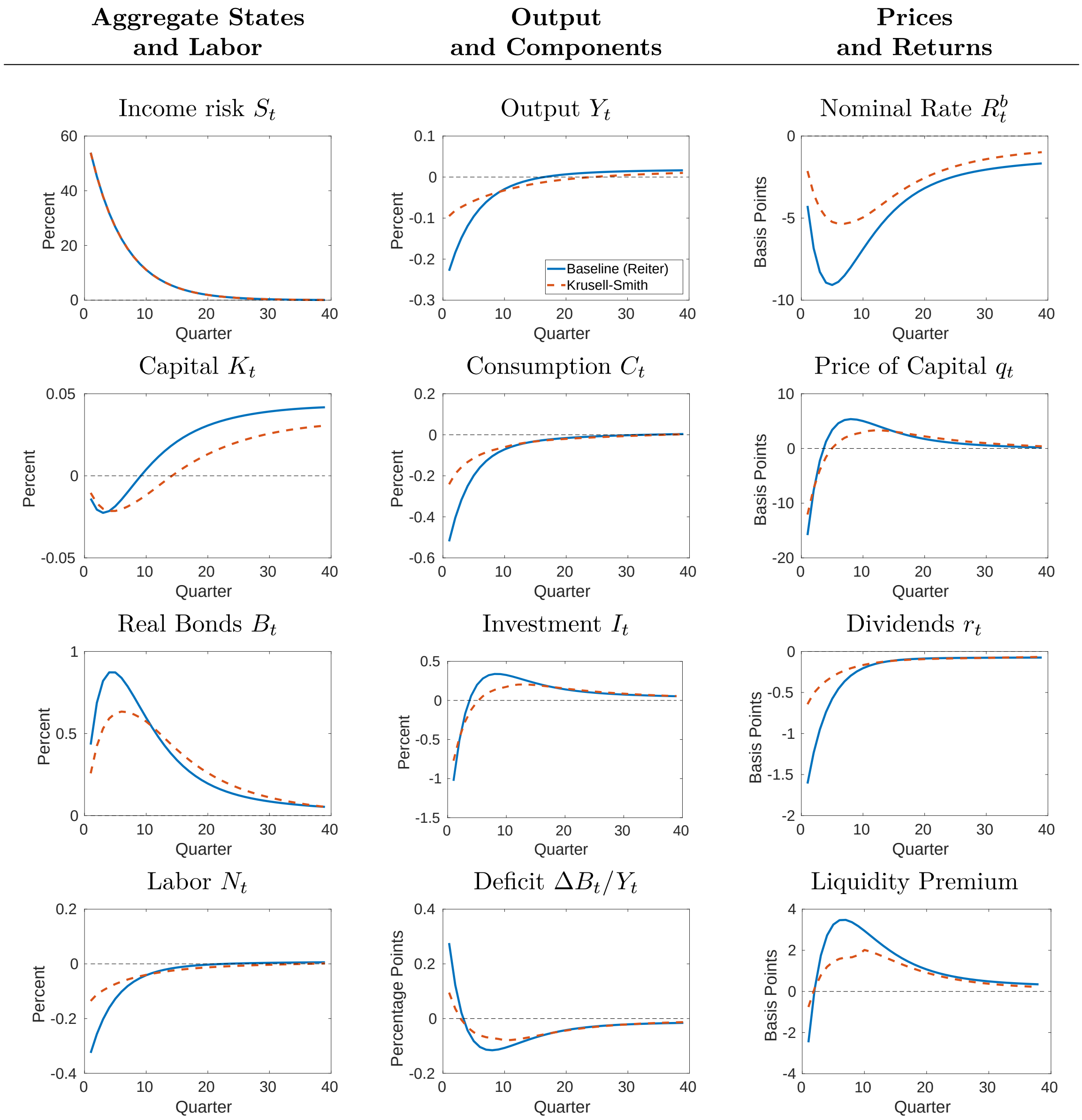
Table 7: Laws of motion for Krusell and Smith

Price of Capital $\log q_t$					(R^2: 99.38)
	s_1	s_2	s_3	s_4	s_5
β_q^1	-1.65	-1.74	-1.84	-1.95	-2.16
β_q^2	-0.02	-0.02	-0.02	-0.02	-0.01
β_q^3	-0.28	-0.28	-0.28	-0.28	-0.28
β_q^4	-0.03	-0.03	-0.03	-0.03	-0.02
β_q^5	1.90	1.90	1.90	1.90	1.90
Inflation $\log \pi_t$					(R^2: 99.95)
	s_1	s_2	s_3	s_4	s_5
β_π^1	-9.65	-9.76	-9.85	-9.95	-10.09
β_π^2	-0.07	-0.06	-0.05	-0.04	-0.03
β_π^3	-0.05	-0.06	-0.06	-0.07	-0.08
β_π^4	-0.01	-0.01	-0.01	-0.01	-0.01
β_π^5	9.96	9.96	9.96	9.96	9.96
Expectation term $\left[\log(1+\pi_{t+1})\frac{Y_{t+1}}{Y_t}\right]$					(R^2: 99.68)
	s_1	s_2	s_3	s_4	s_5
$\beta_{E\pi}^1$	-8.54	-8.62	-8.69	-8.77	-8.88
$\beta_{E\pi}^2$	-0.06	-0.05	-0.05	-0.04	-0.04
$\beta_{E\pi}^3$	-0.04	-0.04	-0.05	-0.05	-0.06
$\beta_{E\pi}^4$	-0.00	-0.01	-0.01	-0.01	-0.01
$\beta_{E\pi}^5$	8.76	8.76	8.76	8.76	8.76

[1] For readability all values are mutiplied by 100.

The impulse responses look qualitatively similar across the two methods. One should take the results and hence the differences, however, with a grain of salt, as we need to approximate the continuous aggregate states in the Krusell and Smith algorithm very coarsely with 3 grid points each for K_t, B_t, R_t^b, H_t and 5 grid points for s_t. In addition, we need to decrease the points on the idiosyncratic assets grids to 40 each, as the total number of nodes with $n_b \times n_k \times n_h \times n_s \times n_R \times n_B \times n_K \times n_H \approx 16E(+6)$ is already very large. This leads to an underestimation of the persistence of the uncertainty shock and the slow moving average idiosyncratic productivity, which decreases the aggregate effects on impact, but makes them somewhat more persistent.

Figure 12: Comparison of Krusell-Smith vs. Reiter method



Notes: Liquidity Premium: $\frac{E_t q_{t+1} + r_t}{q_t} - \frac{R_t^b}{E_t \pi_{t+1}}$.
Impulse responses to a one standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized.

F Estimation of the Stochastic Volatility Process for Household Income

F.1 Data

We estimate the income process based on the Survey of Income and Program Participation (SIPP) panels 1984, '85, '86, '87, 90', 91', '92, '93, '96, 2001, '04, and '08. We do not use the 1988 and 1989 surveys because of their known deficiencies due to the survey design and small sample size that resulted from budgetary constraints.

The SIPP panels provide monthly individual income data for up to three years (more in the 2008 survey) for each household member for each wave. The waves we use span the period 1983Q4 to 2013Q1. We constrain the sample to households with two married adults whose head is between 30 and 55 years of age and calculate for each household the labor income after taxes and transfers using NBER TAXSIM. We aggregate income to quarterly frequency and restrict the sample to households that supply at least 260 hours of work (both spouses together) per quarter (50% of full-time).

We then estimate the predictable part of log household income, based on age and education dummies and a linear quadratic term in age for each education level. Furthermore, we control for time effects, ethnicity and the number of dependent children. The residuals from this regression form the basis of our subsequent analysis. We eliminate the top-bottom 0.5% of the residuals from each age-quarter cell to remove outliers.

Then, we construct a sequence of quarterly panels containing, for each household in the panel, current residual income and two lags thereof. We use these data to calculate for each quarter and age (expressed in quarters) the variance and the first two autocovariances of residual income. We estimate the sampling variance-covariance of the empirical variance-autocovariance estimates for each quarter and age cell by bootstrapping, where we stratify by age and quarter.

F.2 Estimation

Our estimation strategy uses the theoretical (autoco-)variances, $\omega^2_{0,j}(c,t)$ for $j = 0, 1, 2,$, as described in equation (4) in Section 2 and their sample counterparts, $\mathbf{ac}^2_{0,j}(c,t), j = 0, 1, 2$, to construct a quasi-maximum likelihood (QML) estimator. It is only a *quasi* ML estimator as we treat sampling error for the variance terms as if normally distributed though they might be not. Let ψ denote the sampling error, then we have

$$\psi_j(c,t) \quad = \quad \omega^2_{0,j}(c,t) - \mathbf{ac}^2_{0,j}(c,t). \tag{40}$$

We estimate the covariance matrix of ψ, $\Sigma_\psi(c,t)$, by bootstrapping age-quarter strata. With these terms at hand, we can specify the log pseudo-likelihood as

$$-2\log L = \sum_{(c,t)\in S} \psi'(c,t)\Sigma_\psi(c,t)^{-1}\psi(c,t) + \sum_{t\in T}(\epsilon_t^s)^2/\sigma_s^2 + \#T\log\sigma_s^2, \tag{41}$$

where S is the set of all cohort-quarter pairs we observe, i.e., the cohorts 1959Q1 - 2013Q1 (denoted by the quarter they turn 30) between 1983Q4 and 2013Q1 and T is the set of quarters for which we estimate shocks, i.e., 1976Q1-2013Q1. We force $\sum_{t\in T}\epsilon_t^s = 0$.

We directly estimate the shock series, ϵ_t, together with the parameters for the persistent income shocks $(\rho_h, \rho_s, \bar{\sigma}_p, \sigma_s)$, the transitory and permanent part, $(\sigma_\tau, \sigma_\mu, \rho_\tau)$, and the time trend (θ_1, θ_2). However, since the data contain only limited information on shocks far before the sample starts, we set all shocks eight years before the first sample year (i.e., before 1976Q1) to their unconditional mean, i.e., to zero, and exclude them from the calculation of the likelihood. Eight years correspond roughly to the half-life of income shocks – $\frac{\log 1/2}{\log \rho_h} \approx 34$ quarters – and thus twice the half-life of deviations in income variances.

F.3 Bootstrapped Standard Errors

Since asymptotic standard errors might be misleading, we bootstrap the standard errors for our estimates. Yet, bootstrapping the estimator is not entirely trivial. The errors $\psi(c,t)$ are heteroscedastic. Cells with more information and more income inequality will have higher sampling variation in the (auto-co-)variances of income. What is more, bootstrapping the micro data to capture sampling error alone also does not suffice, since the sampling uncertainty also regards the time period we sampled, not only the individuals in the sample.

Therefore we proceed as follows to obtain bootstrapped standard errors for Table 1: We draw $b = 1\ldots B$ bootstrap samples of shocks $\{\epsilon_{t,b}^{s}{}^*\}_{t\in T}^{b=1\ldots B}$ from the estimated shock series $\{\hat{\epsilon}_t^s\}_{t\in T}$. We then feed the shocks through the model under the estimated parameters to obtain bootstrapped theoretical autocovariances $\omega_{0,j}(c,t)_b^*$ for $b = 1\ldots B$. We then use a wild bootstrap using a Rademacher distribution (see Davidson and Flachaire, 2008) for ν^* to draw from the estimated sampling errors $\hat{\psi}_j(c,t)$ generating the bootstrapped sampling errors

$$\psi_{j}{}_b^*(c,t) = \nu_b^*(c,t)\hat{\psi}_j(c,t),$$

i.e. we draw the entire vector of measurement error for all three autocovariances for a cohort-year cell. We then generate the re-sampled data to re-estimate the parameters and shocks from

$$\mathbf{ac}^{2}_{0,j_b}{}^{*}(c,t) = \omega_{0,j_b}^{*}(c,t) + \psi_{j_b}^{*}(c,t).$$

This leaves us with B samples to estimate parameters and shocks from. To calculate the standard deviation for each individual income risk shock ϵ_t^s we subtract the actual value of the shocks $\epsilon_{t\,b}^{s*}$ in each single bootstrap replication from the estimated shock value for that bootstrap.

F.4 Results

Table 8 summarizes the estimated parameter values. The estimated income risk and income risk shocks have been displayed in Section 2. There is a positive but decreasing trend in income risk. We take this trend into account in our model by including the average trend term in the baseline uncertainty.

Table 8: Parameter estimates

ρ_h	ρ_s	$\bar{\sigma}$	σ_s	
0.979	0.839	0.059	0.539	
(0.060)	(0.065)	(0.029)	(0.097)	
$\rho_\mathcal{T}$	$\sigma_\mathcal{T}$	σ_μ	θ_1	θ_2
0.339	0.115	0.271	3.385	-4.401
(0.009)	(0.002)	(0.005)	(0.999)	(1.280)

Notes: Bootstrapped standard errors in parenthesis. Time trend parameters are estimated coding the time 1959Q1-2013Q1 as $-1, \ldots, 1$. The estimate for the average uncertainty $\bar{\sigma}$ includes the average time-trend effect for 1983-2013.

G Wealth Distribution, Asset Classes, and Other Aggregate Variables

G.1 Data from the Flow of Funds

We can map our definition of liquid assets to the quarterly Flow of Funds (FoF), Table Z1. The financial accounts report the aggregate balance sheet of the U.S. household sector (including nonprofit organizations serving households) and are used in our analysis to quantify changes in the aggregate ratio of net liquid to net illiquid assets on a quarterly basis. Net liquid assets are defined as total currency and deposits, money market fund shares, various types of debt securities (Treasury, agency- and GSE-backed, municipal, corporate and foreign), loans (as assets), and total miscellaneous assets net of consumer credit, depository institution loans n.e.c., and other loans and advances.

Net illiquid wealth is composed of real estate at market value, life insurance reserves, pension entitlements, equipment and non-residential intellectual property products of nonprofit organizations, proprietors' equity in non-corporate business, corporate equities, mutual fund shares subtracting home mortgages as well as commercial mortgages. The Flow of Funds computes proprietors' equity in non-corporate business as the sum of all capital expenditures and financial assets of that business minus its liabilities. Therefore, and this is in line with our assumption of non-tradable pure profits, it does not contain any goodwill.

G.2 Data from the Survey of Consumer Finances

We use eleven waves of the Survey of Consumer Finances (SCF, 1983-2013) to calibrate our model and to compare the cross-sectional implications of our model with the data.

Net liquid assets are classified as all households' savings and checking accounts, call and money market accounts (incl. money market funds), certificates of deposit, all types of bonds (such as savings bonds, U.S. government bonds, Treasury bills, mortgage-backed bonds, municipal bonds, corporate bonds, foreign and other tax-free bonds), and private loans net of credit card debt.

All other assets are considered to be illiquid. Most households hold their illiquid wealth in real estate and pension wealth from retirement accounts and life insurance policies. Furthermore, we identify business assets, other non-financial and managed assets and corporate equity in the form of directly held mutual funds and stocks as illiquid, because a large share of equities owned by private households is not publicly traded nor widely circulated (see Kaplan et al., 2017). From gross illiquid asset holdings, we subtract all debt except for credit card debt.

We exclude cars and car debt from the analysis altogether. What is more, we exclude from the analysis households that hold massive amounts of credit card debt such that their net liquid assets are below minus one month of average household income - the debt limit we use in our model. Moreover, we exclude all households whose equity in illiquid assets is below the negative of one average annual income. This excludes roughly 5% of U.S. households on average from our analysis and amounts to a debt limit on unsecured debt of 9,273 US$ in 2013, for example. Table 9 displays some key statistics of the distribution of liquid and illiquid assets in the population.

We estimate the asset holdings at each percentile of the net worth distribution by running a local linear regression that maps the percentile rank in net worth into the net liquid and net illiquid asset holdings. In detail, let LI_{it}, IL_{it} be the value of liquid and illiquid assets of household i in the SCF of year t, respectively. Let ω_{it} be its sample weight. Then we first sort the households by total wealth $(LI + IL)$ and calculate the percentile rank of a household i as $\mathbf{prc}_{it} = \sum_{j<i} \omega_{jt} / \sum_j \omega_{jt}$. We then run for each percentile, $prc = 0.01, 0.02, \ldots 1$, a local linear regression. For this regression, we calculate the weight of household i as $w_{it} = \sqrt{\phi(\frac{\mathbf{prc}_{it} - prc}{h})\omega_{it}}$, where ϕ is the probability density function of a standard normal, and $h = 0.05$ is the bandwidth. We then estimate the liquid and illiquid asset holdings at percentile prc at time t as the intercepts $\lambda^{LI,IL}(prc,t)$ obtained from the weighted regressions for year t:

$$w_{it} LI_{it} = \lambda^{LI}(prc,t) w_{it} + \beta^{LI}(prc,t)(\mathbf{prc}_{it} - prc) w_{it} + \zeta_{it}^{LI}, \tag{42}$$
$$w_{it} IL_{it} = \lambda^{IL}(prc,t) w_{it} + \beta^{IL}(prc,t)(\mathbf{prc}_{it} - prc) w_{it} + \zeta_{it}^{IL}, \tag{43}$$

where ζ is an error term.

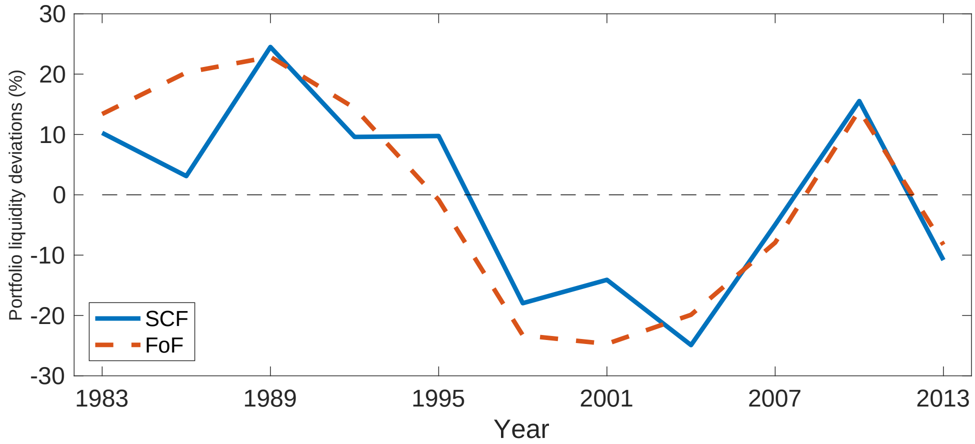
Table 9: Household portfolio composition:
Survey of Consumer Finances 1983-2013
Married households with head between 30 and 55 years of age

Moments	Model	Data
Fraction with $b < 0$	0.16	0.16
Fraction with $k > 0$	0.88	0.89
Fraction with $b \leq 0$ and $k > 0$	0.15	0.14
Gini liquid wealth	0.84	0.86
Gini illiquid wealth	0.79	0.79
Gini total wealth	0.78	0.78

Notes: Averages over the SCFs 1983-2013 using the respective cross-sectional sampling weights. Households whose liquid asset holdings fall below minus half quarterly average income are dropped from the sample. Ratios of liquid to illiquid wealth are estimated by first estimating local linear functions that map the percentile of the wealth distribution into average liquid and average illiquid asset holdings for each year, then averaging over years and finally calculating the ratios.

We can use these estimates for example to calculate average portfolio liquidity at time t as $\sum_{prc} \lambda^{LI}(prc,t) / \sum_{prc} \lambda^{IL}(prc,t)$. Figure 13 compares the percentage deviations of

Figure 13: Percentage deviation of portfolio liquidity from mean in SCF and FoF



these average portfolio liquidity measures from their long-run mean and to those obtained from the FoF data for the years 1983 to 2013. The figure reveals that both data sources capture the very similar changes in the liquidity ratio over time.

However, it is important to note that the SCF, like many comparable surveys on wealth, systematically underestimates gross financial assets, and consequently, the average liquid to illiquid assets ratio in the FoF is roughly 20%, about twice as high as the one in the SCF. This is because households are more likely to underreport their financial wealth and especially deposits and bonds due to a larger number of potential asset items. In contrast, they tend to overestimate the value of their real estate and equity (compare also Table C.1. in Kaplan et al., 2017).

G.3 Other Aggregate Data

In Section 2, we depicted the impulse response functions of the log of real GDP, real personal consumption, real private investment, real wages and the real government deficit. These variables are taken from the national accounts data provided by the Federal Reserve Bank of St. Louis (Series: GDPMC1, PCECC96, GPDIC1, GCEC1, AHETPI) and data on government deficits from the NIPA tables for the U.S. (Table 3.1, BEA).

Data on house prices, Treasury bill returns and the liquidity premium stem from the same source. We use the secondary market rate of the 3-month Treasury bill (DTB3) as a measure for the short-term nominal interest rate. House prices are captured by the Case-Shiller S&P U.S. National Home Price Index (CSUSHPINSA) divided by the all-items CPI (CPIAUCSL). The liquidity premium we construct from nominal house prices, the CPI for rents, and the rate on 3-month Treasuries. We measure the liquidity premium as the excess realized return on housing. This is composed of the rent-price ratio in t, $\frac{r_{h,t}}{q_t^{house}}$ plus the quarterly growth rate of house prices, $\frac{q_{t+1}^{house}}{q_t^{house}}$ in $t+1$, over the nominal return on riskless 3-month Treasury bills R_t^b (converted to a quarterly rate):

$$LP_t = \frac{r_{h,t}}{q_t^{house}} + \frac{q_{t+1}^{house}}{q_t^{house}} - (R_t^b)^{\frac{1}{4}}. \tag{44}$$

Rents are imputed on the basis of the CPI for rents on primary residences paid by all urban consumers (CUSR0000SEHA) fixing the rent-price ratio in 1981Q1 to 4%.

The Solow residual series we use is taken from the latest version (date of retrieval 2017-11-01) of Fernald's raw TFP series (Fernald, 2012). We construct an index from the reported growth rates and use the log of this index.

H Details on the Empirical Estimates of the Response to Shocks to Household Income Risk

H.1 Local Projection Method

In Figure 3 of Section 2 we presented impulse response functions based on local projections (see Jordà, 2005). This method does not require the specification and estimation of a vector autoregressive model for the true data-generating process. Instead, in the spirit of multi-step direct forecasting, the impulse responses of the endogenous variables X at time $t+j$ to uncertainty shocks, ϵ_t^s, at time t are estimated using horizon-specific single regressions, in which the endogenous variable shifted ahead is regressed on the current normalized uncertainty shock ϵ_t^s, a time trend, lagged income risk s_{t-1} and controls $\mathbf{X}_{t-1}$. These controls are specified as the return on T-bills R_{t-1}^b and the log of GDP Y_{t-1}, of consumption C_{t-1}, of investment I_{t-1}, of TFP, A_{t-1}, and of real wages, w_{t-1}, as well as the GDP share of the government deficit $\Delta B_{t-1}/Y_{t-1}$:

$$X_{t+j} = \beta_{j,0} + \beta_{j,\epsilon}\epsilon_t^s/\sigma_s + \beta_{j,t}t + \beta_{j,X}\mathbf{X}_{t-1} + \beta_{j,s}s_{t-1} + \nu_{t+j}, \; j = 0 \dots 12. \tag{45}$$

Hence, the impulse response function $\beta_{j,\epsilon}$ is estimated just as a sequence of projections of X_{t+j} in response to the standardized shock ϵ_t^s/σ_s, local to each forecast horizon $j = 0, \dots, 12$ quarters. We focus on the post-Volcker disinflation era and use aggregate time series data from 1983Q1 to 2016Q2.

H.2 Alternative Identification Schemes

An important assumption made for employing the local projection method, which directly regresses the shocks on the endogenous variable of interest, is that the identified uncertainty shocks ϵ_t^s obtained from SIPP data are purely exogenous and orthogonal to all other structural shocks ν_{t+j} in the economy.

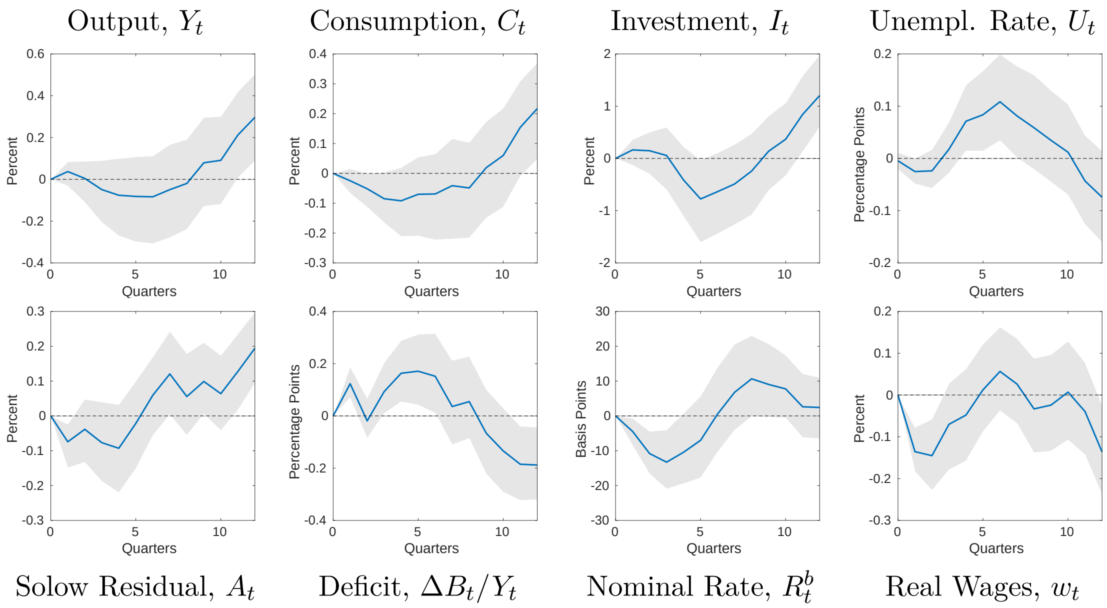
While this method allows for an identification that is fully consistent with our model, where all uncertainty fluctuations are exogenous, this identification strategy is arguably not very conservative. Therefore, we present additional evidence based on two alternative identification schemes.

Our baseline scheme can be understood as ordering income risk first in a Cholesky-identified SVAR. Our first robustness check therefore takes the opposite extreme assumption and assumes that none of the variables in Figure 3 except for income risk itself reacts to an income risk shock, i.e., we estimate

$$X_{t+j} = \beta_{j,0} + \beta_{j,\epsilon}\epsilon_t^s/\sigma_s + \beta_{j,t}t + \beta_{j,X_t}\mathbf{X}_t + \beta_{j,X_{t-1}}\mathbf{X}_{t-1} + \beta_{j,s}s_{t-1} + \nu_{t+j},\ j = 0\ldots 12. \quad (46)$$

Results can be found in Figures 14 and 15. The estimated output response is slightly smaller and all responses are somewhat more delayed as, by construction, the immediate impact is zero for output and its components, for measured productivity, real wages, and the government's policy variables. Still we find that the liquidity of household portfolios increases on impact. The results for house prices and the liquidity premium are slightly more mixed.

Figure 14: Empirical response to household income risk shock: alternative identification

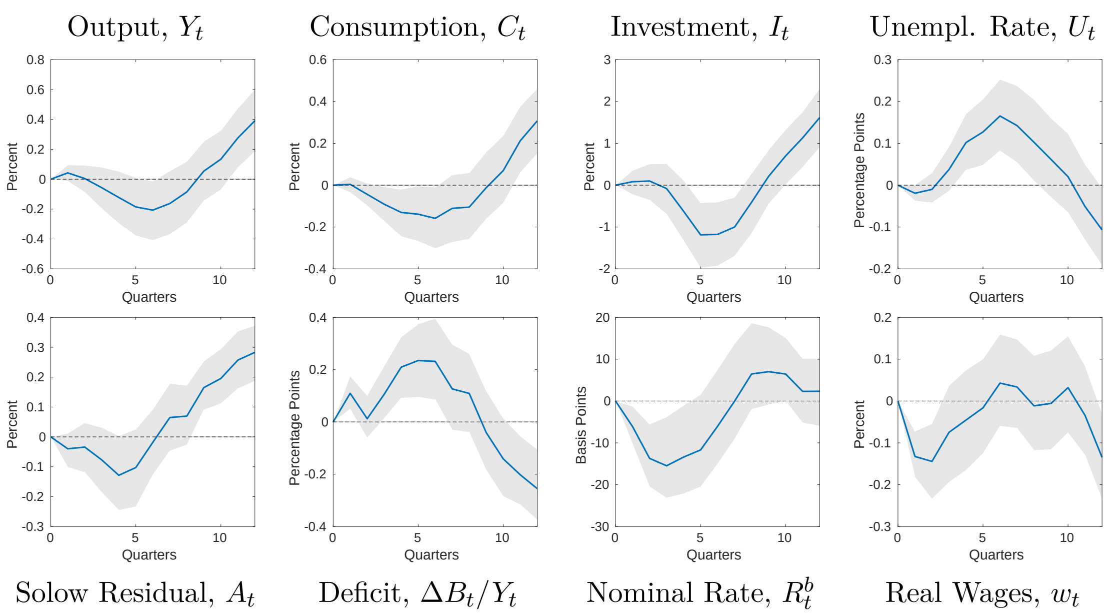


Estimated response of $\mathbf{X}_{t+j}, j = 0 \ldots 12$, where $\mathbf{X}_t = [Y_t, C_t, I_t, A_t, \Delta B_t/Y_t, w_t, R_t^b]$, to the estimated shocks to household income risk, ϵ_t^s. The regressions control for the current and lagged state of the economy $\mathbf{X}_t, \mathbf{X}_{t-1}$ and lagged levels of income risk s_{t-1}. The nominal rate is the 3-month T-bill rate. Bootstrapped 66% confidence bounds in gray (block bootstrap).

Figure 15: Response of household portfolios, house prices and the liquidity premium to household income risk shock: alternative identification

Estimated response of the liquidity of household portfolios, the price of houses (Case-Shiller S&P Index), and the difference between the return on housing and the nominal rate (Liquidity Premium) to income risk using local projections. The set of control variables is as in Figure 14. Bootstrapped 66% confidence bounds in gray (block bootstrap).

Figure 16: Empirical response to household income risk shock: alternative identification



Estimated response of $\mathbf{X}_{t+j}, j = 0 \ldots 12$, where $\mathbf{X}_t = [Y_t, C_t, I_t, A_t, \Delta B_t/Y_t, w_t, R_t^b]$, to the estimated shocks to household income risk, ϵ_t^s. The regressions control for current output, the current value of the variable of interest and the lagged state of the economy $X_t, Y_t, \mathbf{X}_{t-1}$ and lagged levels of income risk s_{t-1}. The nominal rate is the 3-month T-bill rate. Bootstrapped 66% confidence bounds in gray (block bootstrap).

Figure 17: Response of household portfolios, house prices and the liquidity premium to household income risk shock: alternative identification

Estimated response of the liquidity of household portfolios, the price of houses (Case-Shiller S&P Index), and the difference between the return on housing and the nominal rate (Liquidity Premium) to income risk using local projections. The set of control variables is as in Figure 16. Bootstrapped 66% confidence bounds in gray (block bootstrap).

Our second alternative identification scheme is somewhat in-between the baseline and the first alternative scheme. In line with the practice to estimate various small SVARs, we estimate the local projection controlling for all lagged variables and only for current output and the current value of the variable of interest. This is a more parsimonious specification but it comes at the cost of identifying a slightly different shock in each regression. Results can be found in Figures 16 and 17. Also here results are very much in line with our baseline treatment of the data.

I Unconditional Business Cycle Statistics

Table 10 reports unconditional business cycle statistics for the quarterly U.S. data we use in the empirical sections and for our model.

Table 10: Business cycle statistics data/model

	GDP	C	I	Deficit
	Time series standard deviation of . . . (in percent)			
Data	1.38	0.98	6.28	1.33
Model TFP	1.38	0.75	6.28	1.33
Model Uncertainty	0.29	0.63	1.33	0.52
	Correlation with GDP			
Data	1.00	0.92	0.92	-0.76
Model TFP	1.00	0.87	0.96	-0.86
Model Uncertainty	1.00	1.00	0.74	0.08

Notes: Real GDP, Consumption (C), Investment (I) in logs. Net government savings (deficit) as a fraction of GDP. All data are HP-filtered with $\lambda = 1600$. Model refers to the baseline model with TFP or income risk shocks only.

J Model Extensions

J.1 Importance of Illiquid Assets

Even when all assets are liquid, households will decrease their consumption demand for precautionary motives when income uncertainty rises. We have seen in Section 3 that the presence of illiquid assets introduces a portfolio adjustment in response to the uncertainty shock, which augments the increase in demand for liquid assets.

To show the importance of the portfolio adjustment channel also in our full model, we solve a version of the model where all assets are liquid. In this case, the household portfolio position between the two assets is indeterminate in the steady state as long as the expected returns of both assets are equal

$$E_t \left[r_{t+1} + \frac{q_{t+1}}{q_t} \right] = E_t \left[\frac{R^b_{t+1}}{\pi_{t+1}} \right], \tag{47}$$

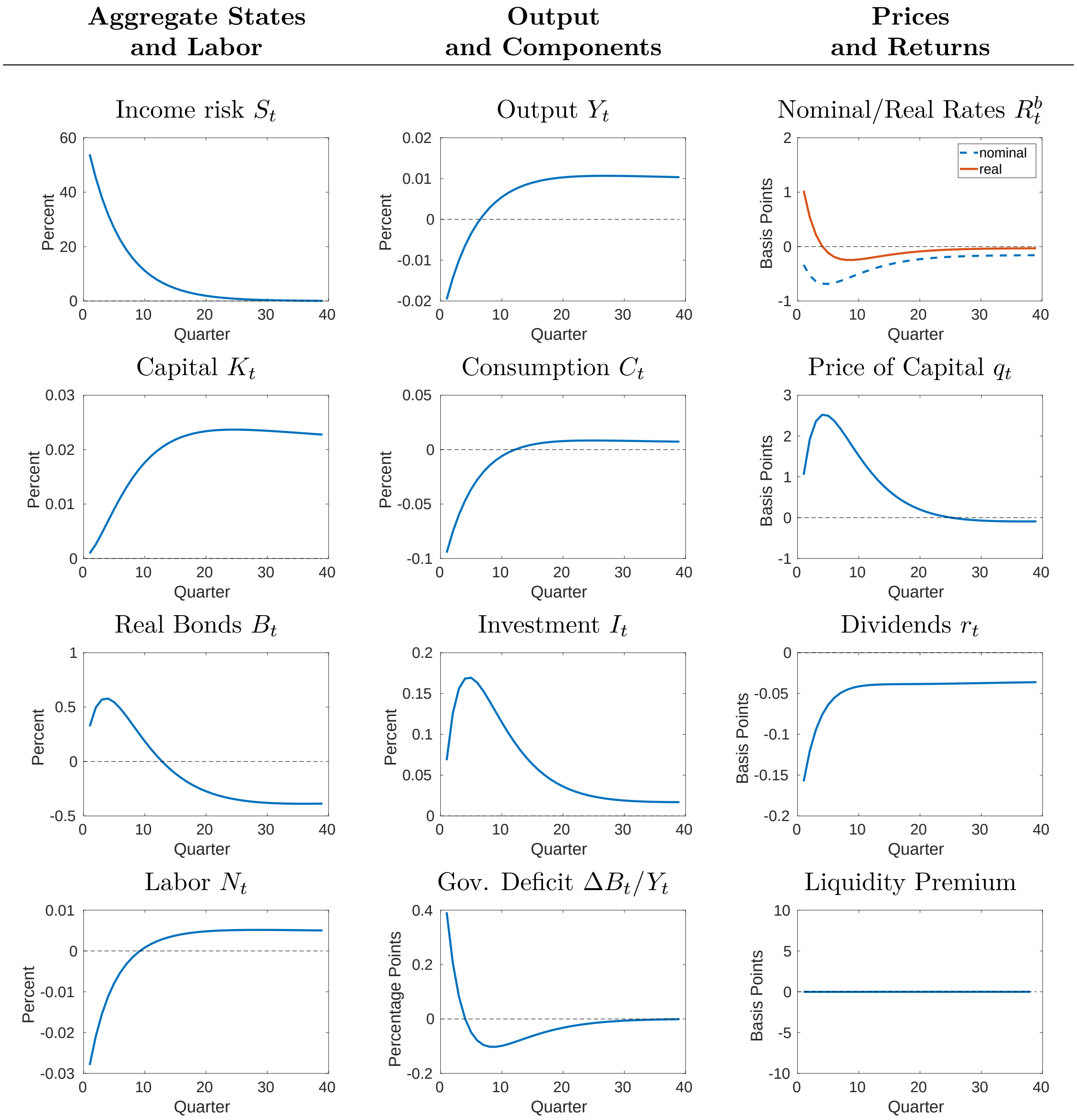
and in equilibrium they must be equal for households to be willing to hold a positive amount of both assets.

Since our solution method linearizes the problem in the presence of aggregate shocks, the portfolio problem remains indeterminate. Therefore, we assume that all households hold the same bond-to-capital ratio, which is in the aggregate determined by (47) and by the supply of government bonds.

We recalibrate the discount factor to match again the capital to output and bond to capital ratio as our baseline model. We also recalibrate the aggregate capital adjustment cost parameter to match again the relative investment volatility in response to TFP shocks but keep all other household, policy and technological parameters as in our baseline.

The response to an income risk shock changes drastically as Figure 18 shows. Output falls much less than in our baseline, but most important, we get an investment boom. The level of price stickiness is not sufficiently strong to drive down the capital return such that, as central bank interest rates fall, the capital stock and hence investment must go up in equilibrium to equate expected capital and bond returns.

Figure 18: Aggregate response to household income risk with liquid capital



Notes: Liquidity Premium: $\frac{Eq_{t+1}+r_t}{q_t} - \frac{R^b_t}{E_t\pi_{t+1}}$.
Impulse responses to a one standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized.

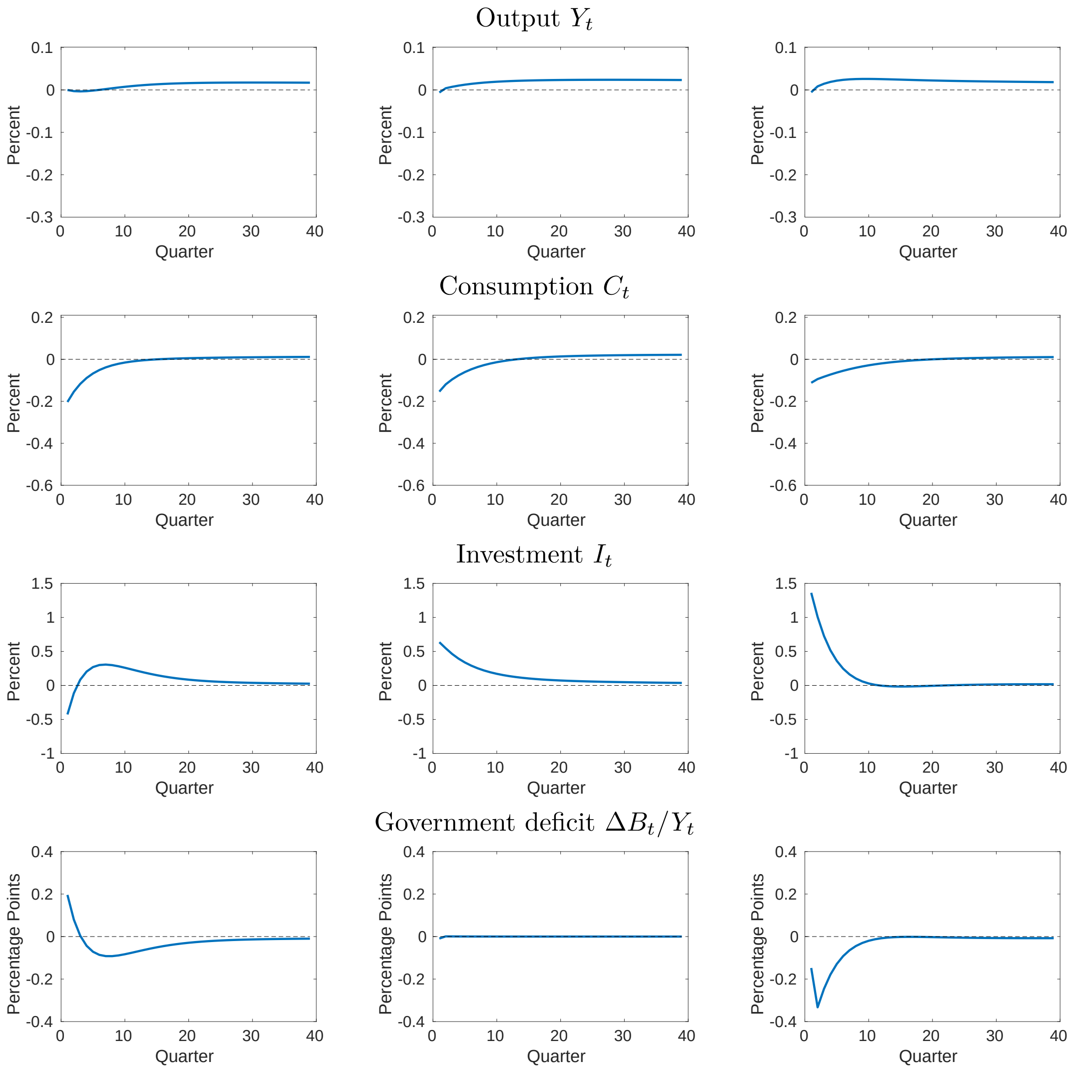
J.2 The Role of Price Stickiness

Next, we assess the importance of price stickiness for our results. For this purpose we set the price adjustment costs to (virtually) zero. We find that the output effects of income risk shocks are negligible in this specification; see Figure 19. The income risk shock in our model creates a slump in private demand, but without any price stickiness this is undone by price-level movements. Inflation falls, and given the Taylor rule, the real interest rate on liquid assets falls, too. Households then shift their portfolios back to the illiquid asset that has a higher relative return now. In summary, price stickiness is essential for the negative output movement that we find.

The exact path of aggregates depends on the fiscal and monetary rules in place. With no fiscal response to inflation the path of aggregates is very similar to the one under perfect monetary stabilization; see Figure 19 columns (b) and (c). When government spending increases in response to falling inflation, the effect on and through the real amount of government debt is different; see Figure 19 column (a). Outstanding government debt increases more persistently and crowds out private consumption and investment on impact.

Figure 19: Aggregate response to household income risk shock with flexible prices

(a) Baseline fiscal and monetary response	(b) No fiscal or monetary response	(c) Only monetary response

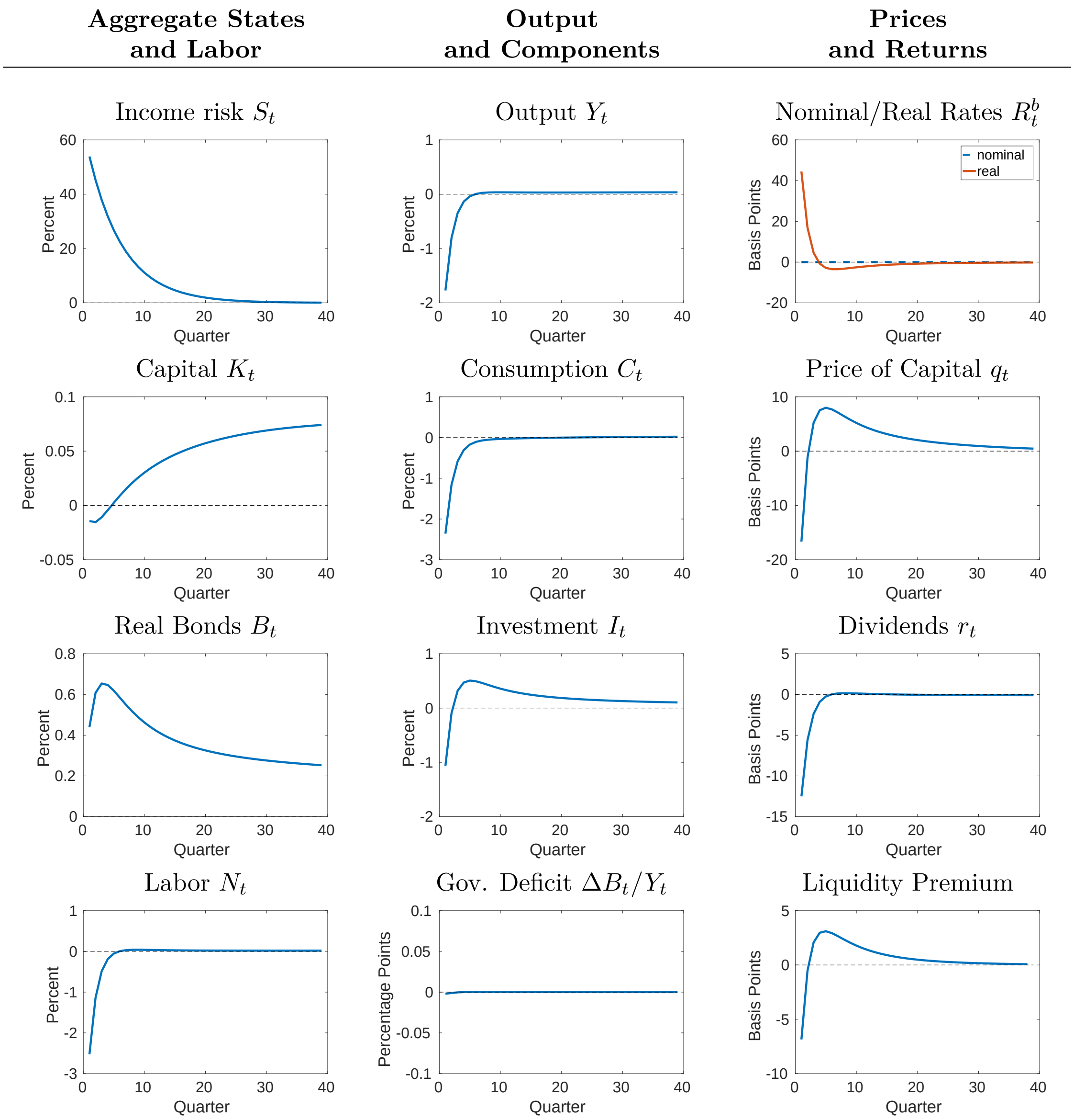


Notes: Liquidity Premium: $\frac{Eq_{t+1}+r_t}{q_t} - \frac{R_t^b}{E_t \pi_{t+1}}$.
Impulse responses to a one standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized.
(a) $\gamma_\pi = 1.5, \gamma_\mathcal{T} = 0.5075, \rho_B = 0.86, \theta_\pi = 1.25, \rho_R = 0.8$
(b) $\gamma_\pi = 0, \gamma_\mathcal{T} = 0, \rho_B = 1, \theta_\pi = 0, \rho_R = 1$
(c) $\gamma_\pi = 0, \gamma_\mathcal{T} = 0, \rho_B = 0.86, \theta_\pi = 1.25, \rho_R = 0.8$

J.3 Response of the Model without Stabilization

To capture what happens if governments do not stabilize, e.g., at the ZLB, we produce the impulse responses for our baseline calibration with an interest rate peg and no fiscal stabilization; see Figure 20. Output, consumption, and investment fall drastically.

Figure 20: Aggregate response to household income risk shock without stabilization



Notes: Liquidity Premium: $\frac{Eq_{t+1}+r_t}{q_t} - \frac{R_t^b}{E_t\pi_{t+1}}$.
Impulse responses to a one standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized. All stabilization policy parameters are set to zero; $\theta_\pi = 0$, $\gamma_\pi = 0$, $\gamma_\mathcal{T} = 0$, and $\rho_B = 1$.

J.4 Introducing Asset-Backed Securities

A potential limitation of our model is that it does not include mortgages – an important way for the banking sector to create liquid assets out of illiquid investments. To capture the effect of mortgages, we assume that any newly created capital good is partly credit financed, i.e., any investment creates an illiquid asset and a liquid asset-backed security at the same time. Let ζ be the number of bonds issued as mortgages per unit of capital. We assume that the fraction ζ cannot be adjusted by the household sector. However, households can buy back the mortgages originated from “their” illiquid assets.

For this purpose, we model a wedge $\underline{R}$ between the borrowing and lending rate on mortgages due to the costs of intermediation. The rate paid to lenders is the central bank's interest rate R_t^b, and the rate borrowers pay is $R_t^b + \underline{R}$. This means that, net of the payments on the asset-backed securities, the dividend stream from illiquid assets decreases to $r_t = F_K(K, N) - \delta - \zeta(R_t^b + \underline{R})$. However, also the price of illiquid assets decreases, which now is $q_t = 1 - \zeta + \phi \frac{K_{t+1} - K_t}{K_t}$ in equilibrium, because the producer of each unit of illiquid assets can sell ζ units of asset-backed securities for each unit of the illiquid capital good in addition to that good itself.

To allow households to adjust the extent to which they effectively draw their mortgages, we assume that the borrowing wedge does not apply to securities backed by assets the household itself owns. This means that the household now faces three marginal interest rates on liquid assets, such that the total payout on liquid assets is:

$$Repayment = \begin{cases} R_t^b(b_{it} - \zeta k_{it}) + \zeta k_{it}(R_t^b + \underline{R}) & \text{if } b_{it} \geq \zeta k_{it} \\ (R_t^b + \underline{R}) b_{it} & \text{if } 0 < b_{it} < \zeta k_{it} \\ (R_t^b + \overline{R}) b_{it} & \text{if } b_{it} < 0. \end{cases}$$

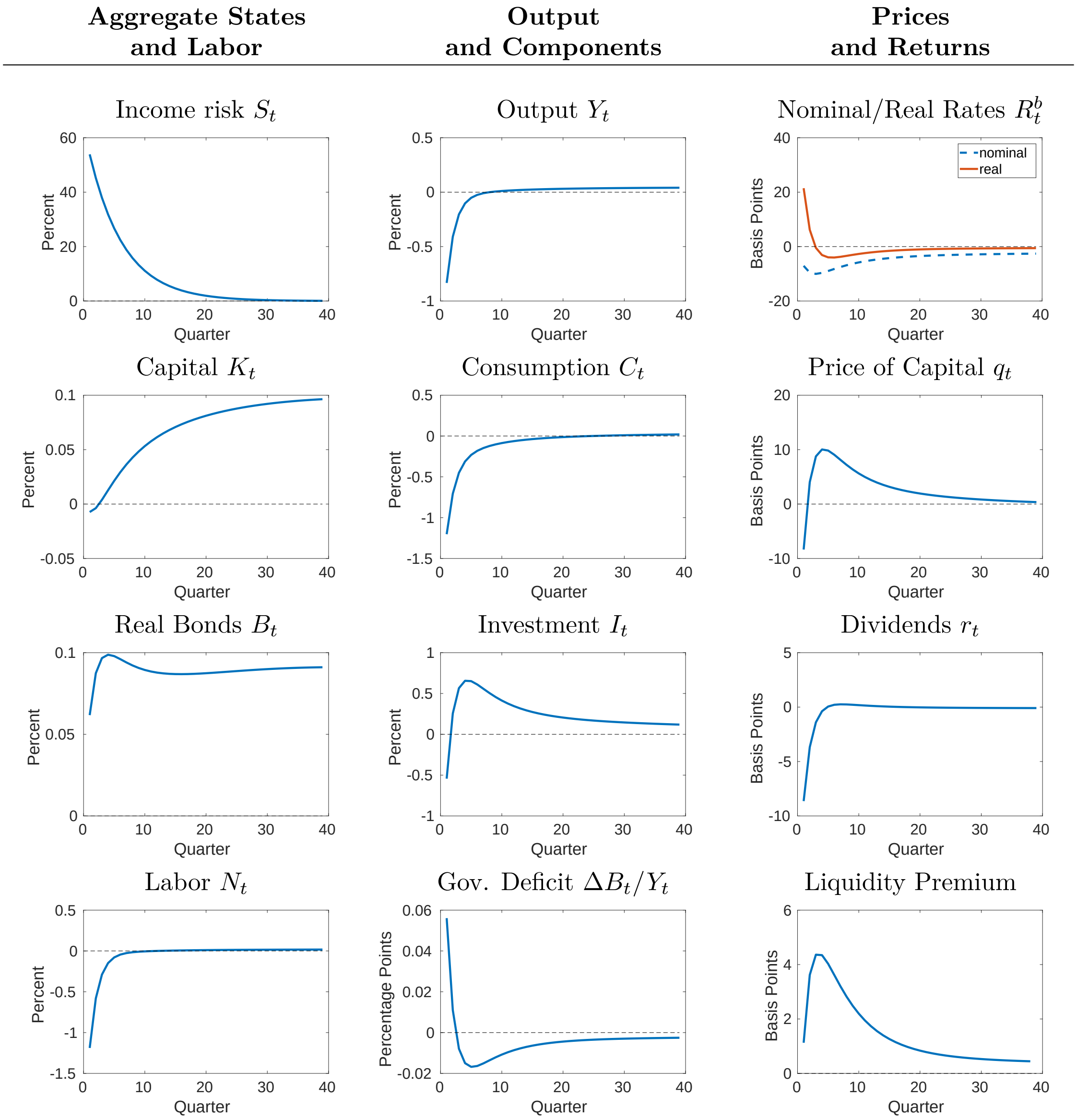
The highest interest rate applies to unsecured borrowing $b < 0$. An intermediate interest rate applies if the household buys back securities originated from the illiquid asset it owns $0 < b < \zeta k$, i.e., that the household saves by paying back a mortgage. The lowest interest rate applies when the household accumulates liquid assets beyond those it has originated.

The bond market equilibrium condition then reads:

$$\begin{aligned} &\zeta K_{t+1} + B_{t+1} \\ = &\int\int\int_{b \geq \underline{B}} \left[\nu^* b_a^*(b, k, h; q_t, \pi_t, R_{t+1}^b) + (1 - \nu^*) b_n^*(b, k, h; q_t, \pi_t, R_{t+1}^b)\right] d\Theta_t(b, k, h), \end{aligned} \tag{48}$$

where ζK_{t+1} is the amount of asset-backed securities in circulation. The market clearing condition for illiquid assets remains unchanged.

Figure 21: Aggregate response to household income risk with asset-backed securities



Notes: Liquidity Premium: $\frac{Eq_{t+1}+r_t}{q_t} - \frac{R_t^b}{E_t\pi_{t+1}}$.
Impulse responses to a one standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized.

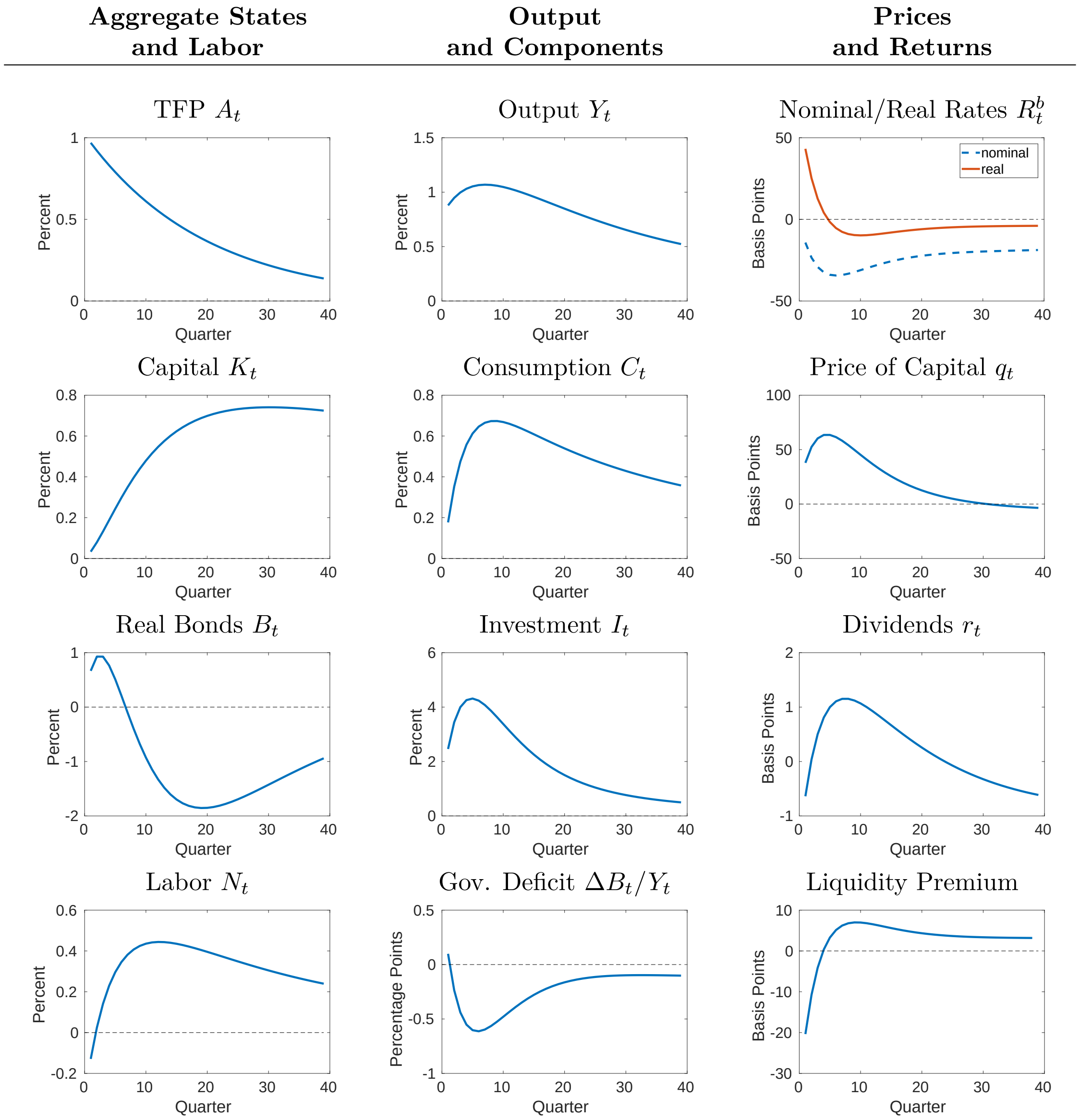
We have re-calibrated the amount of government debt to keep the average portfolio liquidity unchanged when not counting securities held by the issuer. The ratio of mortgage liabilities of households to their net worth in the Flow of Funds (Table Z1-B.101) is roughly 10%. We set $\zeta = 10\%$ and calibrate $\underline{R} = 1\%$ p.a.

Figure 21 shows the impulse responses for our baseline calibration with ABS. Compared to our baseline scenario, the recessionary impact of uncertainty is larger, because the rebalancing of portfolios implies a decline in the supply of liquid assets as households reduce the stock of capital.

J.5 Response of the Model to TFP Shocks

Given our solution technique, it is straightforward to extend the model by other shocks. For our calibration we use an extension with time-varying total factor productivity in production, such that $Y_t = A_t F(K_t, L_t)$, where A_t is total factor productivity and follows an AR(1) process in logs with a persistence of 0.95 and a standard deviation of 0.00965. We use this model variant to calibrate capital adjustment costs. Figure 22 below shows the IRFs to a TFP shock.

Figure 22: Aggregate response to a TFP shock



Notes: Liquidity Premium: $\frac{E_t q_{t+1}+r_t}{q_t} - \frac{R_t^b}{E_t \pi_{t+1}}$.
Impulse responses to a one standard deviation increase in TFP. All rates (dividends, interest, liquidity premium) are *not* annualized.

K Individual Consumption Responses to Persistent and Transitory Income Shocks

In order for the model to provide a useful framework for welfare analysis, it is important that the model replicates the empirical evidence on consumption responses to persistent and transitory income shocks (in partial equilibrium). For this purpose, we consider the average consumption elasticity to a persistent increase in income and an increase in liquid assets proportional to income (transitory income shock). These two elasticities are key to understanding the consumption smoothing behavior of an incomplete markets model; see Kaplan and Violante (2010) and Blundell et al. (2008). Table 11 provides these statistics for our model.

The model replicates the fact that transitory income shocks are well insured, while persistent income shocks are much less insured. Given the below unit-root autocorrelation of persistent income, our model predicts persistent income shocks to be somewhat better insured in comparison to assuming permanent shocks.

Table 11: Consumption smoothing in model and data

Elasticity of consumption to transitory and persistent income shocks		
	Data	Model
Transitory income change	0.05	0.05
Persistent income change	0.43	0.44

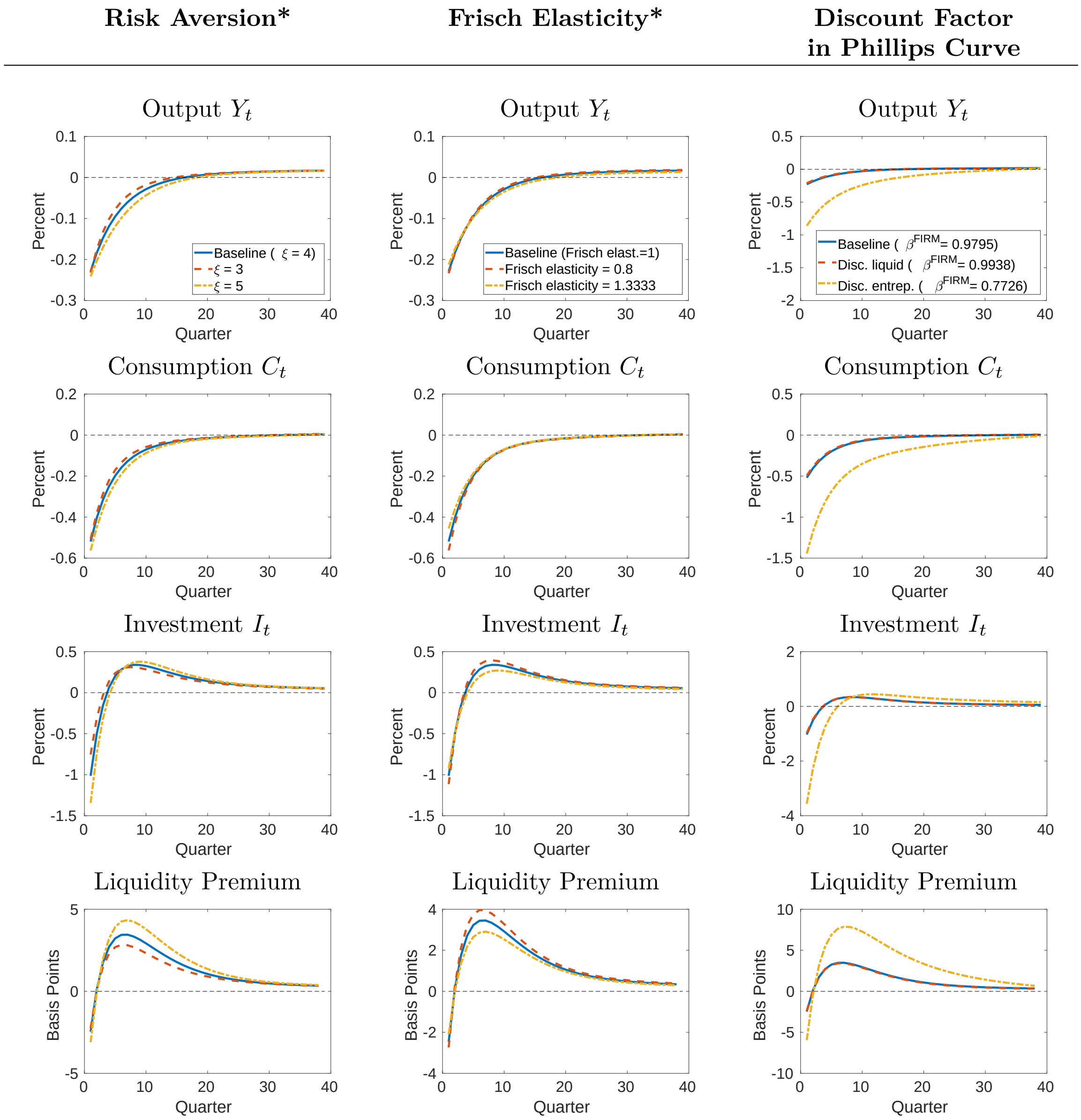
Data correspond to Kaplan and Violante (2010).

L Robustness Checks

For the risk aversion parameter and the Frisch elasticity of labor supply, we take standard values from the literature as there is no direct counterpart in the data. To account for this calibration strategy, we check the robustness of our findings with respect to the assumed parameter values. We do so by varying one of the parameters at a time while recalibrating to match the moments of Table 3 by adjusting the discount factor, the mean and variance of the distribution of adjustment costs, and the borrowing penalty.

We find our results are qualitatively robust to all the considered parameter variations. The impulse response functions for output, consumption, investment, and the liquidity

Figure 23: Robustness A: Aggregate response to household income risk shock



Notes: Liquidity Premium: $\frac{Eq_{t+1}+r_t}{q_t} - \frac{R_t^b}{\pi_{t+1}}$.

Impulse responses to a one standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized.
* Recalibrated to match the moments of Table 3 by adjusting the discount factor, the mean and variance of the distribution of adjustment costs, and the borrowing penalty.

premium are displayed in Figure 23. When we reduce the risk aversion households do not decrease investment demand as much as in the baseline and conversely the liquidity premium increases less. The illiquidity of capital is less important to households. An increase in the inverse Frisch elasticity is very similar to an increase in risk aversion. As can be seen from the household budget constraint when labor supply is maximized out, the lower the inverse Frisch elasticity, the less do the resources the household has for composite consumption fluctuate with productivity h. The recalibration of the illiquidity of capital only partially offsets this, because the return movements through central bank policy become relatively more important when households are effectively less affected by changes in income risk (either because they are less risk averse or better insured through the labor market).

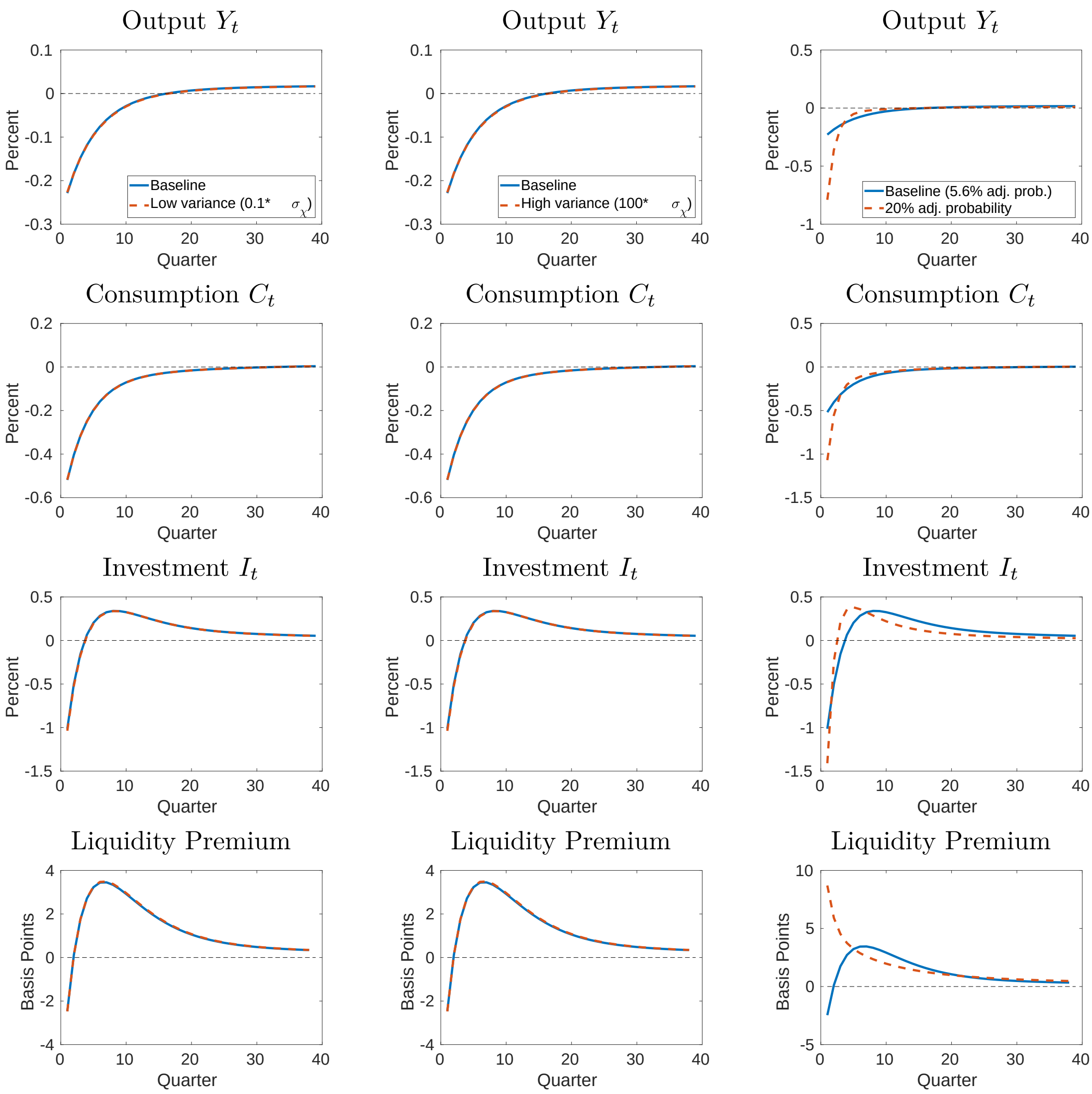
Furthermore, we set the discount factor used in the firms' maximization problem to the median stochastic discount factor for entrepreneurs taking into account that an entrepreneur household becomes a worker household with a certain probability. The resulting discount factor is roughly 77% quarterly. Such an extreme discounting has some impact on results, making the recessionary effects of income risk larger because future expected deflation does help less to stabilize output, hence rendering monetary policy with interest rate smoothing less effective.

As a second robustness check, we vary the utility costs of portfolio adjustment. First, we make the adjustment probability more reactive to the value gained from adjustment by lowering the variance of the logistic distribution from which households draw the adjustment cost. Second, we consider a case of almost fixed adjustment probabilities by increasing the variance of the logistic distribution drastically. Third, we lower the mean of the logistic distribution such that the average adjustment probability goes up to 20% (and the average portfolio liquidity falls). All three cases show results qualitatively similar to our baseline; see Figure 24.

Making adjustment more state dependent yields quantitatively very similar results, the investment response is only slightly muted. When adjustment probabilities are fully exogenous, the investment response is slightly larger. When the illiquid asset is more liquid, the effect of a shock to income risk becomes stronger in the short run but also shorter lived. The economic intuition seems to be the following: When the illiquid asset is very liquid, the demand for liquid assets becomes smaller but also less elastic to the return differences between the assets. Therefore, the central bank's intervention that cuts rates can stabilize less. The supply of liquid assets itself becomes more important, but with a small stock of outside liquidity, the same relative growth in government bonds stabilizes aggregate demand less. However, as soon as the stock of liquid funds has increased sufficiently, households start to invest in illiquid assets again.

Figure 24: Robustness B: Aggregate response to household income risk shock

Low variance of portfolio adj. costs	**High variance of portfolio adj. costs**	**High Liquidity (low mean adj. costs)**



Notes: Liquidity Premium: $\frac{E_t q_{t+1} + r_t}{q_t} - \frac{R_t^b}{E_t \pi_{t+1}}$.

Impulse responses to a one standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized.