Institutional Choice: A Contract-Theoretic Approach

by

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Abstract

This paper distinguishes various settings according to what kind of contracts they allow for. The same model of sequential binary choice is used to illuminate different aspects of the contract-theoretic approach to transaction costs economics. Selective intervention as the notion which leads to Williamson's Puzzle is explored in a framework of incomplete contracts. It is shown that agents committed by contracts that are second-best or worse cannot safely be relied upon to administer selective intervention. In a final section, the model is used critically to revisit the theory of ownership structure which is due to Grossman and Hart (1986).

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1. Introduction

The Coase Theorem claims that parties linked through external effects manage to coordinate their activities in an efficient way provided that transaction costs are sufficiently low. Therefore, as far as efficiency is concerned, the assignment of property rights among parties does not matter. The market failure literature in the tradition of Pigou, on the other hand, views externalities as an omnipresent source of concern. External effects are seen as leading to a discrepancy between private and social gains from individual actions which, in turn, causes markets to fail. Hereby, the notion of transaction costs plays no role at all. To reconcile the two approaches it is useful to view contracts as a means to reduce that discrepancy as I have argued elsewhere (Schweizer, 1988)). In this sense, it seems natural to distinguish various situations according to what kind of contracts they allow for. This is the contract-theoretic approach to transaction costs economics.

Inman (1987) correctly summarizes the market failure literature when he claims market failure to be due to noncooperative behavior of, in essence, the Prisoners' Dilemma type. Recall, however, that the essential message of this paradigm rests on two assumptions. First, the noncooperative Nash-equilibrium of a game, even if it is one in dominant strategies, need not be first best. This fact is commonly used as a game-theoretic foundation of the discrepancy argument. Second, the paradigm refers to prisoners who are locked up in separate cells and who, for that reason, cannot enter a contractual relationship. The inefficiency of the predicted outcome is jointly due to both aspects of the paradigm. The market failure literature has paid little attention to the assumption which has parties sitting in separate cells. To be sure, the paradigm may fit in a figurative if not a literal sense such that market exchange may indeed fail to achieve a first best solution. Nonetheless, it would be premature to speak of market failure as long as no other institutional arrangement has been found which actually outperforms voluntary contracting. In particular, it should never be taken for granted that governmental intervention would automatically be such a superior arrangement because the very fact which prevents parties from writing complete contracts could impede other corrective measures as well. Actually, traditional remedies such as Pigouvian taxation which are claimed to lead to a first best outcome are very similar to complete contracts. The question then arises why, under laissez-faire, the paradigm of solitary confinement should fit whereas, after mere governmental intervention, it becomes possible to make use of complete contracts. Unfortunately, the public interest theory (PIT) which predicts
such first best remedies to emerge as a response to market failure does not give an answer. In fact, the PIT if taken at face value would lead to the conclusion that intervention is the optimum arrangement because it paves the way for complete contracts. Such a conclusion, however, is not plausible. Therefore, the logic behind the PIT leads to a puzzle, henceforth referred to as the PIT-Puzzle, which is akin to Williamson's Puzzle on the limitations of firm size. Coase (1937) has argued that there is a cost of using the price mechanism. Therefore it might be profitable to establish a firm which economizes on these costs. What then would be responsible for limitations of firm size or, as Williamson (1985) has sharply phrased it: Why not organize everything in one large firm? It seems that a combined firm could do everything that autonomous parts of it could do previously and, by selective intervention, the combined firm could accomplish more as compared to the decentralized case. Williamson's question, indeed, is a puzzling one.

The present paper reports on a contract-theoretic approach to the notion of transaction costs on which it then builds a theory of institutional choice. Grossman and Hart (1986) investigate how the ownership structure may affect incentives to invest in managerial efforts. This theory provides a reply to Williamson's puzzle. To resolve the PIT-Puzzle, the present paper heavily borrows from this approach. Well in line with methodological individualism, public intervention is assumed to require political agents who act in a selfish way. Rules may be used to limit their discretionary power. In a world of positive transaction costs, however, where political agents can only be committed through contracts that are second best or worse, selective intervention may remain difficult to administer.

The paper is organized as follows: Section 2 introduces a model of sequential binary choice. As far as mathematical sophistication is concerned, the setting is extremely simple: Optimizing linear functions over an interval is the most intricate technique that is needed! Yet the model captures, in a qualitative if not a quantitative sense, quite many situations. Moreover, with respect to contract-theoretic aspects, the apparently simple model turns out to produce a surprisingly rich variety of phenomena. This latter fact justifies, I think, the extensive use which is made of the model throughout the paper. Obviously, if the approach proves to be conceptually sound then it definitely could be extended to more complicated allocation problems. The present paper, however, attempts to pair conceptual sophistication with formal simplicity. Adding mathematical sophistication is left for future research.
Section 3 explores properties that constitute market exchange. Implicitly, a narrow as well as a more extensive interpretation seems to be relied upon in the literature. As for the narrow view, market exchange refers to the allocation procedure which is governed by price taking behavior. Arrow (1969) has introduced an ingenious reinterpretation of the commodity space such that externalities can be regarded as ordinary commodities for which, in principle, the Welfare Theorem would apply. His construction, however, is based on a notion of personalized commodities each of which has precisely one buyer and one seller. In this case, the hypothesis of price taking behavior becomes rather implausible such that the Welfare Theorem can no longer serve to justify the institution of market exchange: The Coase Theorem is not a corollary of the Welfare Theorem! Instead, to validate the Coase Theorem, market exchange must be interpreted in an extensive sense beyond simple price taking behavior. The present paper advocates the view that Arrow prices in the above sense result from voluntary contracting provided that transaction costs are sufficiently low. This view seems to be well in line with Coase's (1960) idea of a pricing system which operates without cost as well as with Buchanan's (1987) notion of voluntary exchange among individuals which his theory of collective choice departs from ("the unanimity rule as the political analogue to freedom of exchange in markets"). In any case, Arrow prices and complete contracts turn out to be objects which are closely related.

Section 4 investigates sequential binary choice under positive transaction costs. These costs arise because, by assumption, it is prohibitively costly to verify the initial part of sequential choice. A theorem is established which fully characterizes the corresponding second best solution. The setting nicely reproduces the well-known effects of underinvestment. It points to the fact that introducing coercive elements at the final stage of sequential choice may serve a meaningful purpose. In other words, second best contracts may fail to be renegotiation-proof such that, at the final stage, exchange might no longer be voluntary. Finally, the setting allows for configurations where closing down an activity would be superior to controlling it by a contract which is second best or worse. Such examples lend themselves to resolve the PIT-Puzzle.

Section 5 takes the process of renegotiation into account. If parties cannot ex ante commit themselves not to renegotiate then recontracting prevents them from reaching a second best solution. They end up in a world which is third best only even though it might be possible to agree ex ante on a particular set of Arrow prices. Therefore, if the design of the renegotiation process allows to reach a first best solution (c.f. Aghion, Dewatripont and Rey (1989,1990)) this result must be due to assumptions beyond the
possibility of ex ante design. The crucial assumption turns out to be whether an agent must be given the proper incentives to affect his choice or whether some particular choice can directly be enforced. To reach an efficient solution the proper default option must be directly enforceable.

Renegotiations and commitment are crucial elements for the theory of ownership structure as proposed by Grossman and Hart(1986). Section 6 critically revisits that theory in the setting of sequential binary choice. Ownership only matters in a world of positive transaction costs where the first best solution is beyond reach. The findings of section 5 then suggest that the assumptions concerning commitment can be expected to be an intricate matter. In any case, a theorem is established which characterizes the optimum ownership structure provided that Grossman's and Hart's assumptions hold. In particular, configurations are identified under which separate ownership proves to be the optimum structure. In this sense, the theory offers a way to resolve Williamson's Puzzle.

2. Sequential Binary Choice

In this section, the simple model of sequential binary choice is introduced. There are n agents as well as n pairs \((x_i,y_i)\) \(\in \{0,1\} \times \{0,1\}\) of binary choices \((i=1,\ldots,n)\). Let \(x = (x_1,\ldots,x_n)\) and \(y = (y_1,\ldots,y_n)\) denote the vectors of these choices. The utility of agent i amounts to

\[
u_i(x,y) = \sum_{k=1}^{n} x_k (a_{ki} + y_k b_{ki}).
\]

The model is meant to capture in a qualitative if not a quantitative sense various situations. For instance, the following interpretations could be thought of.

First, imagine that \(x_i\) corresponds to the decision of agent i whether to buy (and use) a car \((x_i=1)\) or not \((x_i=0)\) and \(y_i\) denotes his decision whether to install a catalytic converter \((y_i=1)\) or not \((y_i=0)\). The second (final) choice \(y_i\) is relevant only if his first (initial) choice is \(x_i=1\). Moreover, the coefficients \(a_{ik}\) and \(b_{ik}\) which, for technical convenience, are assumed to be constant describe the external effects that are involved.

Second, suppose that agent i is a firm which has the choice of being present at some particular market \((x_i=1)\) or not \((x_i=0)\) and, if it chooses to be present, whether to engage
in some potentially illegal activity (y_i=1) or not (y_i=0) that might be in conflict with antitrust policy. Here, again, misconduct y_i=1 only matters if the firm is present, i.e. if x_i=1. The coefficients a_{ik} and b_{ik} capture the effects on all other participants k=1,...,n.

Third, think of agent i as a political authority which is in charge of some regulatory task. Here, x_i corresponds to the level of effort which the authority spends on observing the market (x_i=1 positive effort, x_i=0 no effort). The choice y_i denotes whether the authority brings a case to court (y_i=1) or not (y_i=0). Again it is assumed that the choice of y_i becomes a feasible alternative only after having spent enough effort (x_i=1). The coefficients a_{ik} and b_{ik} capture the effects on all agents k=1,...,n which arise from the regulatory authority's activities. Irrespective of any particular interpretation, the following assumption is made:

Assumption 1:

Agent i is indispensably needed to choose x_i. Moreover, the initial property rights are such that agent i is free to choose y_i.

Suppose, for reasons whatsoever, parties fail to accept any contract such that the paradigm of the Prisoner's Dilemma (solitary confinement!) would fit. Assumption 1 then implies that rational players make the following choices:

\[ y_i = y_i^* = 1 \text{ iff } b_{ii} \geq 0 \text{ and } x_i = x_i^* = 1 \text{ iff } a_{ii} + y_i^0 b_{ii} \geq 0 \]  

(1)

Here and elsewhere, for any binary choice z, "z=1 iff condition C is met" is taken as a shorthand for "z=1 if C is met whereas z=0 if C is not met"! The first best solution, however, would be

\[ y_i = y_i^* = 1 \text{ iff } \beta_i \geq 0 \text{ and } x_i = x_i^* = 1 \text{ iff } \alpha_i + y_i^* \beta_i \geq 0 \]  

(2)

where

\[ \alpha_i = \sum_{k=1}^{n} a_{ik} \text{ and } \beta_i = \sum_{k=1}^{n} b_{ik} \]

denote the social effects arising from the choices of x_i and y_i. The above model of sequential binary choice involves externalities at potentially two stages. Following
Pigouvian tradition, a discrepancy between private and social benefits is said to arise at the y-stage if \( \text{sign}(b) \neq \text{sign}(\beta) \) such that \( y^0_i \neq y^* _i \). Similarly, such a discrepancy arises at the x-stage if \( x^0 _i \neq x^*_i \). The traditional market failure literature would claim markets to fail whenever such discrepancies arise. The Coase Theorem, on the other hand, takes the level of transaction costs into account. Under vanishing transaction costs, the first best solution would be claimed to emerge. To reconcile the two theories, the contract-theoretic approach distinguishes conditions that allow for complete contracts from those where parties have access to incomplete contracts only. The next section recalls the theory of complete contracts for the above model of sequential binary choice. The remaining part of the paper will then be devoted to positive transaction costs that arise from the incompleteness of contracts. By assumption, contracts remain incomplete because the initial part of sequential choice cannot be verified by courts. In this sense, the paper takes aspects of moral hazard (hidden action) into account. Adverse selection (asymmetric information), however, is not covered at all. Rather, the following assumption is maintained throughout the paper.

Assumption 2:

All coefficients \( a=(a_{ik}) \) and \( b=(b_{ik}) \) are common knowledge among the involved agents.

3. Complete Markets and Complete Contracts

This section departs from Arrow(1969) who has described how, in the presence of external effects, a complete set of markets could serve to allocate resources efficiently. In the case of sequential binary choice, the following personalized markets are needed. At the y-stage, markets \( Y(i,k) \) are required where the externality which choice \( y_i \) inflicts upon agent \( k \) is traded at price \( q_{ki} \). At the x-stage, corresponding markets \( X(i,k) \) allow to trade at price \( p_{ki} \) the externality that arises from the choice \( x_i \) in favor of agent \( k \). As for market \( Y(i,k) \), supply \( y_i \) is chosen by a profit maximizing Lindahl firm according to

\[
y_i \in \arg \max y_i \sum_{k=1}^{n} q_{ki}
\]

whereas demand \( y_{ik} \) is chosen by agent \( k \) according to

\[
y_{ik} \in \arg \max y_{ik} b_{ik} - (y_{ik} - y^0_i) q_{ki} .
\]
Hereby, $q_{ki}$ denotes the price which agent $k$ must pay to induce a renunciation of initial property rights (c.f. (1)). These markets are balanced if $y_{ik} = y_i$ holds for $k = 1, \ldots, i, \ldots, n$. To describe market $X(i,k)$, let
\[
c^*_{ik} = a_{ik} + y^*_{i} b_{ik} - (y^*_{i} - y^0_{i}) q_{ki}
\]
denote the total effect at the $x$-stage which includes the effect at the $y$-stage as evaluated for the efficient choice $y_i = y^*_i$ (see (2)). At this market, supply $x_i$ is chosen according to
\[
x_i \in \arg \max_n x_i \sum_{k=1}^n p_{ki}
\]
whereas demand $x_{ik}$ is chosen according to
\[
x_{ik} \in \arg \max n x_{ik} c^*_{ik} - (x_{ik} - x^0_i) p_{ki}.
\]
For these markets to be in equilibrium it must hold that $x_{ik} = x_i$ for $k = 1, \ldots, i, \ldots, n$.

Equilibrium prices can be constructed as follows. If $\beta_i \geq 0$, choose prices $b_{ik} \geq q_{ki}$ ($k = 1, \ldots, n$) such that $\sum_k q_{ki} = 0$ holds. Similarly, if $\beta_i < 0$, take prices $b_{ik} \leq q_{ki}$ ($k = 1, \ldots, n$) such that $\sum_k q_{ki} = 0$ holds. At such prices, the Lindahl firms earn zero profits. Moreover, demand $y_{ik}$ by agent $k$ is equal to the efficient level, i.e. $y_{ik} = y^*_i$ for $k = 1, \ldots, n$ and the Lindahl firm is willing to supply what is demanded ($y_i = y^*_i$). Prices $q$ as constructed above are referred to as Arrow prices. To describe demand and supply at market $X(i,k)$, let $y_i$ denote the aggregate social effect
\[
y_i = \sum_{k=1}^n c^*_{ik} = \alpha_i + y^*_i \beta_i.
\]

Hereby, use of Arrow prices is made at the $y$-stage. Arrow prices $p$ which clear the markets at the $x$-stage can be constructed similarly. If $\gamma_i \geq 0$ take prices $c^*_{ik} \geq p_{ki}$ ($k = 1, \ldots, n$) for which $\sum_k p_{ki} = 0$ holds. If, however, $\gamma_i < 0$ then select prices $c^*_{ik} \leq p_{ki}$ ($k = 1, \ldots, n$) for which $\sum_k p_{ki} = 0$ holds. At such prices, again, the Lindahl firm earns zero profits. Moreover, in competitive equilibrium, the efficient levels $x_i = x_{ki} = x^*_i$ ($k = 1, \ldots, n$) are chosen at the $x$-stage as well. In other words, if sufficiently many markets are introduced then the welfare theorem holds even though external effects are present at
both stages. Notice, however, that the price taking behavior behind competitive equilibrium can hardly be considered to be a plausible hypothesis if markets are as thin as they are under a system of personalized markets.

To make use of them nevertheless, Arrow prices must be given a contract-theoretic interpretation. For that purpose, let

\[ g_{ik}(x_i, y_i) = x_i(a_{ik} + y_i b_{ik}) - x_i(y_i - y_i^0) q_{ki} - (x_i - x_i^0) p_{ki} \]  \hspace{1cm} (3)

\[ g_{ik}(x_i, y_i) = x_i(a_{ik} + y_i b_{ik}) - x_i(y_i - y_i^0) q_{ki} - (x_i - x_i^0) p_{ki} \]

denote the gain which accrues to agent k from activities \( x_i \) and \( y_i \). The gain is taken net of payments which, according to the contract, are due at levels \( (x_i, y_i) \) and at prices \( (p, q) \). If the contract relies on a pair of Arrow prices then the following proposition holds.

\textbf{Proposition 1}

For all \( x_i, y_i, i \) and \( k \),

(i) \( g_{ik}(x_i, y_i^*) \geq g_{ik}(x_i, y_i) \)

(ii) \( g_{ik}(x_i^*, y_i^*) \geq g_{ik}(x_i, y_i^*) \)

(iii) \( g_{ik}(x_i^*, y_i^*) \geq g_{ik}(x_i^0, y_i^0) \).

\textbf{Proof}

Since \( g_{ik}(x_i, y_i^*) - g_{ik}(x_i, y_i) = x_i(y_i^* - y_i)(b_{ik} - q_{ki}) \geq 0 \), (i) is obvious. Moreover, since \( g_{ik}(x_i^*, y_i^*) - g_{ik}(x_i, y_i^*) = (x_i^* - x_i) [a_{ik} + y_i b_{ik} - (y_i^* - y_i^0) q_{ki} - p_{ki}] = (x_i^* - x_i) (c_{ik} - p_{ki}) \geq 0 \), (ii) is obvious. As for (iii), it follows from (ii) that \( g_{ik}(x_i^*, y_i^*) \geq g_{ik}(x_i^0, y_i^0) \) and from (i) that \( g_{ik}(x_i^0, y_i^*) \geq g_{ik}(x_i^0, y_i^0) \) as was to be shown.

\textbf{Q.E.D.}

Proposition 1 leads to the following extensive interpretation of market exchange (see Introduction). Suppose the contract makes use of Arrow prices. It then follows from (i) and (ii) that the efficient levels of sequential binary choice are agreed upon by unanimous consent. Moreover, it follows from (iii) that it is individually rational for all
agents to subscribe to the contract because, without contract, initial property rights would lead to a Pareto-inferior arrangement.

Buchanan (1987) considers voluntary participation as the distinctive feature of market exchange. Unanimous consent with respect to the choice at both stages paired with individual rationality captures that feature in a precise way. Moreover, if transaction costs are low, the Coase Theorem is used to predict that contracts based on Arrow prices will emerge from the process of bargaining. In this sense, Arrow's construction leads to the extensive interpretation of market exchange in a natural way.

4. Second Best Contracts

In section 2, three different interpretations of models involving sequential binary choice were given. For the first one (catalytic converter), the choices $x_i$ and $y_i$ at both stages could probably be verified at little cost such that the setting is one of low transaction costs. Therefore, rational parties have access to complete contracts which are sustained by Arrow prices. In case of the second interpretation (firm engaging in illegal activity), however, higher transaction costs are likely to be involved. While it remains easy to verify whether the firm is present or not ($x_i=1$ or $x_i=0$), it could be very costly to verify whether it actually does engage in the illegal activity ($y_i=1$) or not ($y_i=0$). At the extreme, where it is impossible to verify $y_i$, contracts could only be based on $x_i$ but not on $y_i$. As for the third interpretation (regulatory authority), it might be prohibitively costly to verify the level of effort $x_i$ whereas it might cause no costs to verify the authority's choice of $y_i$. Contracts could then only be based on the choice of $y_i$ which, as a consequence of the assumed structure, only matters if choice of the x-stage is $x_i=1$. It turns out to be this last setting of transaction costs which produces the richest variety of phenomena and on which, for that reason, the remaining analysis concentrates.

Assumption 3:

Contracts can only be based on $x_i(y_i-y_i^0)$.

As for the third interpretation, the assumption means that it cannot be distinguished whether the authority has decided not to proceed to court ($y_i=y_i^0$) because it has found the case to be without merit after having spent enough effort ($x_i=1$) or whether it has not spent any such effort ($x_i=0$) and, for that reason alone, it refrains from bringing the case to court.
Under assumption 3, contracts can specify a price $q_{ki}$ which agent $k$ must pay in case of $x_i(y_i - y_i^0) = 1$. The gain net of payments which arises from activities $x_i$ and $y_i$ in favor of agent $k$ then amounts to

$$g_{ik}(x_i, y_i) = x_i(a_{ik} + y_i^0 b_{ik}) - x_i(y_i - y_i^0)q_{ki}.$$  \hspace{1cm} (4)

Notice that the gain according to (4) is simply derived from the gain according to (3) evaluated at prices $p=0$. To evaluate the incentives which arise from such a contract, assumption 1 is strengthened in the following way:

Assumption 1a:

Agent $i$ is indispensably needed to choose both $x_i$ and $y_i$.

Under this assumption, agent $i$ can then be predicted to choose

$$y_i = y_i^S \in \arg \max_{y_i} g_{ii}(x_i, y_i)$$

and

$$x_i = x_i^S \in \arg \max_{x_i} g_{ii}(x_i, y_i^S)$$

or, equivalently,

$$y_i = y_i^S = 1 \text{ iff } b_{ii} q_{ii}$$  \hspace{1cm} (5)

and

$$x_i = x_i^S = 1 \text{ iff } a_{ii} + y_i^S b_{ii} - (y_i^S - y_i^0)q_{ii} \geq 0.$$  \hspace{1cm} (6)

The second best problem then amounts to

$$W_i^S = \max \sum_{k=1}^{n} g_{ik}(x_i^S, y_i^S)$$
subject to (5), (6) and \( \sum_k q_{ki} = 0 \) where the maximum is taken with respect to \( x^i_S \) and \( y^i_S \).

The following theorem whose proof is given in the appendix fully characterizes the solution of the second best problem.

**Theorem 1**

There exists a solution of the second best problem such that

\[
g_{ik}(x^i_S, y^i_S) \geq g_{ik}(x^0_i, y^0_i)
\]

holds for all agents \( k=1,\ldots,n \). Moreover,

\[
W^s_i = \begin{cases} 
\text{Max} \{ \alpha_i + \beta_i, 0 \} & \text{if } b_{ii} < 0, \ a_{ii} < 0 \\
\text{Max} \{ \alpha_i + \beta_i, \alpha_i \} & \text{if } b_{ii} < 0, \ a_{ii} \geq 0 \\
\text{Max} \{ \alpha_i + \beta_i, \alpha_i \} & \text{if } b_{ii} \geq 0, \ a_{ii} + b_{ii} \geq 0 \\
\text{Max} \{ 0, \alpha_i \} & \text{if } b_{ii} \geq 0, \ a_{ii} + b_{ii} < 0
\end{cases}
\]

Let us now discuss the case for which the second best solution would strictly be outperformed by the first best solution. Consider, first, the configuration, where \( a_{ii} + b_{ii} < b_{ii} < 0 \) holds and where, in the absence of a contract, \( y_i^0 = 0 \) and \( x_i^0 = 0 \) (see (1)). Here, a deviation from the first best occurs if \( \alpha_i > \text{Max} \{ \alpha_i + \beta_i, 0 \} \) as follows from (8). In this case it would be first best to choose \( x^* = 1 \) but \( y^*_i = 0 \). The second best solution, however, requires \( x^s = 0 \) if \( \alpha_i + \beta_i \leq 0 \) such that second best leads to underinvestment as compared to first best. If, however, \( \alpha_i + \beta_i > 0 \) then the second best solution requires \( x^s = 1 \) and \( y^s_i = 1 \). Here, second best requires inefficiency at the \( y \)-stage which, in turn, means that second best prices fail to be Arrow prices and that the choice \( y^s_i \) is not agreed upon by unanimous consent. In this sense, exchange would not be voluntary at the \( y \)-stage. Therefore, restricting the liberty of contract at the \( y \)-stage could serve a meaningful purpose in this setting of second best. Notice, however, that the condition (7) of individual rationality is met ex ante such that, at the \( x \)-stage, participation would still be voluntary. In this sense, coercion at the \( y \)-stage has been legitimized ex ante by the involved parties.

Consider, second, the case where \( b_{ii} < 0 \leq a_{ii} \) holds such that, without contract, \( y_i^0 = 0 \) but \( x_i^0 = 1 \). In this case, a deviation from first best occurs if \( 0 > \text{Max} \{ \alpha_i + \beta_i, \alpha_i \} \) as follows from (8). Moreover, it would be first best to choose \( x^*_i = 0 \) and \( y^*_i = 0 \). The second best solution, however, requires \( x^s = 1, y^s = 1 \) if \( \beta_i \geq 0 \) and \( y^s_i = 0 = 0 \) if \( \beta_i < 0 \),
respectively. In other words, closing down the activity or - in the case of a regulatory authority - not installing the political agency would here be first best whereas, if installed, the agency can no longer be prevented - not even by the second best contract - from choosing the inefficient level \( x_i^S = 1 \). This example nicely illustrates the point that a political agency which is only to be committed by a second best contract or worse should not be relied upon for selective intervention. In this sense, the existence of such situations allows for a resolution of the PIT-puzzle.

Suppose, third, that \( b_i \geq 0 \) and \( a_i + b_{ii} \geq 0 \). In the absence of any contract, agent \( i \) chooses \( y_i^0 = 1 \) and \( x_i^0 = 1 \). Here, again, the second best solution fails to be first best if \( 0 > \max \{ \alpha_i + \beta_i, \alpha_i \} \) as follows from (8). It would be first best to close down the activity, i.e. \( x_i^* = 0 \), but no second best contract allows to achieve this. Rather second best incentives are such that agent \( i \) always chooses \( x_i^S = 1 \) whereas he chooses \( y_i^S = 1 \) if \( \beta_i \geq 0 \) and \( y_i^S = 0 \) if \( \beta_i < 0 \), respectively. Notice that such cases may occur even if there does not arise any direct externality at the \( x \)-stage (\( a_{ik} = 0 \) for \( i \neq k \)).

Take, fourth, the case, where \( b_i \geq 0 \) but \( a_i + b_{ii} < 0 \). Without contract, agent \( i \) would select \( y_i^0 = 1 \) but \( x_i^0 = 0 \) such that the choice at the \( y \)-stage would become irrelevant. According to (8), the second best solution requires \( x_i^S = 0 \) if \( \alpha_i \leq 0 \) and \( x_i^S = 1 \) but \( y_i^S = 0 \) if \( \alpha_i > 0 \), respectively. Therefore, if \( \alpha_i + \beta_i > \max \{ 0, \alpha_i \} \) the second best solution would strictly be outperformed by the first best solution because, in this case, it would be required that \( x_i^* = 1 \) and \( y_i^* = 1 \). Again, such inefficiency may arise even if there is no direct externality at the \( x \)-stage.

The above discussion hopefully has convinced the reader that the simple model of sequential binary choice produces a surprisingly rich variety of second best phenomena. The main conclusions are as follows. First, political agents which are committed by contracts that fail to be first best cannot be used for selective intervention. Second, even if there is no direct externality at the \( x \)-stage, the second best solution may be strictly worse as compared to the first best solution. Third, second best contracts may introduce coercive elements at the \( y \)-stage which do not allow to rely on Arrow prices at that stage. Therefore, second best prices may fail to be renegotiation-proof. Put differently, if prices are required to be renegotiation-proof, i.e. to be Arrow prices, we may arrive at a third best world.

5. Renegotiation
In this section, contracts are studied which are renegotiation-proof in the sense that, no matter what the initial contract, parties expect actual prices to be of the Arrow type. Assumption 3 is maintained such that the net gain from activities $i$ in favor of agent $k$ amounts to (c.f. (4))

$$g_{ik}(x_i, y_i | q) = x_i(a_{ik} + y_i b_{ik}) - x_i(y_i - y_i^0)q_{ki}.$$ 

Hereby, $q$ denotes actual prices which emerge from the process of renegotiation. Under such Arrow prices, the efficient level $y_i = y_i^*$ would be chosen by unanimous consent such that agent $i$, in particular, is given the incentive to select $y_i = y_i^*$ at the $y$-stage.

While renegotiations necessarily lead to the efficient choice at the $y$-stage, the division of the surplus may remain a subject of dispute. Grossman and Hart (1986) have proposed to divide the surplus according to the Nash (cooperative) bargaining solution which has the surplus equally split among parties. The Arrow prices which sustain this bargaining procedure are given by the following equations:

$$b_{ik} - q_{ki}^B = \beta_i / n \quad (i = 1, \ldots, n) \quad (9)$$

Under these prices, the net gain flowing from $i$ to $k$ amounts to

$$g_{ik}(x_i, y_i | q^B) = x_i \left[ a_{ik} + (y_i - y_i^0)\beta_i / n + y_i^0 b_{ik} \right].$$

Therefore, since $y_i = y_i^*$, the incentives to invest at the $y$-stage are such that

$$x_i = 1 \quad \text{iff} \quad a_{ii} + (y_i^* - y_i^0)\beta_i / n + y_i^0 b_{ii} \geq 0. \quad (10)$$

Hart and Moore (1988) follow a different route as far as the process of renegotiations is concerned. Adapted to the framework of sequential binary choice, their approach would be as follows. Under some initial contract, parties agree on prices $q^0$ which may or may not be Arrow prices. If they are then the process of renegotiation amounts to a zero sum game such that actual prices $q$ and initial prices $q^0$ coincide, i.e. $q = q^0$. If, however, the initial prices fail to be Arrow prices, then there possibly is scope for renegotiation. The initial prices affect how the surplus will actually be divided among parties in a rather complicated way (see Hart and Moore (1987)). Details do not matter here. What matters is that actual prices turn out to be some particular set of Arrow prices in any case.
In section 4, situations have been identified for which the second best solution fails to be sustainable by Arrow prices and for which, at the same time, the second best solution is strictly outperformed by the first best solution. In such situations, no matter whether the cooperative bargaining solution or the more elaborate solution proposed by Hart and Moore is taken, relying on Arrow prices would lead to a third best solution which is strictly outperformed by the second best, let alone the first best solution. In this sense, if parties cannot commit themselves not to renegotiate, incentives to invest at the x-stage may well be insufficient as has been stressed by Hart and Moore.

Aghion, Dewatripont and Rey (1989,1990) allow parties bindingly to commit themselves ex ante to some particular scheme of renegotiation. Their renegotiation design approach can easily be adapted to the setting of sequential binary choice in the following way. Agent i is given the right to propose prices \( q_{ki} (k= 1,..,n) \) on a take-it-or-leave-it basis. If everybody accepts and if thereafter all parties agree on the choice of \( y_i \) by unanimous consent, then \( y_i \) is the actual choice and payments are due according to the prices which agent i has proposed. Otherwise, some default option \((y_i^0,q_{ki}^0)\) which is part of the initial contract does apply. Under this scheme, agent i will propose prices \( q_{ki} (k= 1,..,n) \) that leave the total surplus to him, i.e.

\[
b_{ii} - q_{ii} = \beta_i \quad \text{and} \quad b_{ik} - q_{ki} = 0. \tag{11}
\]

Such prices are of course Arrow prices. In section 4, situations have been identified where there are no externalities at the x-stage, i.e. where \( a_k=0 \) for \( k\neq i \), but where still the second best solution fails to be first best. In such situations, the design of renegotiation processes would seem to be of little help. Therefore, the efficiency result due to Aghion, Dewatripont and Rey must actually depend on their assumption that parties are able to commit to some default option ex ante. As for the above example, suppose that parties commit to the default option \( y_i^0=0 \) and \( q_{ki}^0=0 \) for \( k= 1,..,n \). Under the prices (see(11)) which agent i can be expected to propose the net gain for agent i then amounts to

\[g_{ii} = x_i \left[ a_{ii} + y_i^{-1}(b_{ii} - q_{ii}) \right] = x_i \left[ a_{ii} + y_i^{-1}\beta_i \right].\]

Agent i, indeed, has the incentive to invest at the first best level, i.e. to choose \( x_i=x_i^* \), provided that there are no direct externalities at the x-stage (\( a_i=\alpha_i \)).
For this result to be true, it is crucial that parties can commit to the default option $y_i^0 = 0$. Otherwise, efficiency cannot be guaranteed as follows from the results of section 4. In fact, under assumption 1a, where agent $i$ is indispensably needed to choose both, $x_i$ and $y_i$, the scheme proposed by Aghion, Dewatripont and Rey need not work. Under this assumption, no choice can be enforced. Rather the agent must be given the proper incentives to affect his choice in some desired way. Put differently, if the scheme actually works then assumptions seem to be needed which come close to ask for verifiability of $y_i$. Under such assumptions, however, parties would have complete contracts at their disposal under which it would cause no difficulties to enforce the first best solution. In any case, the assumption whether some agent is indispensably needed or not affects the outcome dramatically. Grossman's and Hart's (1986) theory of ownership structure to which the next section is devoted adopts a middle course as far as this assumption is concerned.
6. Theory of Ownership Structure

The model of sequential binary choice is now used to illuminate the theory of vertical and lateral integration due to Grossman and Hart (1986). As pointed out at the end of the previous section, the assumption concerning what parties can commit themselves ex ante to is crucial for any such theory. To adopt the ideas to the present setting, imagine that some assets are used to carry out the y-activities. While agent i is indispensably needed to choose x, it is the owner of the assets who has the right to control the choice of y. The owner, however, may be prepared to give up this right as part of renegotiation. Well in line with assumption 3, Grossman and Hart assume that the x-decision is noncontractible because it reflects some managerial effort decision which is not verifiable to courts. Conceptually more difficult seems to be their assumption that no aspects of the y-decision are contractible ex ante if it is paired with the additional assumption that the right to control the y-decision actually can be transferred ex ante through a change of ownership. To worsen matters, it is assumed that parties cannot commit themselves to some default option y^0. Rather, while choosing y^0, the owner of the assets is solely guided by his incentives. No doubt, this set of somewhat conflicting assumptions remains difficult to justify, particularly, if the findings of previous sections are taken into account. The assumptions, however, are needed in essence because, otherwise, we return to a first best world for which, according to the Coase Theorem, the ownership structure would not matter at all. The assumption can be summarized as follows.

Assumption 1b:

Agent i is indispensably needed to choose x while it is the owner of the assets who controls y. No aspect of y, however, is ex ante contractible except for a change of ownership.

To illustrate the idea, it is enough to focus on a very simple setting of sequential choice. Let us assume, first, that there are only two agents {i,k} =\{1,2\}, second, that direct externalities do not arise at the x-stage (a_k=0 for k≠i), third, that positive external effects are present at the y-stage (b_k<0 but \(\beta_i = b_i + b_k > 0\)) and, fourth, that the total gain is positive as well (\(\alpha_i + \beta_i > 0\)). As for the first best solution it then follows from (2) that

$$y_i^* = 1 \quad \text{and} \quad x_i^* = 1 \quad \text{for} \quad i = 1,2 \ .$$

\[ (12) \]
Grossman and Hart (1986) identify three different ownership structures. Under separate ownership, agent \( i \) controls all assets behind his activity \( y_i \). Therefore, without renegotiations, the owners would be guided by incentives to choose

\[
y_i^0 = 0 \quad \text{for} \quad i = 1, 2
\]  

because, by assumption, \( b_\alpha < 0 \) (c.f.(1)). Under i-ownership (\( i = 1, 2 \)), however, it is agent \( i \) who controls the assets behind both activities \( y_i \) and \( y_k \). Therefore, without renegotiation, agent \( i \) as the owner would choose

\[
y_i^0 = 0 \quad \text{but} \quad y_k^0 = 1 \quad \text{for} \quad (i, k) = (1, 2)
\]  

because, by assumption, \( b_{ik} > 0 \). However it follows from (12) that, irrespective of the ownership structure, there exist gains from renegotiations. Following Grossman and Hart, it is assumed that parties renegotiate according to the Nash cooperative bargaining solution for which Arrow prices (9) are predicted to emerge. Under these prices, incentives to invest at the x-stage have been shown to be (10). Notice that the default option \( y_i^0 \) depends on the ownership structure and, for that reason, it affects the incentives to invest. To evaluate the effect consider, first, separate ownership. The default option is given by (13). It then follows from (10) that (recall that \( \alpha_i = a_i \))

\[
1 \quad \text{iff} \quad \alpha_i + \beta_i / 2 \geq 0 \quad \text{and} \quad x_k = 1 \quad \text{iff} \quad \alpha_k + \beta_k / 2 \geq 0
\]  

Consider, next, i-ownership. In this case, the default options are given by (14) such that

\[
1 \quad \text{iff} \quad \alpha_i + \beta_i / 2 \geq 0 \quad \text{and} \quad x_k = 1 \quad \text{iff} \quad \alpha_k + b_{ik} \geq 0
\]  

Under k-ownership, finally, it follows from (10) that

\[
1 \quad \text{iff} \quad \alpha_i + b_{ii} \geq 0 \quad \text{and} \quad x_k = 1 \quad \text{iff} \quad \alpha_k + \beta_k / 2 \geq 0
\]  

The aggregate social gain \( W = x_i (\alpha_i + y_i \beta_i) + x_k (\alpha_k + y_k \beta_k) \) depends on the ownership structure. The theory predicts that structure to emerge which produces the maximum social gain. The following theorem describes the optimum ownership structure or at least one of them if ties should arise.

**Theorem 2**
The optimum ownership structure is as follows.

(I) Suppose $\alpha_k + \beta_k/2 \geq 0$. Then separate ownership is optimal if $\alpha_i + \beta_i/2 \geq 0$ whereas, otherwise, $k$-ownership is optimal.

(II) Suppose $\alpha_k + \beta_k/2 < 0$. If $\alpha_i + \beta_i/2 \geq 0$ then $i$-ownership is optimal. If, however, $\alpha_i + \beta_i/2 < 0$ then $k$-ownership is optimal unless, both, $\alpha_k + b_{kk} \geq 0$ and $\alpha_k + \beta_k < \alpha_i + \beta_i$ or $\alpha_i + b_{ii} < 0$ hold in which case again $i$-ownership is optimal.

Proof

The proof is elementary but instructive. To begin with, suppose (I) that $\alpha_k + \beta_k/2 \geq 0$. If $\alpha_i + \beta_i/2 \geq 0$ then separate ownership leads to the first best solution $x_1 = x_1^*$ and $x_2 = x_2^*$ as follows from (15). Here the discrepancy between private and social returns is not sufficient to distort the decisions. If, however, $\alpha_i + \beta_i/2 < 0$ then separate ownership as well as $i$-ownership lead to underinvestment with respect to $x_i$, i.e. $x_i = 0 < x_i^* = 1$. Since $k$-ownership provides first best incentives to invest with respect to $x_k$ ($x_k = x_k^* = 1$), $k$-ownership (weakly) outperforms, both, separate ownership and $i$-ownership as follows from (17). Notice that, for $\alpha_i + b_{ii} \geq 0$, $k$-ownership leads to the first best solution, for $\alpha_i + b_{ii} < 0$ however, $k$-ownership as the optimum ownership structure fails to be first best. In any case, (I) is established.

Consider, next, case (II) for which it is assumed that $\alpha_k + \beta_k/2 < 0$. In this case, separate ownership is (weakly) outperformed by $i$-ownership and, hence, it is sufficient to compare $i$-ownership versus $k$-ownership. If $\alpha_i + \beta_i/2 \geq 0$ then, under (II), $i$-ownership (weakly) outperforms $k$-ownership as follows from (16). If, however, $\alpha_i + \beta_i/2 < 0$ then further subcases must be distinguished. (i) Suppose $\alpha_k + b_{kk} \geq 0$ and $\alpha_i + b_{ii} \geq 0$. Then there is underinvestment under $i$-ownership for both activities ($x_i = 0 < x_i^* = 1$ and $x_k = 0 < x_k^* = 1$) such that $k$-ownership (weakly) outperforms $i$-ownership. (ii) Suppose $\alpha_k + b_{kk} \geq 0$ and $\alpha_i + b_{ii} \geq 0$. Then, under $i$-ownership, $x_i = 0 < x_i^* = 1$ but $x_k = x_k^* = 1$ whereas, under $k$-ownership, $x_k = 0 < x_k^* = 1$ but $x_i = x_i^* = 1$. In this case, obviously, $k$-ownership outperforms $i$-ownership if $\alpha_k + \beta_k \geq \alpha_i + \beta_i$ and, vice versa, if $\alpha_k + \beta_k \leq \alpha_i + \beta_i$. (iii) Suppose $\alpha_k + b_{kk} \geq 0$ but $\alpha_i + b_{ii} < 0$. Then there is underinvestment under $k$-ownership for both activities ($x_i = x_k = 0$) whereas, under $i$-ownership, there is efficient investment with respect to $x_k$ ($x_k = x_k^* = 1$). Therefore, $i$-ownership outperforms $k$-ownership in this subcase. (II) is established.
Q.E.D.

The ownership structure affects the division of surplus under renegotiation and, hence, the incentives to invest. For certain configurations, as we have seen, even the optimum ownership structure fails to lead to the first best solution. Therefore, under assumption 1b, positive transaction costs are involved such that the assignment of property rights does matter. In particular, separate ownership could well turn out to be the unique optimum ownership structure. Both, i-ownership and k-ownership, which correspond to some form of integration are strictly outperformed. In this sense, the theory of Grossman and Hart manages to resolve Williamson's Puzzle.

As a final remark, remember that separate ownership departs from the default options \( y_i^0 = 0 \) and \( y_k^0 = 0 \). Therefore, if parties could commit ex ante to Arrow prices according to (11), then such prices would be renegotiation-proof and they would provide first-best incentives to invest. In other words, the inefficiency is due to the assumption that parties cannot ex ante commit to prices not even if they are renegotiation-proof. Instead they just have to wait until the Nash cooperative bargaining solution emerges under which the incentives to invest may be insufficient as a consequence of how the surplus is divided among parties. Unfortunately, this means, that assumption 1b is essentially needed, no matter how unpleasant to accept it might appear.

7. Concluding Remarks

If selective intervention were available at no costs, then, indeed, Williamson's Puzzle concerning the limitation of firm size as well as the PIT-Puzzle on the limitation of regulatory intervention would be difficult to resolve. Therefore, to develop some plausible theory of institutional choice, selective intervention must be viewed as performing below the first best level. Otherwise the framework would be the frictionless world of the Coase Theorem for which institutional choice does not matter.

To capture the costs that arise from selective intervention, the paper adopts the premises of methodological individualism which accepts individuals as the only evaluating, choosing and acting units. As a consequence, selective intervention must be seen as relying on selfish individuals. To be sure, rules may serve to limit the discretionary power of individuals who are given the authority to interfere. In a world of positive
transaction costs, however, such rules correspond to contracts that are second best at most and, hence, do not allow to administer selective intervention in a frictionless way.

If the tradeoff that arises from the choice of market versus non-market allocation is at stake it seems plausible to assume that collectivization creates political agencies which otherwise would be absent. Moreover, since such agencies are guided by incentives deviating from first best levels it is easy to identify situations where intervention fails to be selective. The approach readily allows to resolve the PIT-Puzzle.

Conceptually, the theory of ownership structure turns out to cause more problems. Firms, irrespective of their ownership structure, have to rely on individual agents. Of course any change of ownership also affects the firm's agency structure. Yet the exact effect remains difficult to capture by some simple hypothesis. Therefore Grossman and Hart (1986) have searched for a theory of integration which need not take resort to any notion of selective intervention whatsoever. Section 6 of the present paper illuminates the assumptions on which their approach is based. Above all, they must require that, except through a change of ownership, no aspects of the final choice is ex ante contractible. On purely logical grounds, their assumption is not easy to justify.

In any case, the present paper concentrates on a simple model of sequential binary choice which allows, both, to describe selective intervention at some second best level and to revisit Grossman's and Hart's theory of ownership structure. In spite of its formal simplicity, the model provides a rich setting of second best phenomena. Such a setting is needed to resolve the two puzzles and, herewith, to pave the way for a rigorous theory of institutional choice.
Appendix

Proof of Theorem 1:

The condition (7) of individual rationality is easily seen to be equivalent to

$$
\epsilon_{ik} = x_i^S (a_{ik} + y_i^S b_{ik}^s) - x_i^0 (a_{ik} + y_i^0 b_{ik}^0) \geq x_i^S (y_i^s - y_i^0) q
$$

(A1)

Moreover, notice that

$$\sum_{k=1}^n \Delta_{ik} \geq 0$$

(A2)

must hold because the social benefit under the second best solution cannot be smaller than under the situation without contract. To establish (8), four subcases have to be distinguished.

(1) Suppose $a_{ii} + b_{ii} < b_{ii} < 0$. It then follows from (1) that $y_{i0}^0 = 0$ and $x_{i0}^0 = 0$. Moreover, for price $q_{ii} \leq a_{ii} + b_{ii}$ incentives are such that $y_i^S = 1$ and $x_i^S = 1$ as follows from (5) and (6). For this price, the social surplus amounts to $W_i = \alpha_i + \beta_i$. For price $q_{ii} > a_{ii} + b_{ii}$, however, it follows from (6) that $x_i^S = 0$ and, hence, that $W_i = 0$. Therefore, indeed, $W_i^S = \max\{\alpha_i + \beta_i, 0\}$ as claimed by (8). To construct second best prices that satisfy the condition (A1) of individual rationality, two subcases must be distinguished.

(1a) If $\alpha_i + \beta_i \leq 0$ then $W_i^S = 0$ and, hence, $q_{ki} = 0$ ($k=1,..,n$) form a second best price system, because, since $x_i^S = x_i^0$ and $y_i^S = y_i^0$, no contract is needed to sustain the second best solution in this case. The condition (A1) of individual rationality obviously holds.

(1b) If $\alpha_i + \beta_i > 0$ then $W_i^S = \alpha_i + \beta_i$ and, hence, the second best choice is $y_i^S = 1$ and $x_i^S = 1$. Any price $q_{ii} \leq a_{ii} + b_{ii}$ provides the second best incentives to invest. Moreover, since $\Delta_{ik} = a_{ik} + b_{ik}$ and since $x_i^S (y_i^S y_i^0) = 1$, the condition (A1) of individual rationality requires that $\Delta_{ik} = a_{ik} + b_{ik} \geq q_{ki}$ holds for $k=1,..,n$. For $k=i$, this is the same condition as the one which provides second best incentives to invest.
It then follows from (A2) that prices must exist for which \( \Delta_{ik} \geq q_{ki} \) (k=1,...,n) and for which \( \sum_k q_{ki} = 0 \) holds. These are prices that satisfy the condition (A1) of individual rationality and which provide the second best incentives to invest.

(2) Suppose \( b_{ii} < 0 \) but \( a_{ii} \leq 0 \). It then follows from (1) that \( y^i_0 = 0 \) but \( x^0_i = 1 \). Moreover, for any price \( q_{ii} \leq b_{ii} \) incentives are such that \( y^S_i = 1 \) and \( x^S_i = 1 \) which, in turn, lead to social surplus \( W_i = \alpha_i + \beta_i \). For any price \( q_{ii} > b_{ii} \), however, it follows from (5) that \( y^S_i = 0 \) and, hence, from (6) that \( x^S_i = 1 \). For any such price, the surplus amounts to \( W_i = \alpha_i \). Therefore, indeed \( W^S_i = \max \{ \alpha_i + \beta_i, \alpha_i \} \). To construct second best prices that satisfy (A1), two subcases must be distinguished.

(2a) If \( \alpha_i + \beta_i \geq \alpha_i \) then \( W^S_i = \alpha_i + \beta_i \) and \( x^S_i = y^S_i = 1 \). Any price \( q_{ii} \leq b_{ii} \) provides second best incentives to invest. Moreover, since \( \Delta_{ik} = b_{ik} \) and since \( x^S_i (y^S_i - y^0_i) = 1 \), it follows again that, for agent \( k=i \), the condition of individual rationality and the incentive constraint are the same such that the required prices can be constructed along the same lines as in subcase (1b).

(2b) If \( \alpha_i + \beta_i < \alpha_i \) then \( W^S_i = \alpha_i \) and, hence, \( x^S_i = 1 \) but \( y^S_i = 0 \). Since \( x^S_i = x^0_i \) and \( y^S_i = y^0_i \), no contract is needed or, equivalently, prices \( q_{ii} = 0 \) (k=1,...,n) sustain the second best solution. The condition (A1) obviously is met under these prices.

(3) Suppose \( b_{ii} \geq 0 \) and \( a_{ii} \geq 0 \). It then follows from (1) that \( y^i_0 = 1 \) and \( x^0_i = 1 \). Moreover, for any price \( q_{ii} \leq b_{ii} \) it follows that \( y^S_i = 1 \) and \( x^S_i = 1 \) and, hence, that \( W_i = \alpha_i + \beta_i \). For any price \( q_{ii} > b_{ii} \), however, it follows that \( y^S_i = 0 \) and \( x^S_i = 1 \) and, hence, that \( W_i = \alpha_i \). Therefore, indeed \( W^S_i = \max \{ \alpha_i + \beta_i, \alpha_i \} \). To investigate the condition of individual rationality, two subcases must be distinguished.

(3a) If \( \alpha_i + \beta_i \geq \alpha_i \) then \( W^S_i = \alpha_i + \beta_i \) and, hence, \( x^S_i = 1 = x^0_i \) and \( y^S_i = 1 = y^0_i \). In this case, prices \( q_{ki} = 0 \) (k=1,...,n) satisfy, both, the condition of individual rationality and the second best incentive constraint.

(3b) If \( \alpha_i + \beta_i < \alpha_i \) then \( W^S_i = \alpha_i \) and, hence, \( x^S_i = 1 \) but \( y^S_i = 0 \). For second best prices, it must hold that \( q_{ii} > b_{ii} \). Moreover, since \( \Delta_{ik} = -b_{ik} \) and since \( x^S_i (y^S_i - y^0_i) = -1 \), second best prices that provide the correct incentives can be constructed in a way which, by now, should be familiar from previous subcases.
(4) Suppose, finally, that $b_{ii} \geq 0$ but $a_{ii} + b_{ii} < 0$. It then follows from (1) that $y_i^0 = 1$ but $x_i^0 = 0$. Moreover, for any price $q_{ii} \leq b_{ii}$ it follows that $y_i^S = 1$ and $x_i^S = 0$ and, hence, $W_i = 0$. For any price $b_{ii} < q_{ii} < -a_{ii}$ it follows that $y_i^S = 0$ and $x_i^S = 0$ and, again, $W_i = 0$. For any price $-a_{ii} \leq q_{ii}$, however, it follows, that $y_i^S = 0$ but $x_i^S = 1$ and, hence, that $W_i = \alpha_i$. Therefore, indeed, $W_i^S = \max \{ 0, \alpha_i \}$.

(4a) If $0 \geq \alpha_i$ then $W_i^S = 0$ and, hence, $y_i^S = y_i^0 = 1$ and $x_i^S = x_i^0 = 0$. In this case, prices $q_{ki} = 0$ (k=1,...,n) sustain the second best solution as required.

(4b) If $0 < \alpha_i$ then $W_i^S = \alpha_i$ and, hence, $x_i^S = 1$ but $y_i^S = 0$ such that prices satisfy the second best constraint provided that $-a_{ii} \leq q_{ii}$ holds. Moreover, since $\Delta_{ik} = a_{ik}$ and since $x_i^S(y_i^S - y_i^0) = -1$, second best prices can now be shown to exist in the same way as in previous subcases.

Q.E.D.
References


