Appendices for online publication

Optimal taxation in a habit formation economy

Sebastian Koehne* Moritz Kuhn†

This document is organized as follows. Appendix A collects the proofs that are omitted from the main text. Appendix B derives a recursive formulation of the social planning problem with habit formation. The setup allows for general recursive habit processes and contains the case of one-period habits discussed in the main text as a special case. Appendix C derives labor and savings wedges when the skill process is persistent.

Appendix A: Proofs

Proof of Lemma 1. Since the constraint set of the unrelaxed problem is a subset of the constraint set of the relaxed problem, it suffices to show that the solution of the relaxed problem is feasible for the unrelaxed problem. In other words, it suffices to show that the solution of the relaxed problem satisfies the upward incentive compatibility constraint.

Without loss of generality, we assume $\theta_t^H > \theta_t^L$. We first show that the downward incentive constraint is binding for the relaxed problem. Suppose to the contrary that the solution of the relaxed problem has a slack downward incentive constraint. By inspection of the Kuhn-Tucker conditions, the solution then takes the form $c_t^H = c_t^L$, $W_t^H = W_t^L$, and $y_t^H > y_t^L$. However, allocations of such form violate the downward incentive constraint. Hence the assumption that the solution of the relaxed problem has a slack downward incentive constraint must be false.

We now show that a binding downward incentive constraint implies that the upward incentive constraint is satisfied. Formally, a binding downward incentive constraint implies

$$u\left(c_t^H, c_{t-1}\right) - v\left(y_t^H/\theta_t^H\right) + \beta W_{t+1}^H = u\left(c_t^L, c_{t-1}\right) - v\left(y_t^L/\theta_t^H\right) + \beta W_{t+1}^L. \tag{1}$$

*Institute for International Economic Studies (IIES), Stockholm University, SE-10691 Stockholm, Sweden, Phone: +46 8 16 35 64, sebastian.koehne@iies.su.se, and CESifo, Munich, Germany.

†University of Bonn, Department of Economics, D-53113 Bonn, Germany, Phone: +49 228 73 62096, mokuhn@uni-bonn.de, and Institute for the Study of Labor (IZA), Bonn, Germany.

Recall that labor disutility v is a convex function. Since $1/\theta_t^L \geq 1/\theta_t^H$, convexity of v implies that the difference $v\left(y/\theta_t^L\right) - v\left(y/\theta_t^H\right)$ increases in y. Moreover, it is easy to see that a binding downward incentive constraint implies $y_t^H \geq y_t^L$. Combining the last two insights, we obtain

$$v\left(y_t^L/\theta_t^L\right) - v\left(y_t^L/\theta_t^H\right) \leq v\left(y_t^H/\theta_t^L\right) - v\left(y_t^H/\theta_t^H\right). \tag{2}$$

We rewrite this inequality as

$$v\left(y_t^L/\theta_t^L\right) - v\left(y_t^H/\theta_t^L\right) \leq v\left(y_t^L/\theta_t^H\right) - v\left(y_t^H/\theta_t^H\right). \tag{3}$$

We combine the binding downward incentive constraint with the above inequality and obtain

$$v\left(y_t^L/\theta_t^L\right) - v\left(y_t^H/\theta_t^L\right) \leq u\left(c_t^L, c_{t-1}\right) - u\left(c_t^H, c_{t-1}\right) + \beta W_{t+1}^L - \beta W_{t+1}^H. \tag{4}$$

Hence the upward incentive constraint is satisfied. □

Proof of Remark 1. Since the incentive compatibility constraint has a Lagrange multiplier of zero in all periods $t \geq t_0$, we have $\mu_t = 0$ for $t \geq t_0$. Now the result follows from Propositions 2 and 3. □

Proof of Proposition 1. See online Appendix B. □

Proof of Proposition 2. The (finite horizon) Bellman equation of the social planner problem is

$$C_t\left(W_t, c_{t-1}\right) = \min_{c_t^i, y_t^i, W_{t+1}^i} \sum_{i=H,L} \left[c_t^i - y_t^i + qC_{t+1}\left(W_{t+1}^i, c_t^i\right)\right] \pi_t\left(\theta_t^i\right) \tag{5}$$

s.t.

$$u\left(c_t^H, c_{t-1}\right) - v\left(y_t^H/\theta_t^H\right) + \beta W_{t+1}^H \geq u\left(c_t^L, c_{t-1}\right) - v\left(y_t^L/\theta_t^H\right) + \beta W_{t+1}^L \tag{6}$$

$$\sum_{i=H,L} \left[u\left(c_t^i, c_{t-1}\right) - v\left(y_t^i/\theta_t^i\right) + \beta W_{t+1}^i\right] \pi_t\left(\theta_t^i\right) = W_t. \tag{7}$$

Problem (5) has the following first-order conditions for consumption

$$0 = \pi_t\left(\theta_t^H\right)\left[1 + qC_{t+1,h}\left(W_{t+1}^H, c_t^H\right)\right] - \lambda_t u_c\left(c_t^H, c_{t-1}\right)\pi_t\left(\theta_t^H\right) - \mu_t u_c\left(c_t^H, c_{t-1}\right), \tag{8}$$

$$0 = \pi_t\left(\theta_t^L\right)\left[1 + qC_{t+1,h}\left(W_{t+1}^L, c_t^L\right)\right] - \lambda_t u_c\left(c_t^L, c_{t-1}\right)\pi_t\left(\theta_t^L\right) + \mu_t u_c\left(c_t^L, c_{t-1}\right), \tag{9}$$

for output

$$0 = -\pi_t\left(\theta_t^H\right) + \lambda_t \frac{v'\left(y_t^H/\theta_t^H\right)}{\theta_t^H}\pi_t\left(\theta_t^H\right) + \mu_t \frac{v'\left(y_t^H/\theta_t^H\right)}{\theta_t^H}, \quad (10)$$

$$0 = -\pi_t\left(\theta_t^L\right) + \lambda_t \frac{v'\left(y_t^L/\theta_t^L\right)}{\theta_t^L}\pi_t\left(\theta_t^L\right) - \mu_t \frac{v'\left(y_t^L/\theta_t^H\right)}{\theta_t^H}, \quad (11)$$

and for continuation utilities

$$0 = \pi_t\left(\theta_t^H\right) qC_{t+1,W}\left(W_{t+1}^H, c_t^H\right) - \lambda_t\beta\pi_t\left(\theta_t^H\right) - \mu_t\beta, \quad (12)$$

$$0 = \pi_t\left(\theta_t^L\right) qC_{t+1,W}\left(W_{t+1}^L, c_t^L\right) - \lambda_t\beta\pi_t\left(\theta_t^L\right) + \mu_t\beta. \quad (13)$$

We begin with the labor wedge of the high-skilled worker. Combine the first-order condition for y_t^H with that for c_t^H to obtain

$$\frac{1 + qC_{t+1,h}\left(W_{t+1}^H, c_t^H\right)}{u_c\left(c_t^H, c_{t-1}\right)} = \frac{\theta_t^H}{v'\left(y_t^H/\theta_t^H\right)}. \quad (14)$$

By the envelope theorem, applied to the Bellman equation (5) at date $t+1$, we have

$$C_{t+1,W}\left(W_{t+1}^H, c_t^H\right) = \lambda_{t+1}^H, \quad (15)$$

$$C_{t+1,h}\left(W_{t+1}^H, c_t^H\right) = -\lambda_{t+1}^H \sum_j u_h\left(c_{t+1}^{Hj}, c_t^H\right)\pi_{t+1}\left(\theta_{t+1}^j\right) - \mu_{t+1}^H\left[u_h\left(c_{t+1}^{HH}, c_t^H\right) - u_h\left(c_{t+1}^{HL}, c_t^H\right)\right] \quad (16)$$

Hence we can rewrite (14) as

$$\frac{\theta_t^H}{v'\left(y_t^H/\theta_t^H\right)}u_c\left(c_t^H, c_{t-1}\right) = 1 - qC_{t+1,W}\left(W_{t+1}^H, c_t^H\right)\sum_j u_h\left(c_{t+1}^{Hj}, c_t^H\right)\pi_{t+1}\left(\theta_{t+1}^j\right) - q\mu_{t+1}^H\left[u_h\left(c_{t+1}^{HH}, c_t^H\right) - u_h\left(c_{t+1}^{HL}, c_t^H\right)\right]. \quad (17)$$

Combine the first-order condition for W_{t+1}^H with the first-order condition for y_t^H to obtain

$$qC_{t+1,W}\left(W_{t+1}^H, c_t^H\right) = \beta\frac{\theta_t^H}{v'\left(y_t^H/\theta_t^H\right)}. \quad (18)$$

Use this to rewrite (17) as follows:

$$\mathbb{E}\left[\tilde{U}_t\left|\theta^{t-1}, \theta_t^H\right.\right] = \frac{v'\left(y_t^H/\theta_t^H\right)}{\theta_t^H}\left(1 - q\mu_{t+1}^H\left[u_h\left(c_{t+1}^{HH}, c_t^H\right) - u_h\left(c_{t+1}^{HL}, c_t^H\right)\right]\right). \quad (19)$$

Therefore the labor wedge is

$$\tau_{y,t}^{H}=-\mu_{t+1}^{H}\frac{qv'\left(y_t^H/\theta_t^H\right)}{\theta_t^H\mathbb{E}\left[\tilde{U}_t\left|\theta^{t-1},\theta_t^H\right.\right]}\left[u_h\left(c_{t+1}^{HH},c_t^H\right)-u_h\left(c_{t+1}^{HL},c_t^H\right)\right]. \tag{20}$$

Using the first-order condition for y_t^H and the identity $q\pi_t\left(\theta_t^H\right)\lambda_{t+1}^H=\beta\left(\lambda_t\pi_t\left(\theta_t^H\right)+\mu_t\right)$, and defining

$$B_t^H=\frac{\beta\left[u_h\left(c_{t+1}^{HH},c_t^H\right)-u_h\left(c_{t+1}^{HL},c_t^H\right)\right]}{\lambda_{t+1}^H\mathbb{E}\left[\tilde{U}_t\left|\theta^{t-1},\theta_t^H\right.\right]}, \tag{21}$$

the labor wedge is $\tau_{y,t}^H=-\mu_{t+1}^H B_t^H$.

We now turn to the labor wedge of the low-skilled worker. First we write the first-order condition for c_t^L as

$$\left[\lambda_t\pi_t\left(\theta_t^L\right)-\mu_t\right]u_c\left(c_t^L,c_{t-1}\right)-\pi_t\left(\theta_t^L\right)=q\pi_t\left(\theta_t^L\right)C_{t+1,h}\left(W_{t+1}^L,c_t^L\right). \tag{22}$$

The envelope theorem, applied to the Bellman equation (5) at date $t+1$, yields

$$C_{t+1,W}\left(W_{t+1}^L,c_t^L\right) = \lambda_{t+1}^L, \tag{23}$$

$$C_{t+1,h}\left(W_{t+1}^L,c_t^L\right) = -\lambda_{t+1}^L\sum_j u_h\left(c_{t+1}^{Lj},c_t^L\right)\pi_{t+1}\left(\theta_{t+1}^j\right)-\mu_{t+1}^L\left[u_h\left(c_{t+1}^{LH},c_t^L\right)-u_h\left(c_{t+1}^{LL},c_t^L\right)\right] \tag{24}$$

Combined with the first-order condition for W_{t+1}^L, we obtain

$$\begin{aligned} q\pi_t\left(\theta_t^L\right)C_{t+1,h}\left(W_{t+1}^L,c_t^L\right) &= -\lambda_t\pi_t\left(\theta_t^L\right)\beta\sum_j u_h\left(c_{t+1}^{Lj},c_t^L\right)\pi_{t+1}\left(\theta_{t+1}^j\right) \\ &\quad +\mu_t\beta\sum_j u_h\left(c_{t+1}^{Lj},c_t^L\right)\pi_{t+1}\left(\theta_{t+1}^j\right) \\ &\quad -\mu_{t+1}^L\pi_t\left(\theta_t^L\right)q\left[u_h\left(c_{t+1}^{LH},c_t^L\right)-u_h\left(c_{t+1}^{LL},c_t^L\right)\right]. \end{aligned} \tag{25}$$

We substitute this in the first-order condition for c_t^L to obtain

$$\lambda_t\pi_t\left(\theta_t^L\right)\mathbb{E}\left[\tilde{U}_t\left|\theta^{t-1},\theta_t^L\right.\right]-\pi_t\left(\theta_t^L\right) \tag{26}$$

$$= \mu_t\mathbb{E}\left[\tilde{U}_t\left|\theta^{t-1},\theta_t^L\right.\right]-\mu_{t+1}^L\pi_t\left(\theta_t^L\right)q\left[u_h\left(c_{t+1}^{LH},c_t^L\right)-u_h\left(c_{t+1}^{LL},c_t^L\right)\right]. \tag{27}$$

Now we use the first-order condition for y_t^L to replace $\pi_t\left(\theta_t^L\right)$:

$$\lambda_t\pi_t\left(\theta_t^L\right)\left\{\mathbb{E}\left[\tilde{U}_t\left|\theta^{t-1},\theta_t^L\right.\right]-\frac{v'\left(y_t^L/\theta_t^L\right)}{\theta_t^L}\right\} \tag{28}$$

$$= \mu_t\left\{\mathbb{E}\left[\tilde{U}_t\left|\theta^{t-1},\theta_t^L\right.\right]-\frac{v'\left(y_t^L/\theta_t^H\right)}{\theta_t^H}\right\}-\mu_{t+1}^L\pi_t\left(\theta_t^L\right)q\left[u_h\left(c_{t+1}^{LH},c_t^L\right)-u_h\left(c_{t+1}^{LL},c_t^L\right)\right]. \tag{29}$$

This can be rewritten as

$$\left(\lambda_t \pi_t\left(\theta_t^L\right) - \mu_t\right)\left\{\mathbb{E}\left[\tilde{U}_t \left|\theta^{t-1}, \theta_t^L\right.\right] - \frac{v'\left(y_t^L/\theta_t^L\right)}{\theta_t^L}\right\} \tag{30}$$

$$= \mu_t\left\{\frac{v'\left(y_t^L/\theta_t^L\right)}{\theta_t^L} - \frac{v'\left(y_t^L/\theta_t^H\right)}{\theta_t^H}\right\} - \mu_{t+1}^L \pi_t\left(\theta_t^L\right) q\left[u_h\left(c_{t+1}^{LH}, c_t^L\right) - u_h\left(c_{t+1}^{LL}, c_t^L\right)\right]. \tag{31}$$

Using the identity $\pi_t\left(\theta_t^L\right) q\lambda_{t+1}^L = \beta\left(\lambda_t \pi_t\left(\theta_t^L\right) - \mu_t\right)$, and defining

$$A_t^L = \frac{\beta}{q\pi_t\left(\theta_t^L\right)\lambda_{t+1}^L \mathbb{E}\left[\tilde{U}_t \left|\theta^{t-1}, \theta_t^L\right.\right]}\left[\frac{v'\left(y_t^L/\theta_t^L\right)}{\theta_t^L} - \frac{v'\left(y_t^L/\theta_t^H\right)}{\theta_t^H}\right], \tag{32}$$

$$B_t^L = \frac{\beta\left[u_h\left(c_{t+1}^{LH}, c_t^L\right) - u_h\left(c_{t+1}^{LL}, c_t^L\right)\right]}{\lambda_{t+1}^L \mathbb{E}\left[\tilde{U}_t \left|\theta^{t-1}, \theta_t^L\right.\right]}, \tag{33}$$

the labor wedge is hence $\tau_{y,t}^L = \mu_t A_t^L - \mu_{t+1}^L B_t^L$. This completes the proof. □

Proof of Proposition 3. We begin with the savings wedge for the high-skilled worker. Combine the first-order condition for consumption (8) and the envelope condition (16) to obtain

$$\frac{\lambda_t \pi_t\left(\theta_t^H\right) + \mu_t}{\pi_t\left(\theta_t^H\right)} u_c\left(c_t^H, c_{t-1}\right) - 1 \tag{34}$$

$$= -q\lambda_{t+1}^H \sum_j u_h\left(c_{t+1}^{Hj}, c_t^H\right)\pi_{t+1}\left(\theta_{t+1}^j\right) - q\mu_{t+1}^H\left[u_h\left(c_{t+1}^{HH}, c_t^H\right) - u_h\left(c_{t+1}^{HL}, c_t^H\right)\right].$$

Using the identity $q\pi_t\left(\theta_t^H\right)\lambda_{t+1}^H = \beta\left(\lambda_t \pi_t\left(\theta_t^H\right) + \mu_t\right)$, we can rewrite the previous equation as

$$\frac{q\lambda_{t+1}^H}{\beta}\mathbb{E}\left[\tilde{U}_t \left|\theta^{t-1}, \theta_t^H\right.\right] = 1 - q\mu_{t+1}^H\left[u_h\left(c_{t+1}^{HH}, c_t^H\right) - u_h\left(c_{t+1}^{HL}, c_t^H\right)\right]. \tag{35}$$

The first-order conditions for consumption in period $t+1$ are

$$0 = \pi_{t+1}\left(\theta_{t+1}^H\right)\left[1 + qC_{t+2,h}\left(W_{t+2}^{HH}, c_{t+1}^{HH}\right)\right] - \lambda_{t+1}^H u_c\left(c_{t+1}^{HH}, c_t^H\right)\pi_{t+1}\left(\theta_{t+1}^H\right) - \mu_{t+1}^H u_c\left(c_{t+1}^{HH}, c_t^H\right) \tag{36}$$

$$0 = \pi_{t+1}\left(\theta_{t+1}^L\right)\left[1 + qC_{t+2,h}\left(W_{t+2}^{HL}, c_{t+1}^{HL}\right)\right] - \lambda_{t+1}^H u_c\left(c_{t+1}^{HL}, c_t^H\right)\pi_{t+1}\left(\theta_{t+1}^L\right) + \mu_{t+1}^H u_c\left(c_{t+1}^{HL}, c_t^H\right). \tag{37}$$

Summing up these conditions and substituting the result into the previous equation yields

$$\begin{aligned}\frac{q\lambda_{t+1}^H}{\beta}\mathbb{E}\left[\tilde{U}_t \left|\theta^{t-1}, \theta_t^H\right.\right] =& -\pi_{t+1}\left(\theta_{t+1}^L\right) qC_{t+2,h}\left(W_{t+2}^{HL}, c_{t+1}^{HL}\right) - \pi_{t+1}\left(\theta_{t+1}^H\right) qC_{t+2,h}\left(W_{t+2}^{HH}, c_{t+1}^{HH}\right) \\ &+\lambda_{t+1}^H\left[u_c\left(c_{t+1}^{HH}, c_t^H\right)\pi_{t+1}\left(\theta_{t+1}^H\right) + u_c\left(c_{t+1}^{HL}, c_t^H\right)\pi_{t+1}\left(\theta_{t+1}^L\right)\right] \\ &-\mu_{t+1}^H\left[u_c\left(c_{t+1}^{HL}, c_t^H\right) - u_c\left(c_{t+1}^{HH}, c_t^H\right)\right] - q\mu_{t+1}^H\left[u_h\left(c_{t+1}^{HH}, c_t^H\right) - u_h\left(c_{t+1}^{HL}, c_t^H\right)\right].\end{aligned} \tag{38}$$

We use the envelope conditions for period $t+2$ to replace $C_{t+2,h}$. This gives, after some algebra,

$$\begin{aligned} q\lambda_{t+1}^{H}\mathbb{E}\left[\tilde{U}_{t}\left|\theta^{t-1},\theta_{t}^{H}\right.\right] \quad = \quad & \beta\lambda_{t+1}^{H}\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_{t}^{H}\right.\right] \\ & -\mu_{t+1}^{H}\beta\left(\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_{t}^{H},\theta_{t+1}^{L}\right.\right]-\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_{t}^{H},\theta_{t+1}^{H}\right.\right]\right) \\ & -q\mu_{t+1}^{H}\beta\left[u_{h}\left(c_{t+1}^{HH},c_{t}^{H}\right)-u_{h}\left(c_{t+1}^{HL},c_{t}^{H}\right)\right] \\ & +q\pi_{t+1}\left(\theta_{t+1}^{H}\right)\mu_{t+2}^{HH}\beta\left[u_{h}\left(c_{t+2}^{HHH},c_{t+1}^{HH}\right)-u_{h}\left(c_{t+2}^{HHL},c_{t+1}^{HH}\right)\right] \\ & +q\pi_{t+1}\left(\theta_{t+1}^{L}\right)\mu_{t+2}^{HL}\beta\left[u_{h}\left(c_{t+2}^{HLH},c_{t+1}^{HL}\right)-u_{h}\left(c_{t+2}^{HLL},c_{t+1}^{HL}\right)\right]. \end{aligned} \tag{39}$$

Setting $i=H$ and defining

$$D_{t}^{i} = \frac{\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_{t}^{i},\theta_{t+1}^{L}\right.\right]-\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_{t}^{i},\theta_{t+1}^{H}\right.\right]}{\lambda_{t+1}^{i}\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_{t}^{i}\right.\right]}, \tag{40}$$

$$E_{t}^{i} = \frac{q\left[u_{h}\left(c_{t+1}^{iH},c_{t}^{i}\right)-u_{h}\left(c_{t+1}^{iL},c_{t}^{i}\right)\right]}{\lambda_{t+1}^{i}\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_{t}^{i}\right.\right]}, \tag{41}$$

$$F_{t}^{ij} = \frac{q\left[u_{h}\left(c_{t+2}^{ijH},c_{t+1}^{ij}\right)-u_{h}\left(c_{t+2}^{ijL},c_{t+1}^{ij}\right)\right]}{\lambda_{t+1}^{i}\mathbb{E}\left[\tilde{U}_{t+1}\left|\theta^{t-1},\theta_{t}^{i}\right.\right]}, \quad j=L,H, \tag{42}$$

the savings wedge is hence

$$\tau_{s,t}^{i}=\mu_{t+1}^{i}D_{t}^{i}+\mu_{t+1}^{i}E_{t}^{i}-\pi_{t+1}\left(\theta_{t+1}^{H}\right)\mu_{t+2}^{iH}F_{t}^{iH}-\pi_{t+1}\left(\theta_{t+1}^{L}\right)\mu_{t+2}^{iL}F_{t}^{iL}. \tag{43}$$

For the savings wedge of the low-skilled worker, we follow the same steps to show that formula (43) applies if we set $i=L$ in definitions (40), (41), (42). This completes the proof. □

Appendix B: Recursive formulation

We rewrite the multiperiod private information problem as a dynamic programming problem with two state variables: promised utility and the agent's habit level. We derive this property in a setting with recursive habit processes: $h_t = H(c_{t-1}, h_{t-1})$, with h_1 being exogenous. Our results extend findings from Spear and Srivastava (1987) and Phelan and Townsend (1991) to the class of habit formation preferences.

We consider the following optimization problem:

$$C_1(W_1,h_1) := \min_{\mathbf{c},\mathbf{y}} \sum_{t=1}^{T} \sum_{\theta^t \in \Theta^t} q^{t-1} \left[c_t\left(\theta^t\right) - y_t\left(\theta^t\right)\right] \Pi^t\left(\theta^t\right) \tag{44}$$

s.t.

$$w_1\left(\mathbf{c},\mathbf{y};h_1\right) \geq w_1\left(\mathbf{c}\circ\sigma,\mathbf{y}\circ\sigma;h_1\right) \quad \forall \sigma \in \Sigma \tag{45}$$

$$w_1\left(\mathbf{c},\mathbf{y};h_1\right) = W_1. \tag{46}$$

First we introduce some notation. A consumption allocation $\mathbf{c}$, combined with a fixed initial habit h_1, generates a unique sequence of habit levels $(h_t(\theta^{t-1}))_{t=1,\ldots,T}$ according to the sequence of equations $h_t = H(c_{t-1},h_{t-1})$, $t = 2,\ldots,T$. Given an allocation $(\mathbf{c},\mathbf{y})$ and a history θ^t, the *continuation allocation* $\left(c_{t+1}^T\left(\theta^t\right), y_{t+1}^T\left(\theta^t\right)\right)$ is defined as the restriction of plans $(c_s,y_s)_{s=t+1,\ldots,T}$ to those histories $\theta^{t+1},\ldots,\theta^T$ that succeed θ^t. The *continuation utility* associated with $\left(c_{t+1}^T\left(\theta^t\right), y_{t+1}^T\left(\theta^t\right)\right)$ is defined as

$$\begin{aligned} &w_{t+1}\left(c_{t+1}^T\left(\theta^t\right), y_{t+1}^T\left(\theta^t\right); h_{t+1}\left(\theta^t\right)\right) \\ &:= \sum_{s=t+1}^{T} \sum_{\theta^s \in \Theta^s} \beta^{s-t-1} \left[u\left(c_s\left(\theta^s\right), h_s\left(\theta^{s-1}\right)\right) - v\left(\frac{y_s\left(\theta^s\right)}{\theta_s}\right)\right] \Pi^s\left(\theta^s|\theta^t\right). \end{aligned} \tag{47}$$

Note that, in contrast to the time-separable case, the continuation utility w_{t+1} depends not only on the continuation allocation but also on the consumption history $c^t(\theta^t)$ as summarized by the one-dimensional statistic $h_{t+1}(\theta^t)$.

For any $h \in \mathbb{R}_+$ we define $\mathrm{dom}_t(h)$ to be the set of time-t continuation utilities W with the property that, given time-t habit level $h_t = h$, there exists an incentive compatible allocation $\left(c_t^T, y_t^T\right)$ that generates utility

$$\mathbb{E}_{t-1}\left[\sum_{s=t}^{T} \beta^{s-t}\left(u(c_s,h_s) - v(y_s/\theta_s)\right)\right] = W, \quad \text{where } h_t = h,\ h_s = H(c_{s-1},h_{s-1}) \text{ for } s > t. \tag{48}$$

The following result transforms the incentive compatibility constraint (45) into a sequence of temporary constraints.

Lemma (One-shot deviation principle)**.** *The allocation* $(\mathbf{c},\mathbf{y})$ *is incentive compatible if and only if it* *satisfies the following condition for all t and all $\theta^t \in \Theta^t$, $\hat{\theta} \in \Theta_t$:*

$$\begin{aligned} & u\left(c_t\left(\theta^t\right), h_t\left(\theta^{t-1}\right)\right) - v\left(\frac{y_t\left(\theta^t\right)}{\theta_t}\right) + \beta w_{t+1}\left(c_{t+1}^T\left(\theta^t\right), y_{t+1}^T\left(\theta^t\right); H\left(c_t\left(\theta^t\right), h_t\left(\theta^{t-1}\right)\right)\right) \\ & \geq\ u\left(c_t\left(\theta^{t-1}, \hat{\theta}\right), h_t\left(\theta^{t-1}\right)\right) - v\left(\frac{y_t\left(\theta^{t-1}, \hat{\theta}\right)}{\theta_t}\right) \\ & \quad + \beta w_{t+1}\left(c_{t+1}^T\left(\theta^{t-1}, \hat{\theta}\right), y_{t+1}^T\left(\theta^{t-1}, \hat{\theta}\right); H\left(c_t\left(\theta^{t-1}, \hat{\theta}\right), h_t\left(\theta^{t-1}\right)\right)\right). \end{aligned} \tag{49}$$

Proof. Since one-shot deviations are special cases of reporting strategies, incentive compatibility clearly implies that the temporary incentive constraint (49) holds for all t and all $\theta^t \in \Theta^t$, $\hat{\theta} \in \Theta_t$.

For the reverse implication, we proceed by induction. Induction basis: Consider any function $\tilde{\sigma}_1 : \Theta_1 \to \Theta_1$. Define reporting strategy $\sigma^{(1)}$ by $\sigma_1^{(1)}(\theta_1) = \tilde{\sigma}_1(\theta_1)$ and $\sigma_t^{(1)}(\theta^t) = \theta_t$ for all $t > 1$. Since the temporary incentive constraint (49) holds for $t = 1$ we obtain the inequality

$$\begin{aligned} & w_1(\mathbf{c}, \mathbf{y}; h_1) \\ & = \sum_{\theta_1 \in \Theta_1} \left[u\left(c_1(\theta_1), h_1\right) - v\left(\frac{y_1(\theta_1)}{\theta_1}\right) + \beta w_2\left(c_2^T(\theta_1), y_2^T(\theta_1);\, H\left(c_1(\theta_1), h_1\right)\right)\right] \pi_1(\theta_1) \\ & \geq \sum_{\theta_1 \in \Theta_1} \left[u\left(c_1\left(\tilde{\sigma}_1(\theta_1)\right), h_1\right) - v\left(\frac{y_1\left(\tilde{\sigma}_1(\theta_1)\right)}{\theta_1}\right)\right] \pi_1(\theta_1) \\ & \quad + \beta \sum_{\theta_1 \in \Theta_1} w_2\left(c_2^T\left(\tilde{\sigma}_1(\theta_1)\right), y_2^T\left(\tilde{\sigma}_1(\theta_1)\right);\, H\left(c_1\left(\tilde{\sigma}_1(\theta_1)\right), h_1\right)\right) \pi_1(\theta_1) \\ & = w_1\left(\mathbf{c} \circ \sigma^{(1)}, \mathbf{y} \circ \sigma^{(1)}; h_1\right). \end{aligned}$$

Hence, truth-telling dominates any strategy $\sigma^{(1)}$ involving deviations only in period 1.

Induction step: Suppose that the inequality $w_1(\mathbf{c}, \mathbf{y}; h_1) \geq w_1\left(\mathbf{c} \circ \sigma^{(t-1)}, \mathbf{y} \circ \sigma^{(t-1)}; h_1\right)$ holds for all strategies $\sigma^{(t-1)}$ involving deviations only in periods $1, \ldots, t-1$. Let $\sigma^{(t)}$ be a reporting strategy that involves deviations only in periods $1, \ldots, t$. Given a history $\theta^{t-1} \in \Theta^{t-1}$, let $\hat{\theta}^{t-1} = \sigma^{(t)}\left(\theta^{t-1}\right) = \left(\sigma_1^{(t)}\left(\theta^1\right), \ldots, \sigma_{t-1}^{(t)}\left(\theta^{t-1}\right)\right)$ be the corresponding history of reports. Let $\sigma^{(t-1)}$ be the strategy that coincides with $\sigma^{(t)}$ in periods $1, \ldots, t-1$ and corresponds to truth-telling in periods $t, \ldots, T$. Since by assumption the temporary incentive constraint (49) holds for all histories $\left(\hat{\theta}^{t-1},\theta_t\right)$, $\theta_t\in\Theta_t$, we obtain

$$
\begin{aligned}
& w_t\left(\left(\mathbf{c}\circ\sigma^{(t-1)}\right)_t^T\left(\theta^{t-1}\right),\left(\mathbf{y}\circ\sigma^{(t-1)}\right)_t^T\left(\theta^{t-1}\right);h_t\left(\hat{\theta}^{t-1}\right)\right)\\
&=\sum_{\theta_t}\left[u\left(c_t\left(\hat{\theta}^{t-1},\theta_t\right),h_t\left(\hat{\theta}^{t-1}\right)\right)-v\left(\frac{y_t\left(\hat{\theta}^{t-1},\theta_t\right)}{\theta_t}\right)\right]\pi_t\left(\theta_t\right)\\
&\quad+\beta\sum_{\theta_t}w_{t+1}\left(c_{t+1}^T\left(\hat{\theta}^{t-1},\theta_t\right),y_{t+1}^T\left(\hat{\theta}^{t-1},\theta_t\right);H\left(c_t\left(\hat{\theta}^{t-1},\theta_t\right),h_t\left(\hat{\theta}^{t-1}\right)\right)\right)\pi_t\left(\theta_t\right)\\
&\geq\sum_{\theta_t}\left[u\left(c_t\left(\hat{\theta}^{t-1},\sigma_t^{(t)}\left(\theta^t\right)\right),h_t\left(\hat{\theta}^{t-1}\right)\right)-v\left(\frac{y_t\left(\hat{\theta}^{t-1},\sigma_t^{(t)}\left(\theta^t\right)\right)}{\theta_t}\right)\right]\pi_t\left(\theta_t\right)\\
&\quad+\beta\sum_{\theta_t}w_{t+1}\left(c_{t+1}^T\left(\hat{\theta}^{t-1},\sigma_t^{(t)}\left(\theta^t\right)\right),y_{t+1}^T\left(\hat{\theta}^{t-1},\sigma_t^{(t)}\left(\theta^t\right)\right);H\left(c_t\left(\hat{\theta}^{t-1},\sigma_t^{(t)}\left(\theta^t\right)\right),h_t\left(\hat{\theta}^{t-1}\right)\right)\right)\pi_t\left(\theta_t\right)\\
&=w_t\left(\left(\mathbf{c}\circ\sigma^{(t)}\right)_t^T\left(\theta^{t-1}\right),\left(\mathbf{y}\circ\sigma^{(t)}\right)_t^T\left(\theta^{t-1}\right);h_t\left(\hat{\theta}^{t-1}\right)\right).
\end{aligned}
$$

This implies

$$
\begin{aligned}
& w_1\left(\mathbf{c}\circ\sigma^{(t-1)},\mathbf{y}\circ\sigma^{(t-1)};h_1\right)\\
&=\sum_{s=1}^{t-1}\beta^{s-1}\sum_{\theta^s\in\Theta^s}\left[u\left(c_s\left(\sigma^{(t-1)}\left(\theta^s\right)\right),h_s\left(\sigma^{(t-1)}\left(\theta^{s-1}\right)\right)\right)-v\left(\frac{y_s\left(\sigma^{(t-1)}\left(\theta^s\right)\right)}{\theta_s}\right)\right]\Pi^s\left(\theta^s\right)\\
&\quad+\beta^{t-1}\sum_{\theta^{t-1}\in\Theta^{t-1}}w_t\left(\left(\mathbf{c}\circ\sigma^{(t-1)}\right)_t^T\left(\theta^{t-1}\right),\left(\mathbf{y}\circ\sigma^{(t-1)}\right)_t^T\left(\theta^{t-1}\right);h_t\left(\hat{\theta}^{t-1}\right)\right)\Pi^{t-1}\left(\theta^{t-1}\right)\\
&\geq\sum_{s=1}^{t-1}\beta^{s-1}\sum_{\theta^s\in\Theta^s}\left[u\left(c_s\left(\sigma^{(t)}\left(\theta^s\right)\right),h_s\left(\sigma^{(t)}\left(\theta^{s-1}\right)\right)\right)-v\left(\frac{y_s\left(\sigma^{(t)}\left(\theta^s\right)\right)}{\theta_s}\right)\right]\Pi^s\left(\theta^s\right)\\
&\quad+\beta^{t-1}\sum_{\theta^{t-1}\in\Theta^{t-1}}w_t\left(\left(\mathbf{c}\circ\sigma^{(t)}\right)_t^T\left(\theta^{t-1}\right),\left(\mathbf{y}\circ\sigma^{(t)}\right)_t^T\left(\theta^{t-1}\right);h_t\left(\hat{\theta}^{t-1}\right)\right)\Pi^{t-1}\left(\theta^{t-1}\right)\\
&=w_1\left(\mathbf{c}\circ\sigma^{(t)},\mathbf{y}\circ\sigma^{(t)};h_1\right),
\end{aligned}
$$

and hence, using the induction hypothesis, we have $w_1\left(\mathbf{c},\mathbf{y};h_1\right)\geq w_1\left(\mathbf{c}\circ\sigma^{(t)},\mathbf{y}\circ\sigma^{(t)};h_1\right)$. Since $\sigma^{(t)}$ was an arbitrary strategy involving deviations only in periods $1,\ldots,t$, the induction step is complete. This completes the proof. □

Equation (49) states that it is not profitable to misreport one's skill in period t and report the truth in all periods thereafter. If this condition holds for all periods and all possible histories, the lemma shows that no reporting strategy (potentially involving deviations in multiple time periods) yields more utility than truth-telling.

Based on definition (47), the promise-keeping constraint (46) can be written as

$$W_1 = \sum_{\theta_1 \in \Theta_1} \left[u\left(c_1\left(\theta_1\right), h_1\right) - v\left(\frac{y_1\left(\theta_1\right)}{\theta_1}\right) + \beta w_2\left(c_2^T\left(\theta_1\right), y_2^T\left(\theta_1\right); H\left(c_1\left(\theta_1\right), h_1\right)\right) \right] \pi_1\left(\theta_1\right). \tag{50}$$

Similarly, for periods $t > 1$ definition (47) is equivalent to

$$\begin{aligned} & w_t\left(c_t^T\left(\theta^{t-1}\right), y_t^T\left(\theta^{t-1}\right); h_t\left(\theta^{t-1}\right)\right) \\ & = \sum_{\theta_t \in \Theta_t} \left[u\left(c_t\left(\theta^{t-1}, \theta_t\right), h_t(\theta^{t-1})\right) - v\left(\frac{y_t\left(\theta^{t-1}, \theta_t\right)}{\theta_t}\right) \right] \pi_t\left(\theta_t\right) \\ & \quad + \beta \sum_{\theta_t \in \Theta_t} w_{t+1}\left(c_{t+1}^T\left(\theta^{t-1}, \theta_t\right), y_{t+1}^T\left(\theta^{t-1}, \theta_t\right); H\left(c_t\left(\theta^{t-1}, \theta_t\right), h_t\left(\theta^{t-1}\right)\right)\right) \pi_t\left(\theta_t\right). \end{aligned} \tag{51}$$

In summary, the incentive compatibility constraint (45) of the social planner problem is equivalent to the sequence of temporary constraints (49), whereas the promise-keeping constraint (46) is equivalent to condition (50) in combination with the sequence (51) of constraints for continuation utilities w_t, $t > 1$.

Since the constraint set can be given the sequential form (49), (50), (51), and since the objective function is a sum of period payoffs, the social planner problem is a standard dynamic programming problem. In particular, the Bellman Principle of Optimality holds. This establishes the following result.[1]

Proposition (Recursive formulation)**.** *Let $W_1 \in \mathrm{dom}_1(h_1)$. The value $C_1(W_1, h_1)$ of the social planner problem* (44) *can be computed by backward induction using the following equation for all t (with the convention $C_{T+1} = W_{T+1} = 0$):*

$$C_t(W_t, h_t) = \min_{c_t, y_t, W_{t+1}} \sum_{\theta \in \Theta_t} \left[c_t(\theta) - y_t(\theta) + qC_{t+1}\left(W_{t+1}(\theta), H(c_t(\theta), h_t)\right)\right] \pi_t(\theta) \tag{52}$$

s.t.

$$u\left(c_t(\theta), h_t\right) - v\left(y_t(\theta)/\theta\right) + \beta W_{t+1}(\theta) \geq u\left(c_t(\theta'), h_t\right) - v\left(y_t(\theta')/\theta\right) + \beta W_{t+1}(\theta') \quad \forall \theta, \theta' \in \Theta_t \tag{53}$$

$$\sum_{\theta \in \Theta_t} \left[u(c_t(\theta), h_t) - v(y_t(\theta)/\theta) + \beta W_{t+1}(\theta)\right] \pi_t(\theta) = W_t \tag{54}$$

$$W_{t+1}(\theta) \in \mathrm{dom}_{t+1}\left(H(c_t(\theta), h_t)\right) \quad \forall \theta \in \Theta_t. \tag{55}$$

Moreover, plans $(c_t, y_t)_{t=1,\dots,T}$ that solve the sequence of problems (52) *constitute an optimal allocation. Conversely, any optimal allocation solves the sequence of problems* (52)*.*

In the numerical section of the paper, it is inevitable to work with compact spaces for consumption and output. For the numerical section we therefore pick bounds $\underline{c}, \overline{c}, \underline{y}, \overline{y} \in \mathbb{R}_{++}$ with $\underline{c} < \overline{c}$, $\underline{y} < \overline{y}$, and add the boundary constraints $\overline{c} \geq c_t \geq \underline{c}$ and $\overline{y} \geq y_t \geq \underline{y}$ for all t to the social planner problem. The

[1] The recursive formulation generalizes without difficulty to infinite time horizons if utilities are bounded.

bounds allow us to find a straightforward expression for the domain restriction $\text{dom}_t(h)$. Based on the monotonicity properties of our preference specification, we obtain the upper bound of $\text{dom}_t(h)$ by simply setting consumption to $\bar{c}$ and output to $\underline{y}$ for all realizations and all remaining periods. Similarly, the lower bound of $\text{dom}_t(h)$ is obtained by setting consumption to $\underline{c}$ and output to $\bar{y}$ for all realizations and all remaining periods. By continuity, all points in the interval between the upper and lower bound of $\text{dom}_t(h)$ are feasible promises.

Appendix C: Persistent skills

We assume that skills form a Markov chain with transition probabilities $\pi_t(\theta_t|\theta_{t-1})$, where $\pi_t(\theta_t^H|\theta_{t-1}^H) > \pi_t(\theta_t^H|\theta_{t-1}^L)$. Following the insights from Fernandes and Phelan (2000), the Markov property imposes two additional state variables (past skill type θ_{t-1}, threat utility $\hat{W}_t$) and one additional constraint (threat-keeping constraint). As usual, we study a relaxed problem in which only the downward incentive compatibility constraints are imposed. With this approach, a high skill report may only come from a high-skilled worker and there is common knowledge of preferences in that case. A low skill report may come from both types of workers. Since those workers face different probability distributions over future uncertainty, we need to impose a threat-keeping constraint in that case.

If the past skill is low, the Bellman equation of the social planning problem is therefore

$$C_t\left(W_t, \hat{W}_t, c_{t-1}, \theta_{t-1}^L\right) = \min_{c_t^i, y_t^i, W_{t+1}^i, \hat{W}_{t+1}^L} \sum_{i=H,L} \left[c_t^i - y_t^i + qC_{t+1}\left(W_{t+1}^i, \hat{W}_{t+1}^i, c_t^i, \theta_t^i\right)\right]\pi_t\left(\theta_t^i|\theta_{t-1}^L\right) \quad (56)$$

s.t.

$$W_t = \sum_{i=H,L}\left[u\left(c_t^i, c_{t-1}\right) - v\left(y_t^i/\theta_t^i\right) + \beta W_{t+1}^i\right]\pi_t\left(\theta_t^i|\theta_{t-1}^L\right) \quad (57)$$

$$\hat{W}_t = \sum_{i=H,L}\left[u\left(c_t^i, c_{t-1}\right) - v\left(y_t^i/\theta_t^i\right) + \beta W_{t+1}^i\right]\pi_t\left(\theta_t^i|\theta_{t-1}^H\right) \quad (58)$$

$$u\left(c_t^H, c_{t-1}\right) - v\left(y_t^H/\theta_t^H\right) + \beta W_{t+1}^H \geq u\left(c_t^L, c_{t-1}\right) - v\left(y_t^L/\theta_t^H\right) + \beta\hat{W}_{t+1}^L. \quad (59)$$

If the past skill is high, the Bellman equation is

$$C_t\left(W_t, c_{t-1}, \theta_{t-1}^H\right) = \min_{c_t^i, y_t^i, W_{t+1}^i, \hat{W}_{t+1}^L} \sum_{i=H,L}\left[c_t^i - y_t^i + qC_{t+1}\left(W_{t+1}^i, \hat{W}_{t+1}^i, c_t^i, \theta_t^i\right)\right]\pi_t\left(\theta_t^i|\theta_{t-1}^H\right) \quad (60)$$

s.t.

$$W_t = \sum_{i=H,L}\left[u\left(c_t^i, c_{t-1}\right) - v\left(y_t^i/\theta_t^i\right) + \beta W_{t+1}^i\right]\pi_t\left(\theta_t^i|\theta_{t-1}^H\right) \quad (61)$$

$$u\left(c_t^H, c_{t-1}\right) - v\left(y_t^H/\theta_t^H\right) + \beta W_{t+1}^H \geq u\left(c_t^L, c_{t-1}\right) - v\left(y_t^L/\theta_t^H\right) + \beta\hat{W}_{t+1}^L. \quad (62)$$

Introduce symbol $\hat{\lambda}$ for the Lagrange multiplier of the threat-keeping constraint (58) and define

$$B_t^H = \frac{\beta\left[u_h\left(c_{t+1}^{HH}, c_t^H\right) - u_h\left(c_{t+1}^{HL}, c_t^H\right)\right]}{\lambda_{t+1}^H \mathbb{E}\left[\tilde{U}_t | \theta^{t-1}, \theta_t^H\right]} \geq 0, \tag{63}$$

$$B_t^L = \frac{\beta\left[u_h\left(c_{t+1}^{LH}, c_t^L\right) - u_h\left(c_{t+1}^{LL}, c_t^L\right)\right]}{\left(\lambda_{t+1}^L + \hat{\lambda}_{t+1}^L\right) \mathbb{E}\left[\tilde{U}_t | \theta^{t-1}, \theta_t^L\right]} \geq 0, \tag{64}$$

$$A_t^L = \beta \frac{\frac{v'\left(y_t^L/\theta_t^L\right)}{\theta_t^L} - \frac{v'\left(y_t^L/\theta_t^H\right)}{\theta_t^H} + \hat{U}_t^L - \mathbb{E}\left[\tilde{U}_t | \theta^{t-1}, \theta_t^L\right]}{q\pi_t\left(\theta_t^L | \theta_{t-1}\right)\left(\lambda_{t+1}^L + \hat{\lambda}_{t+1}^L\right) \mathbb{E}\left[\tilde{U}_t | \theta^{t-1}, \theta_t^L\right]} \geq 0. \tag{65}$$

Proceeding as in the proof of Proposition 2, the labor wedges can be represented as $\tau_{y,t}^H = -\mu_{t+1}^H B_t^H$ and $\tau_{y,t}^L = \mu_t A_t^L - \mu_{t+1}^L B_t^L$. Note that the habit effects B_t^H, B_t^L are exact analogies to the case with transitory shocks. The instantaneous labor distortion A_t^L includes one additional term:

$$\hat{U}_t^L - \mathbb{E}\left[\tilde{U}_t | \theta^{t-1}, \theta_t^L\right] \tag{66}$$

$$= \beta \sum_j u_h\left(c_{t+1}^{Lj}, c_t^L\right) \pi_{t+1}\left(\theta_{t+1}^j | \theta_t^H\right) - \beta \sum_j u_h\left(c_{t+1}^{Lj}, c_t^L\right) \pi_{t+1}\left(\theta_{t+1}^j | \theta_t^L\right) \tag{67}$$

$$= \beta\left[\pi_{t+1}\left(\theta_{t+1}^H | \theta_t^H\right) - \pi_{t+1}\left(\theta_{t+1}^H | \theta_t^L\right)\right]\left[u_h\left(c_{t+1}^{LH}, c_t^L\right) - u_h\left(c_{t+1}^{LL}, c_t^L\right)\right] \geq 0. \tag{68}$$

Savings wedges can be derived by following the proof of Proposition 3. For the high-skilled worker $(i = H)$ we define

$$D_t^i = \frac{\mathbb{E}\left[\tilde{U}_{t+1} | \theta^{t-1}, \theta_t^i, \theta_{t+1}^L\right] - \mathbb{E}\left[\tilde{U}_{t+1} | \theta^{t-1}, \theta_t^i, \theta_{t+1}^H\right]}{\lambda_{t+1}^i \mathbb{E}\left[\tilde{U}_{t+1} | \theta^{t-1}, \theta_t^i\right]}, \tag{69}$$

$$E_t^i = \frac{q\left[u_h\left(c_{t+1}^{iH}, c_t^H\right) - u_h\left(c_{t+1}^{iL}, c_t^i\right)\right]}{\lambda_{t+1}^i \mathbb{E}\left[\tilde{U}_{t+1} | \theta^{t-1}, \theta_t^i\right]}, \tag{70}$$

$$F_t^{ij} = \frac{q\left[u_h\left(c_{t+2}^{ijH}, c_{t+1}^{ij}\right) - u_h\left(c_{t+2}^{ijL}, c_{t+1}^{ij}\right)\right]}{\lambda_{t+1}^i \mathbb{E}\left[\tilde{U}_{t+1} | \theta^{t-1}, \theta_t^i\right]}, \quad j = L, H, \tag{71}$$

and obtain the savings wedge

$$\tau_{s,t}^i = \mu_{t+1}^i D_t^i + \mu_{t+1}^i E_t^i + \sum_j \pi_{t+1}\left(\theta_{t+1}^j | \theta_t^i\right) \mu_{t+2}^{ij} F_t^{ij}. \tag{72}$$

This is again an exact analogy to the case with transitory shocks. For the low-skilled worker $(i = L)$ we replace λ_{t+1}^{i} by the sum $\lambda_{t+1}^{L}+\hat{\lambda}_{t+1}^{L}$ in the definitions of D_t^i, E_t^i, F_t^{ij} and we define

$$\hat{D}_t^L = \frac{\sum_j \left[\pi_{t+1}\left(\theta_{t+1}^j|\theta_t^L\right) - \pi_{t+1}\left(\theta_{t+1}^j|\theta_t^H\right)\right] \mathbb{E}\left[\tilde{U}_{t+1}|\theta^{t-1},\theta_t^L,\theta_{t+1}^j\right]}{\left(\lambda_{t+1}^L+\hat{\lambda}_{t+1}^L\right)\mathbb{E}\left[\tilde{U}_{t+1}|\theta^{t-1},\theta_t^L\right]}, \tag{73}$$

$$\hat{E}_t^L = \frac{q\sum_j u_h\left(c_{t+1}^{Lj},c_t^L\right)\left[\pi_{t+1}\left(\theta_{t+1}^j|\theta_t^H\right) - \pi_{t+1}\left(\theta_{t+1}^j|\theta_t^L\right)\right]}{\left(\lambda_{t+1}^L+\hat{\lambda}_{t+1}^L\right)\mathbb{E}\left[\tilde{U}_{t+1}|\theta^{t-1},\theta_t^L\right]}. \tag{74}$$

The savings wedge is then

$$\tau_{s,t}^L = \mu_{t+1}^L D_t^L + \hat{\lambda}_{t+1}^L \hat{D}_t^L + \mu_{t+1}^L E_t^L + \hat{\lambda}_{t+1}^L \hat{E}_t^L + \sum_j \pi_{t+1}\left(\theta_{t+1}^j|\theta_t^L\right)\mu_{t+2}^{Lj}F_t^{Lj}. \tag{75}$$

The concavity/wealth effect is captured by the sum $\mu_{t+1}^L D_t^L+\hat{\lambda}_{t+1}^L\hat{D}_t^L$. Note that $\hat{D}_t^L$ is zero if $c_{t+1}^{LH}=c_{t+1}^{LL}$. Hence, even though the Lagrange multiplier μ_{t+1}^L does not show up directly, the part $\hat{\lambda}_{t+1}^L\hat{D}_t^L$ vanishes if $\mu_{t+1}^L=0$. If $\mu_{t+1}^L>0$, then due to concavity and $\pi_{t+1}\left(\theta_{t+1}^L|\theta_t^L\right)>\pi_{t+1}\left(\theta_{t+1}^L|\theta_t^H\right)$, the term $\hat{\lambda}_{t+1}^L\hat{D}_t^L$ is positive, just like $\mu_{t+1}^L D_t^L$. The immediate habit effect consists of the terms $\mu_{t+1}^L E_t^L+\hat{\lambda}_{t+1}^L\hat{E}_t^L$. The term $\mu_{t+1}^L E_t^L$ is familiar and looks just like in the case of the high-skilled worker. The term $\hat{\lambda}_{t+1}^L\hat{E}_t^L$ goes in the same direction, since $\pi_{t+1}\left(\theta_{t+1}^H|\theta_t^H\right)>\pi_{t+1}\left(\theta_{t+1}^H|\theta_t^L\right)$ and $u_h\left(c_{t+1}^{LH},c_t^L\right)>u_h\left(c_{t+1}^{LL},c_t^L\right)$ due to complementarity. Hence $\hat{\lambda}_{t+1}^L\hat{E}_t^L$ is also an immediate habit effect. Even though μ_{t+1}^L does not show up directly, we note that this term will be zero if $\mu_{t+1}^L=0$, or equivalently if $c_{t+1}^{LH}=c_{t+1}^{LL}$. Finally we have the subsequent habit effect, consisting of the terms $\mu_{t+2}^{Lj}F_t^{Lj}$ just like in the case of the high-skilled worker.

References

Fernandes, A., and C. Phelan (2000): "A recursive formulation for repeated agency with history dependence," *Journal of Economic Theory*, 91(2), 223–247.

Phelan, C., and R. M. Townsend (1991): "Computing Multi-Period, Information-Constrained Optima," *Review of Economic Studies*, 58(5), 853–881.

Spear, S., and S. Srivastava (1987): "On repeated moral hazard with discounting," *Review of Economic Studies*, 54(4), 599–617.