Modeling Monetary Policy

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Abstract

In an otherwise standard macroeconomic model, we model the central bank as providing money only in exchange for eligible assets in open market operations. The relationship between the policy interest rate and expected inflation and consumption growth is affected by money market conditions, i.e. the varying liquidity value of eligible assets, and aggregate risk. This induces a data-consistent systematic wedge between the policy and consumption Euler rates that standard models equate. Moreover, as the central bank distributes back only its interest earnings and not its entire wealth to households, the relative evolution of eligible assets induces persistent and hump-shaped consumption responses to shocks.

JEL classification: E52; E58; E43; E32.
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1 Introduction

In monetary policy analysis the focus has shifted away from monetary aggregates towards short-run nominal interest rates. Consequently, the money market is widely neglected in the analysis of transmission and optimality of monetary policy, and money demand is treated as a redundant element. This link between the monetary instrument and the private sector is replaced in current monetary macro-models by the consumption Euler equation, which is also called the new IS-curve. It relates the policy rate to expected consumption growth and inflation, and has become essential for monetary transmission and for the implementation of optimal monetary policy. There are however issues with the empirical reliability of this relationship.

Studies in finance provide broad evidence that consumption Euler equations fail when they are applied to asset prices or the rate of returns on bonds (see Weil, 1989). This should already cast doubt on the common practice in monetary policy analysis to assume that the real central bank interest rate, which is close to the risk-free bond rate, is tightly related to consumption growth. But what is more worrying, in our view, are recent studies unveiling substantial failures of (implied) consumption Euler interest rates to match money market rates: interest rates generated by standard consumption Euler equations are negatively related to US money market rates, while their spread is negatively related to the stance of monetary policy, i.e. with the level of the money market rates (see Canzoneri et al., 2007, and Atkeson and Kehoe, 2008). Thus there seems to exist a non-negligible systematic wedge that separates interest rates, which are claimed to be identical in macroeconomic theory. Put differently, observed policy rates do not seem to be related to consumption growth and inflation in the way standard models characterize.

In this paper we take a closer look at the implementation of monetary policy and show that an explicit specification of central bank operations can contribute to the resolution of this problem. We thereby aim at explaining the puzzling relationship between the policy rate and the Euler rate, i.e. expected values of consumption growth and inflation.

We develop a macro-model with three interest rates: a discount rate for open market operations controlled by the central bank (the *repo* or *policy* rate), an interest rate on government bonds (the *bond* rate), and the Euler rate (the rate on private *debt*). The model can explain systematic movements of the spreads with the monetary policy stance and with aggregate uncertainty. Specifically, the liquidity premium on bonds, i.e. the spread between Euler and bond rates, varies endogenously with the expected costs of transforming bonds into means of payment (money). Consistent with empirical evidence, we show that the liquidity premium and the Euler rate can be negatively related to the policy rate. Thus changes in the policy rate are linked to aggregate demand and inflation to a smaller extend than implied by a conventional framework where the central bank sets the Euler rate.

The model is based on a general equilibrium framework, where money demand is introduced by a cash-in-advance constraint. The model mainly differs from standard monetary macro-models by three assumptions: First, we assume that financial markets are separated. The asset market, where agents trade interest bearing assets and cash, opens at the end of each period. Before, the money market opens, where agents can acquire cash from the central bank in exchange for interest bearing assets discounted with the rate set by the central bank, i.e., the repo rate. Bonds bought today can be cashed in the next period at the repo rate. The bond rate is therefore closely linked to the expected future repo rate in open market operations, while the spread between these rates increases on average with aggregate uncertainty and investors' relative risk aversion. Thus, the bond rate exhibits a risk premium.

Second, we assume that only government bonds are eligible in open market operations, while other assets (here, privately issued debt) cannot be cashed at the central bank. The main property is that the amount of eligible assets is not unlimited. Access to money is thus bounded by private sector government bond holdings and cannot be eased by holdings of other securities issued by the private sector. Due to this property, government bonds are perceived as a closer substitute for cash, which gives rise to a liquidity premium. Thus, in equilibrium we observe a spread between the bond rate and the interest rate on privately issued debt, which are not eligible for open market operations.[2] The debt rate, which corresponds to the above mentioned consumption Euler rate, thus differs from the bond rate, while the spread depends on the state of the economy. In particular, a higher repo rate raises the price of money in terms of bonds, i.e. reduces the amount of money per unit of bonds supplied to the central bank, and leads to a decline in the liquidity premium.

Third, we assume that the central bank transfers its revenues to the fiscal authority. Following central bank practice (see Meulendyke, 1998), we assume that it reinvests payoffs from maturing securities in new interest bearing assets. The associated interest rate earnings are then transferred to the fiscal authority, while financial wealth is held by the central bank as the counterpart of outstanding money.[3] As a consequence, the distribution of eligible securities between the private sector and the central bank changes over time and, in particular, varies with the monetary policy stance. This property exerts an additional effect of monetary policy on the private sector behavior.

When we examine the transmission of monetary policy shocks, which will be substantially affected by these assumptions, we consider prices to be set in an imperfectly flexible way, to allow for realistic inflation dynamics. When the constraint in open market operations ("discounted value of bonds held by the private sector $\geq$ new money") is binding, the model's

[2] Bansal and Coleman (1996) endogenously derive a liquidity premium by assuming bonds reduce transactions costs.

[3] This differs from the common assumption in general equilibrium macro-models that the central bank transfers seigniorage (defined as the change in the monetary base) to the fiscal authority.

predictions substantially differs from results generated by standard models. Consider, for example, an unexpected increase in the repo rate, i.e. a positive innovation to a Taylor-type feedback rule for the repo rate. Since aggregate demand is constrained by the amount of short-term bonds discounted with the repo rate (plus money carried over from the previous period), which represents the amount of money the private sector can get through open market operations, the higher repo rate has a negative effect on the level of nominal consumption. Under sticky prices, monetary policy affects real consumption. However, monetary policy rather impacts on the level of real consumption than on its growth rate, as implied by the consumption Euler equation in standard models. Here monetary policy has a smaller initial impact on the level of consumption than in standard models. When the central bank increases its policy rate, part of that increase reflects a decrease in liquidity premium such that expected consumption growth is less affected than in a standard model.

Moreover, due to the third assumption above, the rise in the repo rate further affects consumption through its impact on the distribution of eligible securities. If, for example, monetary policy is persistently tightened by a higher repo rates, the central bank demands more bonds in exchange for money. With reduced bond holdings, the constraint in the money market tends to become even tighter in the next period, which is responsible for a hump-shaped consumption response. Hence, an inertial *rise* in the repo rate leads to a decline in the consumption growth rate, which – together with lower expected inflation – implies the Euler-rate to *fall*, consistent with empirical evidence (see Canzoneri et al., 2007).

Finally, the model provides a simple explanation for the existence and – at the same time – for the lack of a liquidity effect: When the central bank controls the money growth rate and the open (or money) market constraint is binding, there is negative relation between newly injected money and the repo rate, since the stock of eligible bonds is predetermined by the last period investment decision.[4] As a consequence, a money injection leads to an unambiguous liquidity effect, i.e. a decline in the repo rate and in the bond rate. In contrast, the debt/Euler rate increases due to the well-known anticipated inflation effect. The latter typically leads to a lack of a liquidity effect in standard sticky price models (see e.g. Christiano, et al., 1997), which we also found for the version of model where the money market constraint is not binding.

The paper is organized as follows. Section 2 presents empirical evidence on short-term interest rates and spreads. In section 3, the model is developed. In section 4, we examine the behavior of interest rates and spreads in the model. Section 5 presents responses to interest rate and money supply shocks, and section 6 concludes.

[4] This is of course due to the first above mentioned assumption on the timing of financial markets.

2 Empirical behavior of interest rates

This section presents the empirical behavior of the different interest rates considered in the model and the relationships between them. The model contains the Euler rate R^d, a policy rate R^m, i.e. the price of money in terms of bonds inside open market operations, and an interest rate R on an asset that the central bank accepts in exchange for money in its open market operations, i.e. the price of money in terms of bonds outside open market operations.

Euler rate vs. policy rate This sub-section compares the empirical behavior of two interest rates that standard models equate, i.e. the Euler rate and the policy rate. In our model there is a third interest rate R, i.e. the interest rate on assets accepted by the central bank in exchange for money in open market operations. In this sub-section we focus on the spread between the fed funds rate and the Euler rate, given that empirically and in the model both R^m and R move relatively close to each other and contrast significantly with the behavior of R^d. Thus for empirical comparison with the Euler rate we can interchangeably use R^m or R, with only negligible quantitative differences (see below).

First, the empirical interest rate implied by standard Euler equations is computed. The methodology is similar to Fuhrer (2000) and Canzoneri et al. (2007). In a standard Euler equation, the inverse of the gross nominal interest rate R_t^d can be expressed as

$$\frac{1}{R_t^d} = \beta E_t \left(\frac{u_{c,t+1}}{u_{c,t}} \frac{P_t}{P_{t+1}} \right), \tag{1}$$

where β is the discount factor, u_c is marginal utility of consumption, and P is the price level. With a standard CRRA utility function, leading to a marginal utility of consumption $c_t^{-\sigma}$, and under conditional log-normality the Euler equation can be written as

$$\frac{1}{1+r_t^d} = \beta \exp \begin{bmatrix} -\sigma \left(E_t \log c_{t+1} - \log c_t \right) - E_t \log \pi_{t+1} \\ +\frac{\sigma^2}{2} var_t \log c_{t+1} + \frac{1}{2} var_t \log \pi_{t+1} + \sigma cov_t \left(\log c_{t+1}, \log \pi_{t+1} \right) \end{bmatrix}, \tag{2}$$

where $\pi_t = P_t/P_{t-1}$. Equation (2) is used to compute the implied standard Euler (net) interest rate r^d, where the conditional moments are estimated from a six-variable VAR, $Y_t = A_0 + A_1 Y_{t-1} + v_t$, assuming $v \sim i.i.d.N\left(0, \Sigma\right)$, $\sigma = 2$ and $\beta = .993$. The variables included in Y (1966Q1-2008Q2) are log per capita real personal consumption expenditures on nondurable goods and services, log change in the deflator of this consumption measure, log price of industrial commodities, log per capita real disposable personal income, federal funds rate, and log per capita real non-consumption GDP. Moreover, a segmented (1974Q1) time trend is included in A_0.

Figure 1 displays the computed standard Euler interest rate r^d and the fed funds rate r, as well as the spread between these two rates, $s_{1,t} = r_t^d - r$, in percent. The Euler rate averages at 11.4 percent, whereas the federal funds rate averages at 6.5 percent; thus the

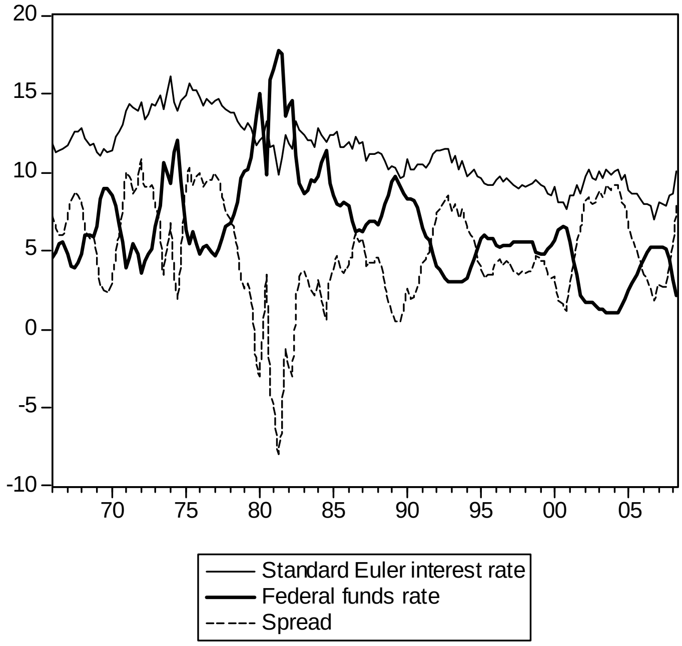

Figure 1: Euler and federal funds rates (%)

average spread is about 5 percentage points. Inflation averages at 4.4 percentage points over the period considered. The federal funds rate and the Euler rate, which should be identical according to standard macroeconomic models, display no apparent co-movement. The fed funds rate is strongly negatively correlated with the spread, a fact that has recently been pointed out by Atkeson and Kehoe (2008), while using Smets and Wouter's (2007) model. Thus, the unexplained wedge between the federal funds rate and the Euler rate are substantially related to the federal funds rate.

At low frequency, the Euler and federal funds rates are positively correlated, which is mainly due to inflation trends (upward in the 1970s and then downward in the 1980s) that move both rates in the same direction. These trends evidently distort the correlation between the Euler and policy rates in comparison to a theoretical environment with constant steady-state inflation. In order to correct for these inflation trends and to isolate short-run (business cycle) interest rate dynamics from longer term movements, we HP-filter ($\lambda = 1600$) the interest rate series. The correlations between HP-filtered variables will be used to assess theoretical moments of our model, which will be examined around a given steady-state inflation rate.

Figure 2 displays the same variables as in Figure 1 but HP filtered. The bold line is *minus* the detrended federal funds rate. Thus, there is an apparent negative comovement between fluctuations of the spread and of the policy rate. Also, the Euler and policy rates are negatively correlated at business cycle frequency.

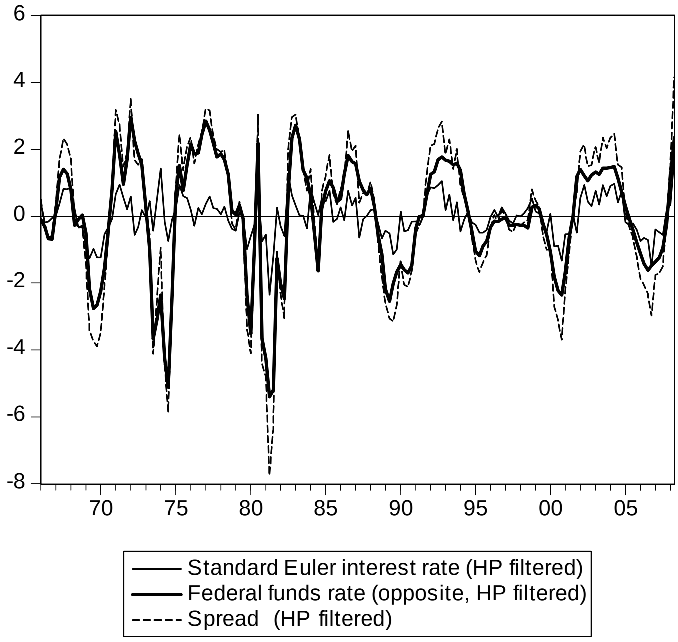

Figure 2: HP-filtered Euler and federal funds rates

Table 1 presents the (unconditional) correlations between the federal funds rate r, the Euler rate r^d, and the spread s_1, using standard Euler equations as well as our own model's Euler equation.[5]

Table 1 Empirical correlations

	Standard Euler equation	Our model's Euler equation
$\text{corr}(s_1, r)$	-0.98	-0.90
$\text{corr}\left(r^d, r\right)$	-0.66	-0.57

There is a strong (close to minus one) negative correlation between the spread and the policy rate. The Euler rate and the policy rate are negatively correlated as well, as in Canzoneri et al. (2007) in the case of real rates.[6] The correlations presented in Table 1 are relatively similar for both Euler rates.

[5] Details on this latter rate can be found derived in the appendix. The difference between the standard Euler equation and our own model Euler equation is mainly due to a cash-in-advance constraint. Overall, these two Euler rates differ only slightly, except in accelerating inflation (late 1970s) and disinflation (early 1980s) episodes, as well as around 1992 and 2003 with the drops in the policy rate.

[6] Canzoneri et al. (2007) reported correlation between real rates is smaller (-0.37) and they find a positive correlation between nominal rates, which comes from the inflation trends, as explained above.

Policy rate vs. money market rate In this subsection we briefly assess the empirical counterpart of the spread between the policy rate R^m and interest rate R, which measure the relative price of money inside and outside open market operations. For this we assess monthly data for the effective federal funds rate and the (overnight and 3-month) US$-LIBOR since January 2001. In general, the LIBOR lies slightly above the policy rate (see fig. **??** in appendix 9). The average spread between the federal funds rate and the overnight (3-month) LIBOR has been 7 (25) basis points, when the recent financial crisis period (back to August 1, 2007) is omitted.

3 The model

In this section we develop a macroeconomic framework where the asset market and the money market are separated. There are four different types of agents: households, firms, the central bank and the government. We abstract from financial intermediation and assume that households directly trade with the central bank in open market operations.

Households can invest in government bonds and non-interest bearing money, and they can borrow and lend among each other using a full set of nominally state contingent claims. Their demand for money is induced by assuming that goods market transactions cannot be conducted by using credit. This is modelled by a cash-in-advance constraint, i.e. by assuming that households have to hold money for goods market purchases. They can get money from the central bank only in exchange for securities in open market operations. To give a preview, financial markets separation will lead to a spread between the government bond rate and the policy (repo) rate, i.e. a risk premium, whereas the spread between the Euler and government bond rates, i.e. a liquidity premium, will be due to the special role of government bonds in open market operation.

Throughout the paper, upper case letters denote nominal variables, lower case letters real variables, and variables without an index (i or j) aggregate variables.

3.1 Timing of events

The timing of markets and the specification of open market operations will be important for our results. We will focus on the case where only government bonds are eligible in open market operations (like in Lacker, 1997, or Schabert, 2004). The timing of events in each period is as follows:

> There is a continuum of infinitely lived households indexed with $i \in [0,1]$. A household i enters a period t with nominal assets carried over from the previous period $t-1$:
>
> $$M^H_{i,t-1} + B_{i,t-1} + D_{i,t-1},$$
>
> where M^H denotes holdings of money, B government bonds, and D private debt.

1. Aggregate shocks materialize, labor is supplied by households, and goods are produced by firms.

2. Households enter the money market, where they can engage in open market operations with the central bank. There, money can be traded only in exchange for eligible securities, and is supplied via outright sales/purchases and via repurchase agreements. The relative price of money R_t^m (for both types of trades) is controlled by the central bank and will be called repo rate:

$$\Delta B_{i,t}^c / R_t^m = I_{i,t},$$

where $I_{i,t}$ is the amount of money received by household i and $\Delta B_{i,t}^c$ the amount of bonds the CB gets. We assume that only government bonds are eligible

$$\Delta B_{i,t}^c \leq B_{i,t-1}. \tag{3}$$

When household i leaves the money market its bonds holdings equal $B_{i,t-1} - \Delta B_{i,t}^c$.

3. Households enter the (final) goods market, where money is assumed to be the only accepted means of payment. Thus goods market expenditures are restricted by money carried over from the previous period plus additional money acquired from the central bank via current period open market operations:

$$P_t c_{i,t} \leq I_{i,t} + M_{i,t-1}^H, \tag{4}$$

where c_i denotes purchases of the final consumption good and P its price level. When household i leaves the goods market, its money stock equals $I_{i,t} + M_{i,t-1}^H - P_t c_{i,t}$.

4. Finally, the asset market opens. Before households trade in the asset market, current labor income and dividends are paid back in cash to households. Further, government bonds can be repurchased from the central bank with cash, i.e. household i can repurchase bonds $B_{i,t}^R$ using money $M_{i,t}^R = B_{i,t}^R$. After repurchase agreements are settled, money and bond holdings of household i equal

$$\widetilde{M}_{i,t} = I_{i,t} + M_{i,t-1}^H + P_t w_t n_{i,t} + P_t \delta_{i,t} - P_t c_{i,t} - M_{i,t}^R,$$
$$\widetilde{B}_{i,t} = B_{i,t-1} - \Delta B_{i,t}^c + B_{i,t}^R,$$

where w_t denotes the real wage rate, n_t working time and $P_t \delta_{i,t}$ dividends. In the asset market, households borrow/lend and trade money and bonds among each other. They can further buy bonds from the government at the price $1/R_t$, such that the price of money in terms of bonds in the asset market equals R_t. Hence, we can summarize the asset market constraint of household i as

$$(B_{i,t}/R_t) + E_t[q_{t,t+1}D_{i,t}] + M_{i,t}^H \leq \widetilde{B}_{i,t} + D_{i,t-1} + \widetilde{M}_{i,t} + P_t\tau_t, \quad (5)$$

where $P_t\tau_t$ denotes lump-sum government transfers and $q_{t,t+1}$ is a stochastic discount factor, which will be defined below.

Money cannot be issued by the private sector, $\int \widetilde{M}_{i,t}di = \int M_{i,t}^H di$, while the total amount of government bonds held by the private sector at the end of the period $\int B_{i,t}di$ will depend on how many bonds are issued by the fiscal authority or held by the central bank. In what follows we describe the model in detail.

3.2 Private sector

Households Households have identical asset endowments and identical preferences. Household i maximizes the expected sum of a discounted stream of instantaneous utilities u :

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{it}, n_{it}\right), \quad (6)$$

where E_0 is the expectation operator conditional on the time 0 information set, and $\beta \in (0,1)$ is the subjective discount factor. The instantaneous utility u is assumed to satisfy $u_t = [(c_{i,t}^{1-\sigma} - 1)(1-\sigma)^{-1}] - \gamma n_{i,t}$.

A household i is initially endowed with money $M_{i,-1}$, government bonds $B_{i,-1}$, and contingent claims $D_{i,-1}$. As described above, it faces three constraints in each period. In the money market, it can acquire money $I_{i,t}$ up to the amount of government bonds carried over from the previous period B_{t-1} discounted by R_t^m. The constraint (3) can be written as

$$I_{i,t} \leq B_{i,t-1}/R_t^m. \quad (7)$$

The constraint (7) will be called the open (or money) market constraint. It should be noted that this model can also be applied to the case where the central bank withdraws money from the private sector $I_{i,t} < 0$. For monetary injections to be positive in equilibrium a sufficiently large fraction of money has to be supplied under repurchase agreements. Throughout the analysis we will restrict our attention to the case where the central bank supplies money in a way that ensures $I_{i,t} \geq 0$.

Households are further assumed to rely on cash for transactions in the goods market. Given that they can first trade with the central bank in open market operations, the cash-in-advance constraint differs from Svensson's (1985) cash-in-advance constraint by $I_{i,t}$:

$$P_t c_{i,t} \leq I_{i,t} + M_{i,t-1}^H. \quad (8)$$

In the asset market, the government issues bonds, and households trade money and bonds with each other. They can further borrow and lend using a full set of nominally state contingent claims. Dividing the period t price of one unit of nominal wealth in a particular state of period $t+1$ by the period t probability of that state gives the stochastic discount factor $q_{t,t+1}$. The period t price of a payoff D_{jt} in period $t+1$ is then given by $E_t[q_{t,t+1}D_{jt}]$. Substituting out the stock of bonds and money held before the asset market opens, $\widetilde{B}_{i,t}$ and $\widetilde{M}_{i,t}$, in (5), the asset market constraint of household i reads

$$(B_{i,t}/R_t) + E_t[q_{t,t+1}D_{i,t}] + M^H_{i,t} + (R^m_t - 1)\, I_{i,t} \tag{9}$$
$$\leq B_{i,t-1} + D_{i,t-1} + M^H_{i,t-1} + P_t w_t n_{i,t} - P_t c_{i,t} + P_t \delta_{i,t} + P_t \tau_t,$$

where household $i's$ borrowing is restricted by the following no-Ponzi game condition

$$\lim_{s\to\infty} E_t q_{t,t+s} D_{i,t+s} \geq 0, \tag{10}$$

as well as $M^H_{i,t} \geq 0$ and $B_{i,t} \geq 0$. The term $(R^m_t - 1)\, I_{i,t}$ measures the costs of money acquired in open market operations: The households receive new cash $I_{i,t}$ in exchange for $R^m_t I_{i,t}$ bonds.

Maximizing the objective (6) subject to the money market constraint (7), the goods market constraint (8), the asset market constraints (9) and (10), for given initial values $M_{i,-1}$, $B_{i,-1}$, and $D_{i,-1}$ leads to the following first order conditions for working time $n_{i,t}$, consumption $c_{i,t}$, open market trades $I_{i,t}$, as well as holdings of contingent claims, government bonds and money:

$$-u_{i,nt}/w_t = \lambda_{i,t}, \tag{11}$$
$$u_{i,ct} = \lambda_{i,t} + \psi_{i,t}, \tag{12}$$
$$R^m_t \left(\lambda_{i,t} + \eta_{i,t}\right) = \lambda_{i,t} + \psi_{i,t}, \tag{13}$$
$$\frac{\beta}{\pi_{t+1}} \frac{\lambda_{i,t+1}}{\lambda_{i,t}} = q_{t,t+1}, \tag{14}$$
$$\beta E_t \left[\left(\lambda_{i,t+1} + \eta_{i,t+1}\right) \pi^{-1}_{t+1}\right] = \lambda_{i,t}/R_t, \tag{15}$$
$$\beta E_t \left[\left(\lambda_{i,t+1} + \psi_{i,t+1}\right) \pi^{-1}_{t+1}\right] = \lambda_{i,t} \tag{16}$$

where $\lambda_{i,t}$ and $\psi_{i,t}$ denote the multiplier on the asset and goods constraint. The first two conditions (11) and (12) show that a binding goods market constraint ($\psi_{i,t} > 0$) distorts the intratemporal consumption-leisure decision in a conventional way, $u_{i,ct} + u_{i,nt}/w_t = \psi_{i,t}$. Combining (11) and (12) with (16), discloses the inflation tax on consumption, which is implied by the cash-in-advance constraint (8):

$$\beta E_t \left[u_{i,ct+1}/\pi_{t+1}\right] = -u_{i,nt}/w_t \tag{17}$$

The open market constraint is associated with the multiplier $\eta_{i,t}$, which measures the liquidity value of bonds. When the goods market constraint is binding, $u_{i,ct} + u_{i,nt}/w_t > 0$, the role of money as a means of payment is positively valued. Likewise, government bonds, as a substitute for money, can also be valued differently from non-eligible assets; for this, the exchange rate R^m has to be sufficiently low, as discussed below.

Combining (11), (12), and (13), we obtain

$$\eta_{i,t} = \frac{u_{i,nt}}{w_t} + \frac{u_{i,ct}}{R_t^m}. \tag{18}$$

The multiplier on the open market constraint $\eta_{i,t}$, which measures the liquidity value of bonds, tends to decline with the policy rate (see 18), since a higher policy rate reduces the amount of money for each unit of bonds supplied to the central bank. The bond pricing equation (15) shows that a rise in this multiplier tends to lower the interest rate on bonds, which can generate a spread between the Euler rate and the bond rate, i.e. a liquidity premium.

The household's investment decisions further links the bond rate to the repo rate. They are willing to hold both assets, money and bonds, if the rate of return on bonds compensates for the costs of acquiring new money in the next period. This can be seen by combining (11), (13), (15), and (16)

$$\frac{E_t\left[\left(1/R_{t+1}^m\right)\left(u_{i,ct+1}/\pi_{t+1}\right)\right]}{E_t\left[\left(u_{i,ct+1}/\pi_{t+1}\right)\right]} = 1/R_t,$$

implying that the interest rate on bonds equals the expected future policy rate up to first order. Throughout, we will repeatedly refer to the rate of return on a nominally risk-free portfolio of claims that deliver one unit of currency in each state. This interest rate R_t^d, which corresponds to the Euler rate in section 2, is given by

$$R_t^d = \left[E_t q_{t,t+1}\right]^{-1}. \tag{19}$$

It should be noted that equation (14), which defines the Euler-rate, can differ from the standard Euler rate (see 1) due to the cash-credit-good friction, $\lambda_{i,t} \leq u_{i,ct}$. Finally, the following complementary slackness conditions are satisfied in the household's optimum

$$\begin{aligned} &i) \quad 0 \leq b_{i,t-1}\pi_t^{-1}/R_t^m - i_{i,t}, \quad \eta_{i,t} \geq 0, \quad \eta_{i,t}\left(b_{i,t-1}\pi_t^{-1}/R_t^m - i_{i,t}\right) = 0, \\ &ii) \quad 0 \leq i_{i,t} + m_{i,t-1}^H\pi_t^{-1} - c_{i,t}, \quad \psi_{i,t} \geq 0, \quad \psi_{i,t}\left(i_{i,t} + m_{i,t-1}^H\pi_t^{-1} - c_{i,t}\right) = 0, \end{aligned}$$

where $m_{i,t}^H = M_{i,t}^H/P_t$, $b_{i,t} = B_{i,t}/P_t$, and $i_{i,t} = I_{i,t}/P_t$, and (9) and (10) hold with equality. In equilibrium households are willing to hold both types of money, i.e. money held under repurchase agreements $M_{i,t}^R$ and under outright sales/purchases $M_{i,t}^H$. Changes in the composition of money supplied to the private sector might however affect the distribution of eligible securities between the private sector and the central bank.

Production To facilitate a reasonable transmission of monetary shocks we will allow for imperfectly flexible prices. We will introduce price stickiness in the standard way following the New Keynesian literature. In particular, we assume that the final consumption good is an aggregate of differentiated goods produced by monopolistically competitive firms indexed with $j \in [0,1]$. The CES aggregator of differentiated goods is $y_t^{\frac{\epsilon-1}{\epsilon}} = \int_0^1 y_{jt}^{\frac{\epsilon-1}{\epsilon}} dj$, with $\epsilon > 1$, where y_t is the number of units of the final good, y_{jt} the amount produced by firm j, and ϵ the constant elasticity of substitution. Let P_{jt} and P_t denote the price of good j set by firm j and the price index for the final good. The demand for each differentiated good is $y_{jt} = (P_{jt}/P_t)^{-\epsilon} y_t$, with $P_t^{1-\epsilon} = \int_0^1 P_{jt}^{1-\epsilon} dj$. A firm j produces good y_j employing the technology: $y_{jt} = a_t n_{jt}^{\alpha}$, where $\alpha \in (0,1)$, a is a stochastic productivity level satisfying $a_t = a_{t-1}^{\rho_a} \exp \varepsilon_{a,t}$, $\rho^a \geq 0$, and ε_t^a is i.i.d. normally distributed with $E_{t-1}\varepsilon_t^a = 0$. Hence, labor demand satisfies:

$$w_t = mc_{jt} \alpha y_{jt}/n_{jt}, \tag{20}$$

where mc_{jt} denotes real marginal costs.

We consider a nominal rigidity in form of staggered price setting as developed by Calvo (1983) and Yun (1995). Each period firms may reset their prices with the probability $1-\phi$ independently of the time elapsed since the last price setting. The fraction $\phi \in [0,1)$ of firms is assumed to adjust their prices with the steady state inflation rate π, where $\pi_t = P_t/P_{t-1}$, such that $P_{jt} = \pi P_{H,jt-1}$. In each period a measure $1-\phi$ of randomly selected firms sets new prices $\widetilde{P}_{jt}$ in order to maximize the expected sum of discounted future dividends $P_t \delta_{jt} = (P_{jt} - P_t mc_t)\, y_{jt}$: $\max_{\widetilde{P}_{jt}} E_t \sum_{s=0}^{\infty} \phi^s q_{t,t+s} (\widetilde{P}_{jt} y_{jt+s} - P_{t+s} mc_{t+s} y_{jt+s})$, s.t. $y_{jt+s} = \widetilde{P}_{jt}{}^{-\epsilon} P_{t+s}^{\epsilon} y_{t+s}$. For $\phi > 0$, the first order condition is given by

$$\widetilde{P}_{jt} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} \phi^s \left[q_{t,t+s} y_{t+s} P_{t+s}^{\epsilon+1} mc_{t+s} \right]}{E_t \sum_{s=0}^{\infty} \phi^s \left[q_{t,t+s} y_{t+s} P_{t+s}^{\epsilon} \right]}. \tag{21}$$

Aggregate output is $y_t = (P_t^*/P_t)^{\epsilon} n_t^{\alpha}$, where $(P_t^*)^{-\epsilon} = \int_0^1 P_{jt}^{-\epsilon} dj$ and thus $(P_t^*)^{-\epsilon} = \phi \left(P_{t-1}^*\right)^{-\epsilon} + (1-\phi)\widetilde{P}_t^{-\epsilon}$. Under flexible prices $\phi = 0$, real marginal costs are given by $mc_{jt} = \frac{\varepsilon - 1}{\varepsilon}$.

3.3 Public sector

The public sector consists of a government and a central bank. The government issues debt B^T, which is held both by households $\int B_{i,t} di = B_t$ and by the central bank $\int B_{i,t}^c di = B_t^c$: $B_t^T \geq B_t + B_t^c$. It further receives payments $P_t \tau_t^m$ from the central bank and transfers financial wealth $P_t \tau_t$ to the households. Its flow budget constraint thus reads

$$\left(B_t^T / R_t\right) + P_t \tau_t^m = B_{t-1}^T + P_t \tau_t.$$

Though taxes are non-distortionary, Ricardian equivalence will not apply when the money market constraint is binding. The supply of government debt will then not be irrelevant for the conduct of monetary policy and for monetary transmission. Given that for many central banks only short-term government debts are eligible, we focus on the supply of t-bills and disregard debt with longer maturity. As we are focusing on monetary policy effects, we assume that the supply of government bonds is exogenously determined and that government debt is issued at a constant growth rate Γ satisfying:

$$\Gamma > \beta : B_t^T = \Gamma B_{t-1}^T.$$

The central bank supplies money in exchange for government bonds in open market operations in form of outright sales/purchases M_t^H and repurchase agreements M_t^R. Before the money market opens, the central bank's stock of government bonds equals B_{t-1}^c and the stock of outstanding money equals M_{t-1}^H. It then receives an amount of bonds ΔB_t^c in exchange for money I_t, and after repurchase agreements are settled its holdings of bonds reduces by B_t^R and the amount of outstanding money by $M_t^R = B_t^R$. Before the asset market opens, where the central bank can invest in government bonds B_t^c, it holds an amount of bonds equal to $\widetilde{B}_t^c = \Delta B_t^c + B_{t-1}^c - B_t^R$. Its budget constraint is given by

$$(B_t^c/R_t) + P_t\tau_t^m = \Delta B_t^c + B_{t-1}^c - B_t^R + M_t^H - M_{t-1}^H - \left(I_t - M_t^R\right).$$

In accordance with the operational practice of central banks we assume that it rolls over its maturing assets (see e.g. Meulendyke, 1998, ch.7). Thus, we assume that the central bank also enters the asset market at the end of each period, and reinvests in bonds to the amount that equals its current stock of maturing debt $B_t^c = \widetilde{B}_t^c$. Further using $B_t^R = M_t^R$ and $\Delta B_t^c = R_t^m I_t$, the budget constraint can be simplified to $(B_t^c/R_t) - B_{t-1}^c = M_t^H - M_{t-1}^H + (R_t^m - 1) I_t - P_t\tau_t^m$.

Following common practice (see Meulendyke, 1998), we assume that the central bank transfers interest earnings from asset holdings to the government.

$$P_t\tau_t^m = B_t^c (1 - 1/R_t).$$

Note that these transfers will not be negative in equilibrium, such that the central bank will never rely on funds from the government.[7] Accordingly, its bond holdings will evolve according to

$$B_t^c - B_{t-1}^c = R_t^m I_t - \left(I_t - M_t^H + M_{t-1}^H\right). \tag{22}$$

Thus the central bank tends to accumulate more bonds, i.e. bonds flow from households to the central bank, when money supply or the policy rate is high.

Regarding the implementation of monetary policy, we assume that the central bank conducts monetary policy by using simple instrument rules, which contain a stochastic element

[7] Note that this is different in models, where central bank tranfers seigniorage (defined as the change in the monetary base) to the government in each period. A discussion of government transfers and central bank independence can be found in Sims (2003).

to allow for monetary policy shocks. We consider two alternatives. For the benchmark specification of monetary policy, we assume that the central bank sets the repo rate R_t^m. It might be set contingent on its own lags and on current inflation to allow for a Taylor-rule-type interest rate setting. To assess the monetary transmission mechanism, we further consider shocks to the interest rate reaction function

$$R_t^m = \left(R_{t-1}^m\right)^\rho \left(R^m\right)^{1-\rho} \left(\pi_t/\pi\right)^{\rho_\pi(1-\rho)} \exp \varepsilon_t^\rho. \tag{23}$$

where $\rho \geq 0$ and ε_t^ρ is normally i.i.d. with $E_{t-1}\varepsilon_t^\rho = 0$ and variance $var_{\varepsilon^\rho} \geq 0$. The long-run repo rate, $R^m > 1$, and the target inflation rate, $\pi > \beta$, can be chosen by the central bank. Alternatively, we will also assume that the central bank controls the growth rate of money.

In contrast to (standard) models, where repurchase agreements are not considered, the central bank has an additional role: It can decide on whether money is traded in form of outright sales/purchases or in form of repurchase agreements. For simplicity, we assume that it controls the ratio of money supply under both types of open market operations Ω:

$$M_t^R = \Omega \cdot M_t^H,$$

or $M_t^R = M_t \frac{\Omega}{1+\Omega}$, where $\Omega \geq 0$ and M_t is the total money supply, $M_t = M_t^H + M_t^R$.

Finally, substituting out central bank transfers in the government budget constraint shows that the government transfers revenues from debt issuance and central bank profits to the households: $P_t \tau_t = \left(B_t^T/R_t\right) - B_{t-1}^T + B_t^c\left(1 - 1/R_t\right)$.

3.4 Rational expectations equilibrium

In equilibrium, there will be no arbitrage opportunities and markets clear, $n_t = \int_0^1 n_{jt} dj = \int_0^1 n_{it} di$ and $y_t = \int_0^1 y_{jt} dj = \int_0^1 c_{it} di = c_t$. Households will not behave differently and aggregate asset holdings satisfy $\forall t \geq 0 : \int D_{i,t} di = 0$,

$$\int M_{i,t}^H di = \int \widetilde{M}_{i,t} di = M_t^H, \qquad \int M_{i,t}^R di = M_t^R, \qquad \int B_{i,t} di = B_t,$$
$$\int I_{i,t} di = I_t = M_t^H - M_{t-1}^H + M_t^R, \qquad B_t^T = B_t + B_t^c.$$

Since government bonds are the single eligible security, its distribution between the central bank and the private sector will matter. Given that the government issues bonds according to a constant growth rate Γ, household bond holdings change according to $B_t - B_{t-1} = (\Gamma - 1)B_{t-1}^T - B_t^c + B_{t-1}^c$. Further using (22), the evolution of bonds held by households satisfies

$$B_t - B_{t-1} = (\Gamma - 1)B_{t-1}^T - R_t^m \left(M_t^H - M_{t-1}^H + M_t^R\right) + M_t^R. \tag{24}$$

Thus, private sector holdings of bonds tend to decrease with a higher price of money R^m and to increase with Γ. For a given injection I_t households further loose less bonds when the fraction of money held under repurchase agreements increases.

Throughout, we will focus on the case where the central bank sets its instrument such that the goods market constraint (8) is strictly binding ($\psi_t > 0$).[8] A rational expectations equilibrium can then be defined as follows:

A rational expectations equilibrium is a set of sequences $\{c_t,\ n_t,\ y_t,\ w_t,\ m_t,\ b_t,\ b_t^T,\ R_t^m,\ R_t^d,\ R_t,\ P_t\}_{t=0}^{\infty}$ satisfying the firms' first order conditions and the production technology, the households' first order conditions (11)-(16) and the transversality condition, the binding goods market constraint $P_t c_t = M_t^H + M_t^R$, the open market constraint

$$\frac{b_{t-1}}{R_t^m \pi_t} \geq m_t^R + m_t^H - m_{t-1}^H \pi_t^{-1},$$

and $b_t - b_{t-1}\pi_t^{-1} = (\Gamma - 1) b_{t-1}^T \pi_t^{-1} - R_t^m \left(m_t^H - m_{t-1}^H \pi_t^{-1}\right) - (R_t^m - 1)\, m_t^R$, for $\Gamma = b_t^T \pi_t / b_{t-1}^T$, for a monetary policy satisfying (23) and initial values $M_{-1} \geq 0$, $B_{-1} > 0$, and $P_{-1} > 0$.

Note that under a non-binding open market constraint, $b_{t-1}/\pi_t > R_t^m \left(m_t^R + m_t^H - m_{t-1}^H \pi_t^{-1}\right)$, the evolution of government bonds will neither affect the equilibrium allocation nor the associated price system. If however the open market constraint is binding, $b_{t-1}/\left(R_t^m \pi_t\right) = m_t^R + m_t^H - m_{t-1}^H \pi_t^{-1}$, household bond holdings matter and (24) reduces to $B_t = (\Gamma - 1)\, B_{t-1}^T + M_t^R$.

3.5 Steady state

In the following analysis, the two cases of a binding and a non-binding open market constraint (7) will be treated separately, which facilitates analyzing the mechanisms that are responsible for the main results.[9] Throughout the analysis, we are particularly interested in the case where the money market constraint is binding. For this we assume that the central bank conducts monetary policy in a way that ensures the rate of return on government bonds to be lower on average than the rate of return on private debt in equilibrium. Households then tend to economize on bond holdings, i.e. they will not hold more government bonds than necessary for their money market trades. If however both returns are identical, households can borrow and invest in government bonds without costs such that the money market constraint will not be binding.[10] As seen from Figure 1, the policy rate has almost always been below the implied Euler rate, which corresponds to the binding money market constraint case.

In order to analyze the two regimes in a separate way, we first examine steady states with a binding and a non-binding open market constraint. We then assume that monetary policy is conducted in a way to implement one particular steady state and that aggregate shocks are sufficiently small, so that we can analyze the properties of the economy in the neighborhood

[8] In the long-run, this is ensured by the nominal interest rate R being larger than one.

[9] The set of equilibrium conditions for both cases can be found in the appendix 8.2.

[10] Likewise, if the central bank simply declares both assets as eligible for open market operations, the private sector can freely create any amount of private debt that can be used in exchange for money, such that the private sector never runs out of eligible securities.

of this steady state. A steady state value of an endogenous variable x_t will not carry a time index, x.

To examine the two cases, we combine (14), (15), and (19), to give the following steady state condition

$$\eta/\lambda = \left(R^d - R\right)/R. \tag{25}$$

The spread between the debt rate R^d and the bond rate R thus determines if the multiplier on the open market constraint is positive $\eta > 0$, which indicates a binding open market constraint.

Before examining the differences between both steady states, we look at common properties. Throughout the paper, we assume that the central bank successfully implements its inflation target π in the long-run. Hence, the steady state Euler rate is as usual determined by (14) and (19), $R^d = \pi/\beta$. In the steady state, consumption is then given by

$$c^{\sigma+1/\alpha-1} = \frac{1}{R^d}\frac{\varepsilon-1}{\varepsilon}\frac{\alpha}{\gamma}$$

where we used (17), (20), and the $c = n^a$. Real balances are then determined by: $m = c$, $m = m^H + m^R$, and $m^R = \Omega m^H$. Thus, for a fixed inflation target π, the steady state values R^d, c, m, m^h and m^R are independent of η, i.e. do not depend on the tightness of the money market constraint. The latter only matters for the steady state values of the bond rate and debt.

i.) If the central bank sets the average repo-rate R^m equal to the debt rate R^d in a steady state, $R^m = \pi/\beta$, the interest rate on government bonds R also equals R^d, as can be seen from (16). The multiplier on the open market constraint will then be equal to zero $\eta = 0$ (see 25) and the steady state is characterized by $R = R^d = R^m$, while bonds are neutral.

ii.) If however the central bank chooses an average repo-rate R^m that is strictly smaller than R^d, which requires $R^m < \pi/\beta$, there exists a steady state with a binding open market constraint, $\eta > 0$ (see 25) satisfying $R = R^m$,

$$\frac{b}{R^m\pi} = m^H\left(1-\pi^{-1}\right) + m^R, \tag{26}$$

and $b\left(1-\Gamma\pi^{-1}\right) = m^R\left(1-\Gamma\pi^{-1}\right)$.

In the case *ii.*), the condition $b\left(1-\Gamma\pi^{-1}\right) = m^R\left(1-\Gamma\pi^{-1}\right)$ together with (26) would only be consistent with $\Gamma \neq \pi$ and $M_t^H \geq 0$ for deflationary equilibria, thus we restrict our attention to the case where the growth rate of bonds equals the steady state inflation rate $\Gamma = \pi$. For this, we assume that the central bank chooses its inflation target and eventually adjusts the set of eligible assets if the growth rate of bonds differs from the inflation target, which is not considered in this paper.

If, for example, $\Gamma < \pi$, the central bank might accept also a fraction of private debt in open market operations. If $\Gamma > \pi$, it might accept only a fraction of government bonds in open market operations. Thus, by deciding on the set of eligible securities, the central bank actually decides on the maximum amount of money that can be traded in open market operations.

4 Interest rates and spreads

In this section, we examine the relation between the three interest rates, i.e., the repo or policy rate R^m, the bond rate R, and the debt rate R^d. The bond rate R_t and the repo rate R_t^m are closely related to each other as can be seen from (16). The spread between these two rates, i.e. a risk premium, will be examined below. Before, we will take a look at the spread between the debt rate R_t^d and the bond rate R_t, i.e. a liquidity premium.

For the analysis of both spreads we will use simple versions of the model, to facilitate the derivation of analytical results. Throughout this section, we assume that production is linear $\alpha = 1$, the production sector is perfectly competitive $\epsilon \to \infty$, and that prices are perfectly flexible $\phi = 0$. We further simplify public policy by assuming that money is only supplied under repurchase agreements $\Omega \to \infty$ and that the supply of government bonds is constant $\Gamma = 1$.

4.1 The liquidity premium

Households are willing to hold government bonds even if the bond rate is lower than the debt rate, since bonds exhibit an additional liquidity value. Due to lower interest earnings, households will economize on bond holdings such that the money market constraint is binding. This property has already been used for the steady state analysis (see 25). The central bank can implement a long-run equilibrium with a binding money market constraint if the repo rate R^m is set at a value lower than $R^d = \pi/\beta$. Outside the steady state, the debt-bond rate spread will not be constant over time and will in particular depend on the monetary policy stance, since the value of liquidity will depend on the money market conditions.

To facilitate the analysis of the liquidity premium, we focus on the case of an exogenous interest rate policy $\rho_\pi = 0$. Since the current bond rate is affected by tomorrow's repo rate rather than today's repo rate, we further assume that the repo rate sequence exhibits inertia $\rho > 0$.[11] A rise in the repo rate R_t^m, i.e. in the relative price of money, has two immediate effects. It reduces nominal consumption for a given stock of household bond holdings B_{t-1}

[11] Under perfectly flexible prices both rates, R^d and R, will be constant if $\rho = 0$. This will not be the case under sticky prices (see section 5).

(see 7 and 8). It further leads to lower end-of-period nominal bond holdings B_t (see 24, which in the simplified version reads $B_t = B_{t-1}/R_t^m$). Thus, both effects tend to reduce inflation. Since the repo rate is raised in an inertial way, inflation is also expected to be lower in the following period, thus households demand a lower debt rate R_t^d (see 14).

The effects can easily be shown for the case where utility is logarithmic in consumption $\sigma = 1$. The results are summarized in the following proposition.

Proposition 1 *Consider the case, where $\sigma = 1$ and the repo rate satisfies $\rho_\pi = 0$ and $R^m < 1/\beta$, such that the open market constraint is binding. The debt rate R_t^d and the ratio R_t^d/R_t decrease with i.) the current level of the repo rate if $\rho > 0$ and ii.) with the variance of repo rate innovations ε_t^ρ.*

Proof. See appendix 8.3. ■

The spread, i.e., the liquidity premium, depends on the ability of bonds to be convertible into means of payments in open market operations before the goods market opens. If these costs of exchanging bonds against money R_t^m are high or more uncertain, the liquidity value of bonds and thus the liquidity premium declines (see 18).

According to the standard Fischer effect, the debt rate falls in response to an increase in the repo rate, when inflation is expected be to lower than average inflation rate in the subsequent period, which requires $\rho > 0$. It should be noted this inflation response is also responsible for an increase in consumption, since the inflation tax on cash goods is then lowered (see 17). This counterfactual consumption response will disappear when prices are assumed to be imperfectly flexible (see section 5).

4.2 The risk premium

As discussed in the previous section, the interest rates on bonds and debt only differ when the open market constraint is binding. In contrast, there can be a spread between the repo rate and the bond rate, regardless whether the open market constraint is binding or not. This can be seen from the household optimality condition (16), which can by using (11) and (12) be rewritten as

$$1/R_t = E_t\left(1/R_{t+1}^m\right) + \frac{cov_t\left[\left(1/R_{t+1}^m\right), \left(u_{ct+1}/\pi_{t+1}\right)\right]}{E_t\left[u_{ct+1}/\pi_{t+1}\right]} \tag{27}$$

Households are willing to hold both, money and bonds, if the rate of return on bonds compensates for the costs of converting bonds against money in next period's open market operations. Up to first order, the current bond price $1/R_t$ in the asset market equals the expected future money-price of bonds in open market operations $E_t\left(1/R_{t+1}^m\right)$. However, the price of a government bond $1/R_t$ will be smaller than $E_t\left(1/R_{t+1}^m\right)$, if the covariance on the RHS of (27) is negative, i.e., if the real repo rate R_{t+1}^m is positively related to the marginal utility of consumption divided by the inflation rate, u_{ct+1}/π_{t+1}.

The spread between the bond rate and the repo rate then tends to be positive and increases with the measure of relative risk aversion σ. It can therefore be interpreted as a risk premium on the nominal rate of return on bonds compared to the expected repo rate. A risk-averse agent who considers investing in bonds in the asset market will ask for a price $1/R_t$ that is lower than the money-price of bonds in next period's open market market, if a lower real repo rate (and thus a higher real pay-off from bonds) is associated with higher consumption.

To establish this result, we again apply a simplified version ($\alpha = \Gamma = 1$, $\phi = 0$, $\epsilon \to \infty$, and $\Omega \to \infty$). We now allow for varying degrees of relative risk aversion, $\sigma > 1$, and we focus on i.i.d. technology shocks, $\rho^a = 0$, as the only source of aggregate uncertainty, such that $var_{\varepsilon^\rho} = 0$, while the repo rate will endogenously be adjusted according to $\rho_\pi > 0$ and $\rho = 0$. The following proposition summarizes the main results.

Proposition 2 *Consider the case where $\sigma > 1$ and $\rho^a = 0$, while the repo rate satisfies $\rho_\pi > 0$, $\rho = 0$ and $R^m < 1/\beta$, such that the open market constraint is binding. The current price of government bonds is smaller than the expected future money-price of bonds $1/R_t < E_t\left(1/R^m_{t+1}\right)$. The average bond rate R_t further increases with the households' relative risk aversion and with the variance of productivity shocks.*

Proof. See appendix 8.4. ■

The covariance term in (27) is strictly negative under a binding open market constraint, where a higher repo rate tends to reduce current consumption times inflation $b_{t-1}/R^m_t = c_t\pi_t$. Hence, the bond rate tends to exceed the repo rate and further increases for a given repo rate, if aggregate uncertainty, $var(\varepsilon^a)$ or the relative risk aversion σ increases. In both cases households only want to invest in bonds at a higher rate of return.

5 Numerical analysis

In this section we apply a numerical analysis of a less simplistic model, using a second order approximation at the deterministic steady state (see Schmitt-Grohé and Uribe, 2004) and standard parameter values as far as possible (see table A1 in appendix 8.5).[12] In the first part, we re-examine the behavior of the interest rates. In the second part we look at the transmission of monetary policy shocks.

To allow for more realistic dynamics we now use a sticky price version of the model. For most of the model's parameter we apply standard values for quarterly data, namely, $\sigma = 2$, $\alpha = 0.66$, $\phi = 0.8$, $\rho^{(a)} = 0.9$, and $\epsilon = 6$. To match the average interest rate values found in the data, we apply an inflation rate of $\pi = 1.0108$ (for an annual rate of 4.4%, see section 2), a low discount factor $\beta = 0.984$, and a target repo rate equal to $R^m = 1.015$, leading to a steady state spread $R^d - R$ equal to 120 basis points per quarter and a spread of 503 basis

[12] For the computation we used dynare.

points per year. We further set the inflation feedback ρ_π either equal to zero or equal to 1.5.[13] Finally, we set the value for the ratio between repo-money and money supplied outright Ω equal to 0.5.

5.1 Interest rate behavior

In this section we again look at the relations between the interest rates, which have already been analysed qualitatively in section 4. To facilitate comparisons with the latter results, we present numerical results for only one type of shock.

Liquidity premium Table 2 presents values for the average spread between the debt rate and the bonds rate, $E_0 s_{1,t} = E_0 \left(R_t^d - R_t\right)$. Starting with a steady state value of 120 basis points, it decreases with larger variances of repo rate innovations ε_t^ρ, which accords to claim *i.)* in proposition 1. When the repo rate is endogenously adjusted ($\rho_\pi = 1.5$), such that its variance is not only directly affected by the innovations ε_t^ρ, this effect is less pronounced.

Table 2 Average spread $Es_{1,t}$ under interest rate shocks

	$var(\varepsilon^\rho) = 0.0001$	$var(\varepsilon^\rho) = 0.0005$	$var(\varepsilon^\rho) = 0.001$
$\rho_\pi = 0$	477 *b.p.*	342 *b.p.*	172 *b.p.*
$\rho_\pi = 1.5$	489 *b.p.*	400 *b.p.*	288 *b.p.*

Table 3 further presents the correlation between the debt rate and the repo rate as well as the correlations between the spread $s_{1,t}$ and the repo rate. The columns refer to only one type of shock. Both, the debt rate and the spread are found to be highly negatively correlated with the repo rate, while the correlations are slightly smaller under technology shocks. Overall, these finding supports claim *ii.)* made in proposition 1. The correlations of the spread further accord to the empirical results presented in section 2 and in other studies (see Atkeson and Kehoe, 2008, and Canzoneri et al., 2007).

Table 3 Unconditional correlations

	Interest rate shocks		Technology shocks
	$\rho_\pi = 0$	$\rho_\pi = 1.5$	$\rho_\pi = 1.5$
$\text{corr}(s_{1,t}, R_t^m)$	-0.996	-0.997	-0.861
$\text{corr}\left(R_t^d, R_t^m\right)$	-0.885	-0.907	-0.830

The model overstates the negative correlation between the debt rate and the repo rate compared to the numbers presented in the empirical analysis (see section 2). Nevertheless, we can conclude that the debt rate hardly mimics the policy rate in all cases.

[13] In contrast to standard sticky price models a passive interest rate policy does not give rise to local equilibrium indeterminacy when the money market constraint is binding. The reason is that nominal debt serves a nominal anchor like a constant money supply. A local determinacy analysis for a simplified model version can be found in Schabert (2004).

Risk premium We now turn to the spread between the repo rate and the bond rate. Applying the parameter values from above (with $\rho_\pi = 1.5$, see table A1), we find small positive numbers for the spread $s_{2,t} = R_t - R_t^m$. As shown in table 4, they lie in between 0.3 basis-points and 1.5 basis-points, where the latter is obtained for a high variance of the technology shock $var(\varepsilon^a)$.[14]

Table 4 Spread $E_0 s_{2,t}$ for technology shocks

	$\sigma = 2$	$\sigma = 5$
$var(\varepsilon^a) = 0.01$	0.34 *b.p.*	0.75 *b.p.*
$var(\varepsilon^a) = 0.02$	0.68 *b.p.*	1.51 *b.p.*

The results for different values for σ and for $var(\varepsilon^a)$ support the claims made in the second part of proposition 2. Overall, the model is able to generate a positive spread between the policy rate and the bond rate. Evidently, the average spread between the federal funds rate and the 3-month Libor presented above (25 b.p.) is much larger than the model's predictions.

5.2 Monetary transmission

In this section we examine responses to repo rate innovations and money supply shocks to disclose the monetary transmission mechanism in our model. Throughout the analysis we report results for the case where the open market constraint is binding, unless the opposite is explicitly mentioned.

5.2.1 Responses to interest rate shocks

Consider a positive innovation to the repo rate satisfying (23) with $\rho = 0.9$. Figure 3 presents the impulse responses of interest rates and macroeconomic aggregates for the case where the repo rate is exogenously set (blue solid line: $\rho_\pi = 0$) and for the case where it follows a Taylor type feedback rule ($\rho_\pi = 1.5$, green marked line). An increase of the repo rate by 1% from its steady state value leads to a rise in the bonds rate by less than one percent, which accords to (27). The debt rate decreases on impact and is closely followed by the rate R-Euler, which is the rate implied by a standard Euler equation, $\beta E_t [u_{c,t+1}/(u_{c,t}\pi_{t+1})] = 1/R_t^{Euler}$; the latter has no meaningful role in our model and is only computed to facilitate comparisons (see section 2). The spread between the debt rate and the bond rate decreases, as predicted in proposition 1. The impact response of the spread almost equals the size of its steady state value.[15]

[14] Note that the variances are small enough so that the multiplier on the open market constraint remains positive in response to a standard deviation productivity innovation.

[15] The multiplier on the open market constraint is thus strictly positive after the interest rate shock.

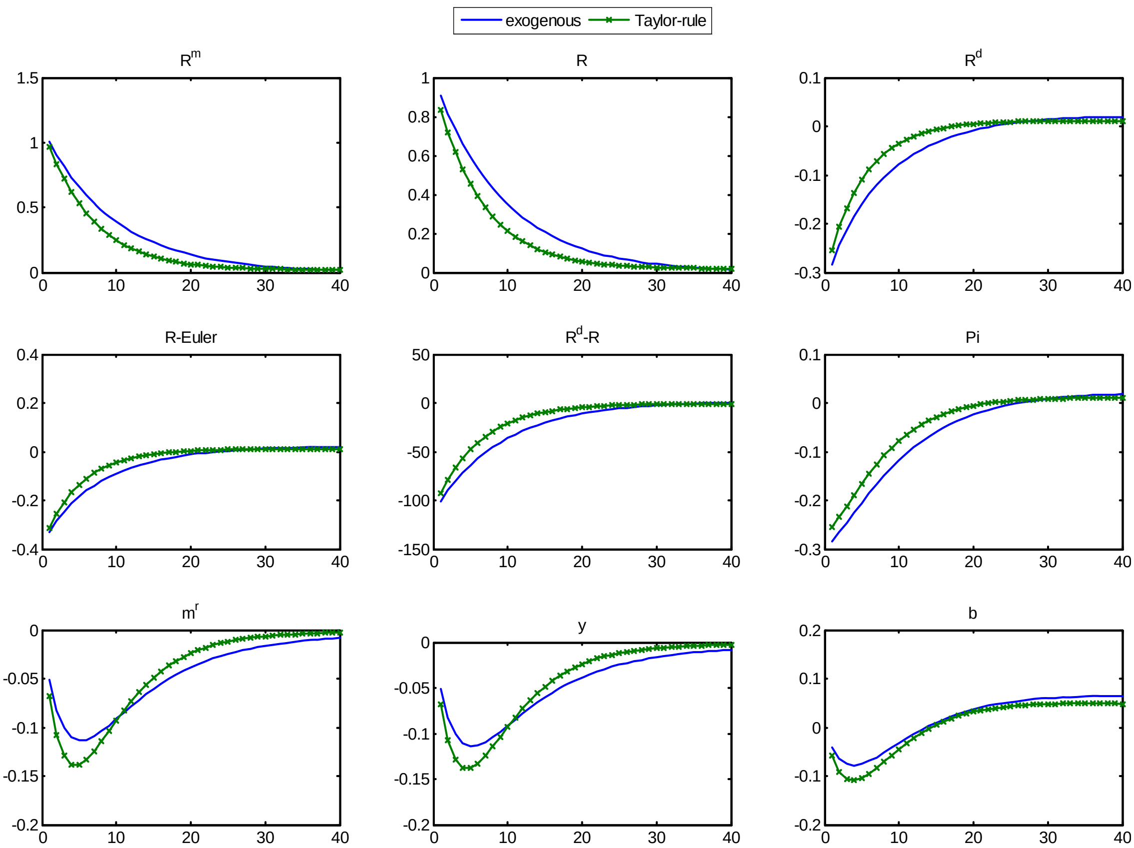

Figure 3: Responses (in % dev. from st.st.) to an interest rate shock

Regarding the responses of macroeconomic aggregates, figure 3 further shows that output decline in a hump-shaped way, which is qualitatively consistent with standard VAR evidence. Hump-shape impulse responses are usually not generated by simple sticky price models (like the version of our model without the money market constraint). However, hump-shaped impulse responses, which are also found in the data, can also be generated by considering additional frictions or rigidities (see Christiano et al., 2005). Here, the shape of the consumption response is affected by the dynamics of households' real bond holdings b_t^H, which falls in response to the monetary contraction.

On the one hand, the real value of government bonds should increase, since inflation falls. Yet, the amount of bonds held by the central bank tends to rise by higher interest rates and by less repo money (see 22). Thus, a monetary tightening does not only lead to contractionary effects on impact, but subsequently shifts the distribution of bond holdings towards the central bank. With depleting eligible securities, households can acquire less money in the subsequent periods, such that the initial contraction in consumption will even be enhanced. Then the inflation effects starts dominating the interest rate effect and real bonds return to their steady-state value. Thus, the dynamics of bond holdings affects the transmission of monetary policy shocks, which relies on the assumption that the central banks does not transfer its wealth to the household at the end of each period. Modeling central bank operations can thus help explaining persistence to monetary policy effects. Even after the policy rate has decreased substantially after its positive impact, for example after 15 periods in this simulation, output is still at the level it was just after the shock.

For a smaller fraction of repo money, $\Omega = M_t^R/M_t^H$, the impact of an interest rate shock, in particular, the responses of the macroeconomic aggregates, are less pronounced. The impulse responses to interest rate shocks for $\Omega = 0.1$ are given in the appendix. Thus, our model predicts that the size of interest rate shock effects depends on the way the central bank conducts open market operations. The impact of an increased policy rate is less pronounced if consumption is based more on money holdings than on bonds discounted in open market operations.

Figure 4 shows impulse responses to a one percent repo rate innovation for a version of the model where the money market constraint is not binding.[16] Since the stock of government

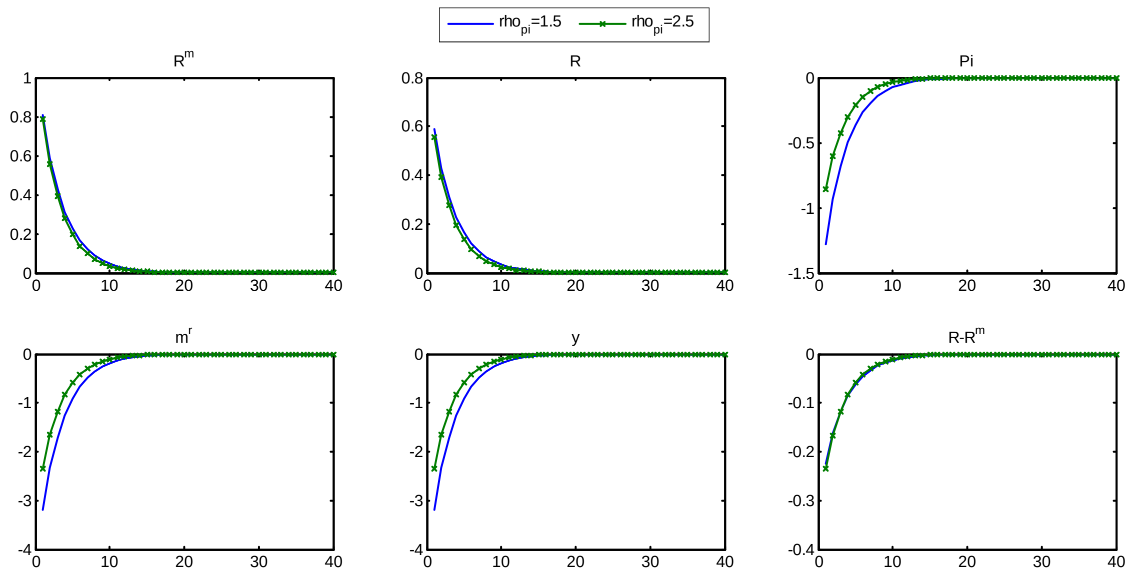

Figure 4: Responses (in % dev. from st.st.) to an interest rate shock for a non-binding money market constraint

bonds is now irrelevant and the bond rate and the debt rate are identical, the responses of the latter, the spread $s_{1,t}$, and of bonds are not presented. The inflation feedback of the policy rule is set equal to $\rho_\pi = 1.5$ (blue solid line) and $\rho_\pi = 2.5$, since determinacy now requires the Taylor-principle. The effects of the same policy shock on inflation and on output

[16] For this version of the model we used a standard value for the subjective discount factor ($\beta = 0.99$).

are much more pronounced than for a binding money market constraint. Here, the output response does not exhibit a hump shape, since the distribution of asset holdings is irrelevant.

5.2.2 Money supply shocks and liquidity effects

This section reports the effects of money injections modelled by innovations to the growth rate of money holdings M^H, i.e.

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + \varepsilon_t^\mu, \qquad \mu_t = M_t^H / M_{t-1}^H,$$

where $\mu = \pi$ and ε_t^μ is normally i.i.d. with $E_{t-1}\varepsilon_t^\mu = 0$ and $\rho \geq 0$. We assume that the amount of money supplied under repurchase agreements satisfies $M_t^R = M_t \frac{\Omega}{1+\Omega}$, as before. Figure 5 shows impulse responses to a positive one percent deviation from the steady state money growth rate for different degrees of serial correlation, $\rho^\mu = 0.5$ and $\rho^\mu = 0.9$.

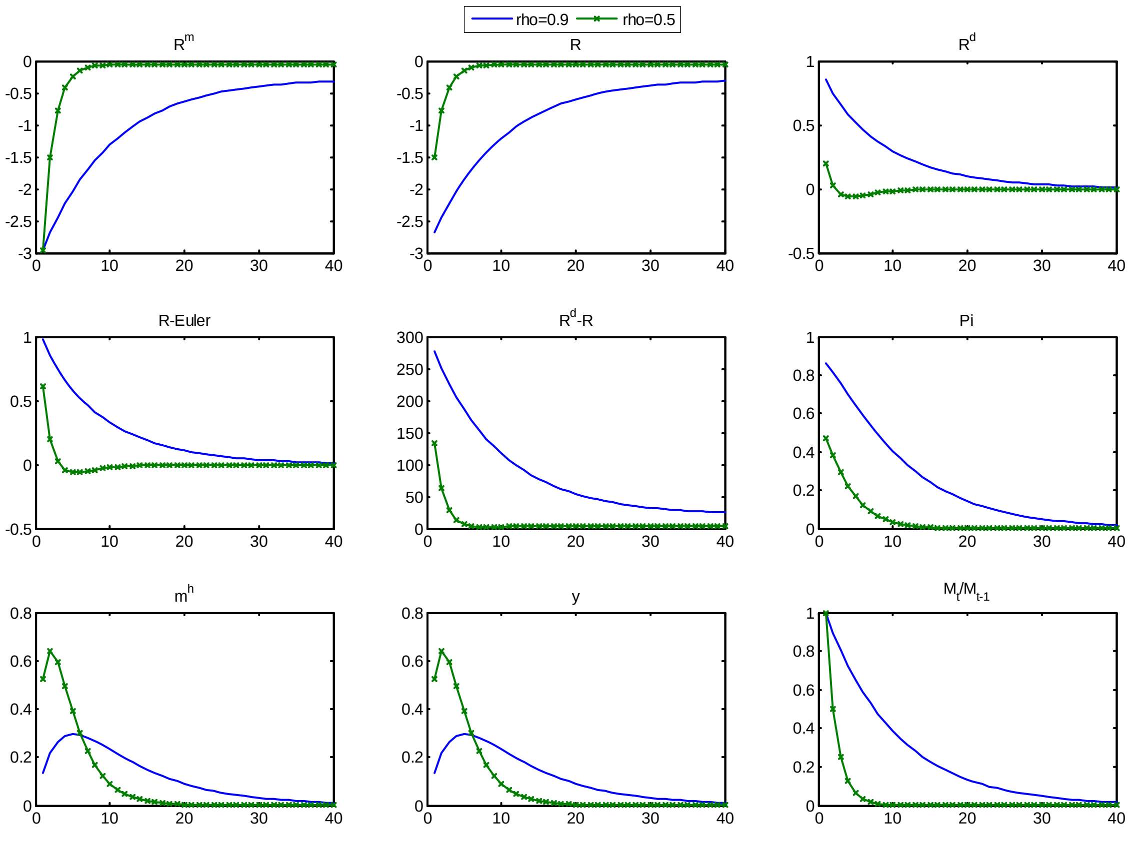

Figure 5: Responses (in % dev. from st.st.) to a money supply shock

A shock to the money growth rate leads to a decline in the repo rate R_t^m and in the closely linked to the bonds rate R_t. The simple reason is that a binding open market constraint implies a negative relation between money injections and the repo rate for a given level of nominal debt, $B_{t-1}/R_t^m = I_t$. Thus, both rates exhibit an unambiguous liquidity effect. At the same time the debt rate R_t^d (as well as the standard Euler-rate R_t^{Euler}) is rising in accordance with to the usual anticipated inflation effect. The spread between the debt rate and the bond rate $R_t^d - R_t$ increases. Thus, the model again predicts a negative relation between the repo rate and the debt rate and between the repo rate and the liquidity premium. Like in the case of interest rate shocks, output displays a hump-shaped impulse response function.

When the constraint is not binding, the impulse responses are closely related to the responses of standard models, as displayed in figure 6, except for the small difference between the repo rate and the bond rates. The impact on inflation and output is identical to the case of

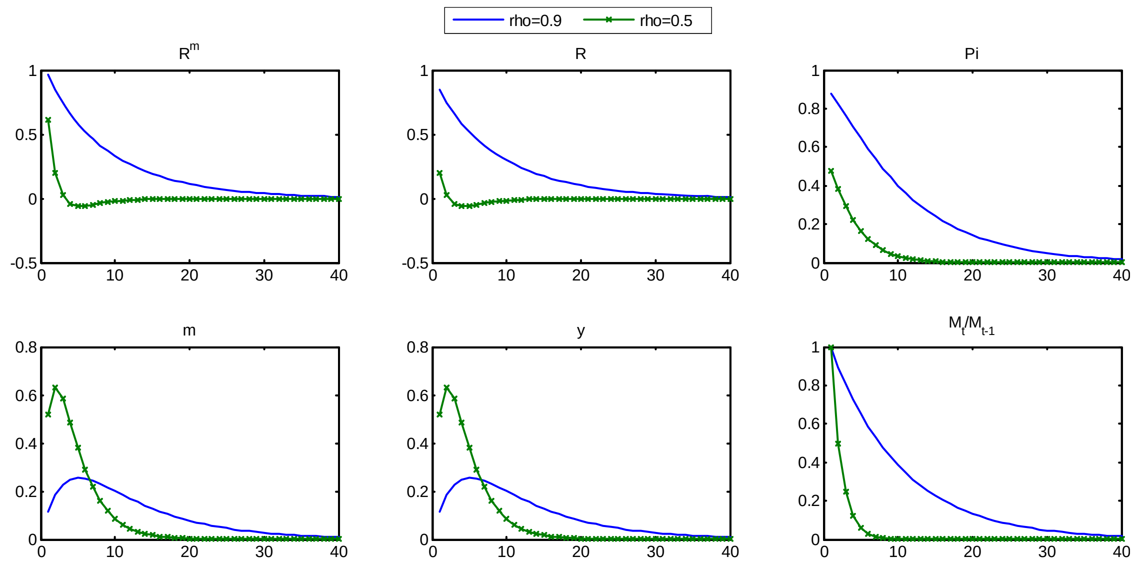

Figure 6: Responses (in % dev. from st.st.) to a money supply shock for a non-binding money market constraint

a binding open market constraint, while the responses of the interest rate substantially differ. The repo rate R_t^m and the bond rate R_t, which equals the debt rate $R_t = R_t^d$, unambiguously rise in response to a money growth shock with a high autocorrelation. If the serial correlation of the money growth rate is smaller, both rates first rise and then fall below their steady state values. Thus, in both cases the model does not generate clear liquidity effects, like in most standard sticky price models (see Christiano et al., 1997).

Productivity shocks. Finally, we look at impulse responses to a 1% innovation to the technology parameter, which are shown in figure 7 for different degrees of price stickiness ($\phi = 0.8$ and $\phi = 0.7$). The increase in productivity leads, as usual, to a rise in output and to an immediate decline in inflation. Given that the Taylor-rule links the repo rate to the

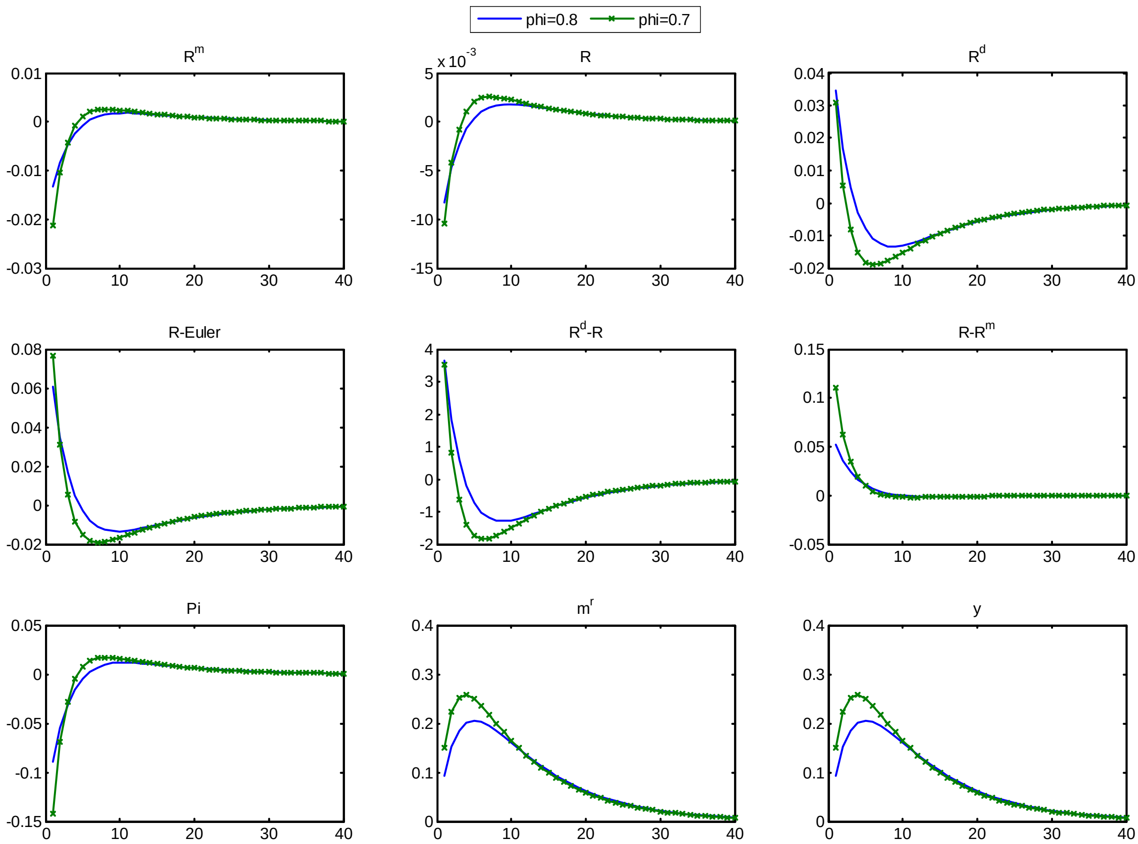

Figure 7: Responses (in % dev. from st.st.) to a 1% productivity shock

inflation rate, the real repo rate decreases. The response of the bond rate is similar, but less pronounced, such that spread s_2 between the bond rate and the repo rate also increases. The output reponse is again hump-shaped (due to the mechanism described above), implying that output growth and thus the real debt rate increase. Hence, the liquidity premium, i.e. the spread $s_1 = R^d - R$, also increases.

6 Conclusion

In this paper we have shown that modeling central bank operations can explain the wedge between the monetary policy interest rate and the Euler rate. Specifying money supply as exchange of asset can further lead to persistent effects of monetary policy and a hump-shaped consumption response to shocks. Contrary to why is claimed by standard models' builders, neglecting the money market is not without consequence for the variables of interest for monetary policy, as this affects the relationship between instruments and targets.

The framework presented in this paper can be used to address consequences of money market (e.g. liquidity) shocks and the appropriate monetary policy responses, and the influence of cental banks on the yield of various assets accepted in open market operations.

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8 Appendix

8.1 Computation of the Euler rate

In section 2, the empirical Euler interest rate r^d implied by our model has been computed as

$$\frac{1}{1+r_t^d}=\beta\exp\left[\begin{array}{l}-\sigma\left(E_t\log c_{t+1}-\log c_t\right)-E_t\log\pi_{t+1}-E_tr_{t+1}^m+r_t^m+\frac{\sigma^2}{2}var_t\log c_{t+1}+\frac{1}{2}var_t\log\pi_{t+1}\\+\frac{1}{2}var_tr_{t+1}^m+\sigma cov_t\left(\log c_{t+1},\log\pi_{t+1}\right)+\sigma cov_t\left(\log c_{t+1},r_{t+1}^m\right)+cov_t\left(\log\pi_{t+1},r_{t+1}^m\right)\end{array}\right].$$

8.2 Equilibrium conditions

A rational expectations equilibrium for a binding money market constraint and a binding goods market constraint is a set of sequences $\{c_t,n_t,y_t,w_t,m_t^R,m_t^H,mc_t,R_t^m,R_t^d,R_t,b_t,\pi_t\}_{t=0}^{\infty}$ satisfying

$$m_t^R+m_t^H=c_t, \tag{28}$$

$$m_t^R=\Omega m_t^H, \tag{29}$$

$$\frac{b_{t-1}}{R_t^m\pi_t}=m_t^R+m_t^H-m_{t-1}^H\pi_t^{-1}, \tag{30}$$

$$\beta E_t\frac{u_{ct+1}}{\pi_{t+1}}=\frac{-u_{nt}}{w_t}, \tag{31}$$

$$w_t=mc_t\alpha y_t/n_t, \tag{32}$$

$$1/\beta=R_t^dE_t\frac{-u_{nt+1}\left(n_{t+1}\right)/w_{t+1}}{-u_{nt}\left(n_t\right)/w_t}\pi_{t+1}^{-1}, \tag{33}$$

$$R_t=\frac{E_tu_{ct+1}\left(c_{t+1}\right)\pi_{t+1}^{-1}}{E_t\left(R_{t+1}^m\right)^{-1}u_{ct+1}\left(c_t\right)\pi_{t+1}^{-1}}, \tag{34}$$

$$y_t=a_tn_t^{\alpha}, \tag{35}$$

and either $mc_t=\frac{\varepsilon-1}{\varepsilon}$ and $y_t=c_t$ for flexible prices or (21) with $\widetilde{P}_{jt}=\widetilde{P}_t$, and $P_t^{1-\epsilon}=\phi\left(P_{t-1}\right)^{1-\epsilon}+(1-\phi)\widetilde{P}_t^{1-\epsilon}$, $y_t=(P_t^*/P_t)^{\epsilon}n_t^{\alpha}$, where $(P_t^*)^{-\epsilon}=\phi\left(P_{t-1}^*\right)^{-\epsilon}+(1-\phi)\widetilde{P}_t^{-\epsilon}$ for sticky prices, and a sequence for household's bond holdings satisfying

$$b_t-b_{t-1}\pi_t^{-1}=(\Gamma-1)b_{t-1}^T\pi_t^{-1}-R_t^m\left(m_t^H-m_{t-1}^H\pi_t^{-1}\right)-\left(R_t^m-1\right)m_t^R, \tag{36}$$

$$b_t^T=\Gamma b_{t-1}^T\pi_t^{-1}, \tag{37}$$

the tvc's, and $\{a_t\}_{t=0}^{\infty}$, for a monetary policy (23) and initial asset endowments. For convenience, we neglect higher order terms of the aggregate supply constraint $\log\left(\pi_t/\pi\right)=\beta E_t\log\left(\pi_{t+1}/\pi\right)+\chi\log\left(mc_t/mc\right)$, where $\chi=(1-\phi)(1-\beta\phi)/\phi$ (for a detailed analysis of aggregate supply under sticky prices, see, e.g., the working paper version of Schmitt-Grohé and Uribe, 2007).

If the money market constraint is not binding, the sequence of bonds is irrelevant and the model can be reduced to a set of equilibrium sequences for $\{c_t, n_t, y_t, w_t, m_t, mc_t, R_t^m, R_t, \pi_t\}_{t=0}^{\infty}$ given by (32)-(35) $m_t = c_t$, $u_{ct} = R_t^m \frac{-u_{nt}}{w_t}$ and either $mc_t = \frac{\varepsilon-1}{\varepsilon}$ for flexible prices or $\log(\pi_t/\pi) = \beta E_t \log(\pi_{t+1}/\pi) + \chi \log(mc_t/mc)$ for sticky prices, the tvc's, and $\{a_t\}_{t=0}^{\infty}$, for a monetary policy (23) and initial values.

8.3 Proof of proposition 1

Applying the parameter restrictions $\phi = \rho_\pi = 0$, $\Gamma = \alpha = \sigma = 1$, $\rho > 0$, and $\Omega = \epsilon \to \infty$, the set of equilibrium conditions given in 8.2 can be reduced to

$$m_t^R = c_t, \quad \frac{b_{t-1}}{R_t^m \pi_t} = m_t^R, \quad \beta E_t \frac{c_{t+1}^{-1}}{\pi_{t+1}} = \frac{\gamma}{w_t}, \tag{38}$$

$$w_t = a_t, \quad y_t = a_t n_t, \quad y_t = c_t, \quad b_t - b_{t-1}\pi_t^{-1} = -\left(R_t^m - 1\right) m_t^R, \tag{39}$$

$$1/R_t^d = \beta E_t \frac{a_{t+1}}{a_t} \pi_{t+1}^{-1}, \quad R_t = E_t R_{t+1}^m, \tag{40}$$

and a monetary policy. Eliminating consumption, the real wage rate, and output, it can further be reduced to a system in b, π, R, and R^d satisfying (40)

$$\beta E_t \frac{R_{t+1}^m}{b_t} = \frac{\gamma}{a_t}, \quad b_t = \frac{1}{R_t^m \pi_t} b_{t-1} \tag{41}$$

and the policy rule $R_t^m = \left(R_{t-1}^m\right)^\rho \left(R^m\right)^{1-\rho} \exp \varepsilon_t^\rho$. Since this system is log-linear, and shocks are log-normally distributed, all variables are also log-normal. Thus, the two conditions in (41) can be written as

$$E_t \log R_{t+1}^m + (1/2) var_t(\log R_{t+1}^m) = \log b_t - \log a_t + \log \gamma / \log \beta$$
$$\log b_t = -\log R_t^m - \log \pi_t + \log b_{t-1}$$

where we used that $\log E_t R_{t+1}^m = E_t \log R_{t+1}^m + (1/2) var_t(\log R_{t+1}^m)$ and $var_t\left(x_{t+i}\right) = E_t var\left(x_{t+i}\right)$. Using the logged policy rule $\log R_t^m = \rho \log R_{t-1}^m + (1-\rho)\log R^m + \varepsilon_t$ and defining $\kappa = \log \gamma - \log \beta$, we get the solutions for real bonds and inflation

$$\log \pi_t = -(1+\rho)\log R_t^m + \log b_{t-1} - \log a_t - (1/2) var_t(\log R_{t+1}^m) - (1-\rho)\log R^m + \kappa \tag{42}$$

$$\log b_t = \rho \log R_t^m + \log a_t + (1-\rho)\log R^m + (1/2) var_t(\log R_{t+1}^m) - \kappa \tag{43}$$

To assess the spread between the debt rate and the bonds rate, we apply the conditions in (40), which can be combined to

$$R_t / R_t^d = \beta E_t R_{t+1}^m E_t \frac{a_t}{a_{t+1}\pi_{t+1}}$$

Taking logs and using $\log E_t R^m_{t+1} = E_t \log R^m_{t+1} + (1/2)var_t(\log R^m_{t+1})$, the ratio R^d_t/R_t can be written as

$$-\log(R^d_t/R_t) = \log a_t + \log\beta + E_t \log R^m_{t+1} + (1/2)var_t(\log R^m_{t+1}) + \log E_t[a^{-1}_{t+1}\pi^{-1}_{t+1}]$$

Rewriting the last term $\log E_t[a^{-1}_{t+1}\pi^{-1}_{t+1}]$ by using $\log E_t[a^{-1}_{t+1}\pi^{-1}_{t+1}] = E_t\left(-\log a_{t+1} - \log\pi_{t+1}\right)+$ $(1/2)var_t\left(-\log a_{t+1} - \log\pi_{t+1}\right)$ and $var_t\left(-\log a_{t+1} - \log\pi_{t+1}\right) = var_t(\log a_{t+1}) + var_t\left(\log\pi_{t+1}\right)+$ $2cov_t\left(\log a_{t+1}, \log\pi_{t+1}\right)$, we get

$$\begin{aligned}-\log(R^d_t/R_t) = &(1-\rho_a)\log a_t + \log\beta + \rho\log R^m_t + (1-\rho)\log R^m - E_t\log\pi_{t+1}\\ &+(1/2)var_t(\log R^m_{t+1}) + var_t(\log a_{t+1}) + var_t\left(\log\pi_{t+1}\right)\\ &+2cov_t\left(\log a_{t+1}, \log\pi_{t+1}\right)\end{aligned}$$

where we used $E_t\log a_{t+1} = \rho_a\log a_t$ and $E_t\log R^m_{t+1} = \rho\log R^m_t + (1-\rho)\log R^m$. Eliminating $E_t\log\pi_{t+1}$ with (42),

$$\begin{aligned}\log(R^d_t/R_t) = &-\log a_t - \rho\left(\rho+2\right)\log R^m_t + \log b_t - (1/2)E_t[var_{t+1}(\log R^m_{t+2})]\\ &-(1/2)var_t(\log R^m_{t+1}) - var_t(\log a_{t+1}) - var_t\left(\log\pi_{t+1}\right)\\ &-2cov_t\left(\log a_{t+1}, \log\pi_{t+1}\right) - 2\left(1-\rho\right)\log R^m + \kappa - \log\beta\end{aligned}$$

and further $\log b_t$ with (43), gives

$$\begin{aligned}\log(R^d_t/R_t) = &-\rho\left(1+\rho\right)\log R^m_t - (1/2)var_t(\log R^m_{t+2}) - \log\beta - (1-\rho)\log R^m\\ &-var_t(\log a_{t+1}) - var_t\left(\log\pi_{t+1}\right) - 2cov_t\left(\log a_{t+1}, \log\pi_{t+1}\right)\end{aligned}$$

Using that (42) implies $var_t\left(\log\pi_{t+1}\right) = (1+\rho)^2\, var_t\left(\log R^m_{t+1}\right) + var_t\left(\log a_{t+1}\right)$ as well as $cov_t\left(\log a_{t+1}, \log\pi_{t+1}\right) = -var_t\log a_{t+1}$, leads to

$$\begin{aligned}\log(R^d_t/R_t) = &-\rho\left(1+\rho\right)\log R^m_t - \log\beta - (1-\rho)\log R^m\\ &-(1/2)var_t(\log R^m_{t+2}) - (1+\rho)^2\, var_t\left(\log R^m_{t+1}\right)\end{aligned}$$

Using $var_t\left(\log R^m_{t+1}\right) = var\left(\varepsilon^\rho\right)$ and $var_t(\log R^m_{t+2}) = \left(1+\rho^2\right) var\left(\varepsilon^\rho\right)$, we get

$$\log\left(R^d_t/R_t\right) = -\rho\left(1+\rho\right)\log R^m_t - \left((1/2)\left(1+\rho^2\right) + (1+\rho)^2\right) var\left(\varepsilon^\rho\right) - \log\beta - (1-\rho)\log R^m \tag{44}$$

implying that the spread decreases with R^m_t and $var\left(\varepsilon^\rho\right)$, and requires $R^m < 1/\beta$. Further eliminating $\log R_t = \log E_t R^m_{t+1}$ using $\log E_t R^m_{t+1} = \rho\log R^m_t + (1-\rho)\log R^m + (1/2)var_t(\log R^m_{t+1})$, gives

$$\log R^d_t = -\rho^2\ln R^m_t - \left((1/2)\rho^2 + (1+\rho)^2\right) var\left(\varepsilon^\rho\right) - \log\beta \tag{45}$$

which together with (44) establish the claims made in the proposition. □

8.4 Proof of proposition 2

We want to establish the claims made in proposition 2. For $\phi = \rho^{(a)} = 0$, $\Gamma = \alpha = 1$, $\sigma > 1$, $\rho_\pi > 0$, $var_{\varepsilon^\rho} = 0$, and $\Omega = \epsilon \to \infty$, the model can be reduced to a system in b, c, π, R, and R^d satisfying (40),

$$\frac{b_{t-1}}{R_t^m \pi_t} = c_t, \quad \beta E_t \frac{u_{ct+1}}{\pi_{t+1}} = \frac{\gamma}{a_t}, \quad b_t = \frac{1}{R_t^m \pi_t} b_{t-1} \tag{46}$$

where $u_{ct} = c_t^{-\sigma}$, and a policy rule satisfying $R_t^m = R^m (\pi_t/\pi)^{\rho_\pi}$. Applying the latter and $u_{ct} = [b_{t-1}/ (R_t^m \pi_t)]^{-\sigma}$, the covariance on the RHS of (27) can easily be shown to satisfy

$$cov_t \left[\left(1/R_{t+1}^m\right), \left(u_{ct+1}/\pi_{t+1}\right) \right] = \left(R^m/\pi^{\rho_\pi}\right)^{\sigma-1} b_t^{-\sigma} cov_t \left[\pi_{t+1}^{-\rho_\pi}, \pi_{t+1}^{\sigma\rho_\pi+\sigma-1} \right] < 0$$

implying $1/R_t < E_t \left(1/R_{t+1}^m\right)$. In order to examine the impact of the relative risk aversion and of aggregate uncertainty on the bond rate, we derive the solutions for b_t and π_t. Eliminating the repo rate in (41), we get two conditions for the equilibrium sequences of b_t and π_t:

$$\begin{aligned} \log b_t &= \rho_\pi E_t \log \pi_{t+1} + \log a_t + (1/2)\rho_\pi^2 var_t(\log \pi_{t+1}) + \kappa_2 \\ (1+\rho_\pi) \log \pi_t &= -\log b_t + \log b_{t-1} + \kappa_3 \end{aligned}$$

where $\kappa_2 = -\log \gamma + \log \beta + \log R^m - \rho_\pi \log \pi$ and $\kappa_3 = -\log R^m + \rho_\pi \log \pi$. Since the model is log-linear, all variables will finally be log-normally distributed. We further know that the solutions can be written as

$$\begin{aligned} \log \pi_t &= \delta_{\pi b} \log b_{t-1} + \delta_{\pi a} \log a_t + \delta_{\pi v} var_t \left(\log a_{t+1}\right) + \delta_\pi \\ \log b_t &= \delta_{bb} \log b_{t-1} + \delta_{ba} \log a_t + \delta_{bv} var_t \left(\log a_{t+1}\right) + \delta_b \end{aligned}$$

where the $\delta' s$ are unknown constants. Inserting these solutions in (41), the unknown coefficients can easily be identified:

$$\log \pi_t = \frac{1}{1+\rho_\pi} \log b_{t-1} - \log a_t - \frac{(1/2)\rho_\pi^2}{1+\rho_\pi} var_t \left(\log a_{t+1}\right) + \kappa_5, \tag{47}$$

$$\log b_t = (1+\rho_\pi) \log a_t + (1/2)\rho_\pi^2 var_t \left(\log a_{t+1}\right) + \kappa_4, \tag{48}$$

where $\kappa_4 = \exp(\log \beta - \log \gamma + \frac{\log R^m - \rho_\pi \log \pi}{\rho_\pi + 1})$ and $\kappa_5 = \exp(-\frac{(\rho_\pi+2)(\log R^m - \rho_\pi \log \pi) + (\rho_\pi+1)(\log \beta - \log \gamma)}{(\rho_\pi+1)^2})$. We now want to solve for the bonds rate, which satisfies (16) or

$$R_t = \frac{E_t c_{t+1}^{-\sigma} \pi_{t+1}^{-1}}{E_t \left(R_{t+1}^m\right)^{-1} c_{t+1}^{-\sigma} \pi_{t+1}^{-1}}$$

Using the solutions for inflation and bonds (47)-(48), we get

$$\begin{aligned}
E_t\left(c_{t+1}^{-\sigma}/\pi_{t+1}\right) &= \left(R^m/\pi^{\rho_\pi}\right)^{\sigma} a_t^{-1} \left(\kappa_5\right)^{\sigma\rho_\pi+\sigma-1} \left(\kappa_4\right)^{-\frac{1}{1+\rho_\pi}} e^{(1/2)var_t(\log a_{t+1})\left((\sigma-1)\left(\sigma+2\sigma\rho_\pi+\sigma\rho_\pi^2-1\right)\right)} \\
E_t\left[\left(1/R_{t+1}^m\right)\left(c_{t+1}^{-\sigma}/\pi_{t+1}\right)\right] &= \left(R^m/\pi^{\rho_\pi}\right)^{\sigma-1} a_t^{-(1+\rho_\pi)} \kappa_4^{-1} \kappa_5^{(\rho_\pi+1)(\sigma-1)} e^{(1/2)\left(((\sigma-1)(1+\rho_\pi))^2-\sigma\rho_\pi^2\right)var_t(\log a_{t+1})}
\end{aligned}$$

The solution for the bond rate can thus be written as

$$R_t = a_t^{\rho_\pi} \cdot \exp\left[\rho_\pi\left(2\sigma-\rho_\pi+2\sigma\rho_\pi-2\right)(1/2)\, var_t\left(\log a_{t+1}\right)\right] \cdot \left(R^m\right)^{\frac{1}{\rho_\pi+1}} \pi^{-\frac{\rho_\pi}{\rho_\pi+1}}$$

Taking unconditional expectations (E_0) and using that $E_0 a_t^{\rho_\pi} = \exp \rho_\pi^2 (1/2) var_t\left(\log a_{t+1}\right) = \exp \rho_\pi^2 (1/2) var\left(\varepsilon_t^a\right)$ for $\rho^a = 0$, the mean of the bond rate is given by

$$E_0 R_t = \exp\left[\rho_\pi\left(\sigma\rho_\pi+\sigma-1\right) var\left(\varepsilon_t^a\right)\right] \cdot \left(R^m\right)^{\frac{1}{\rho_\pi+1}} \pi^{-\frac{\rho_\pi}{\rho_\pi+1}}$$

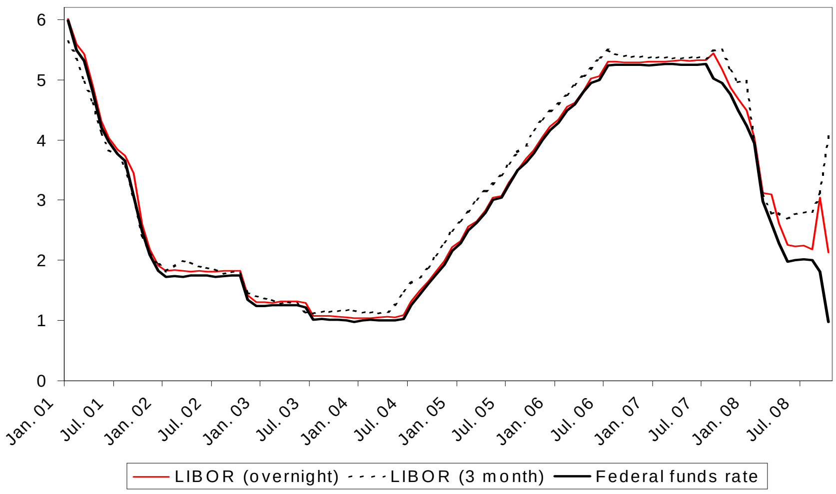
and thus increases with $var\left(\varepsilon_t^a\right)$ and σ. □

8.5 Parameter values

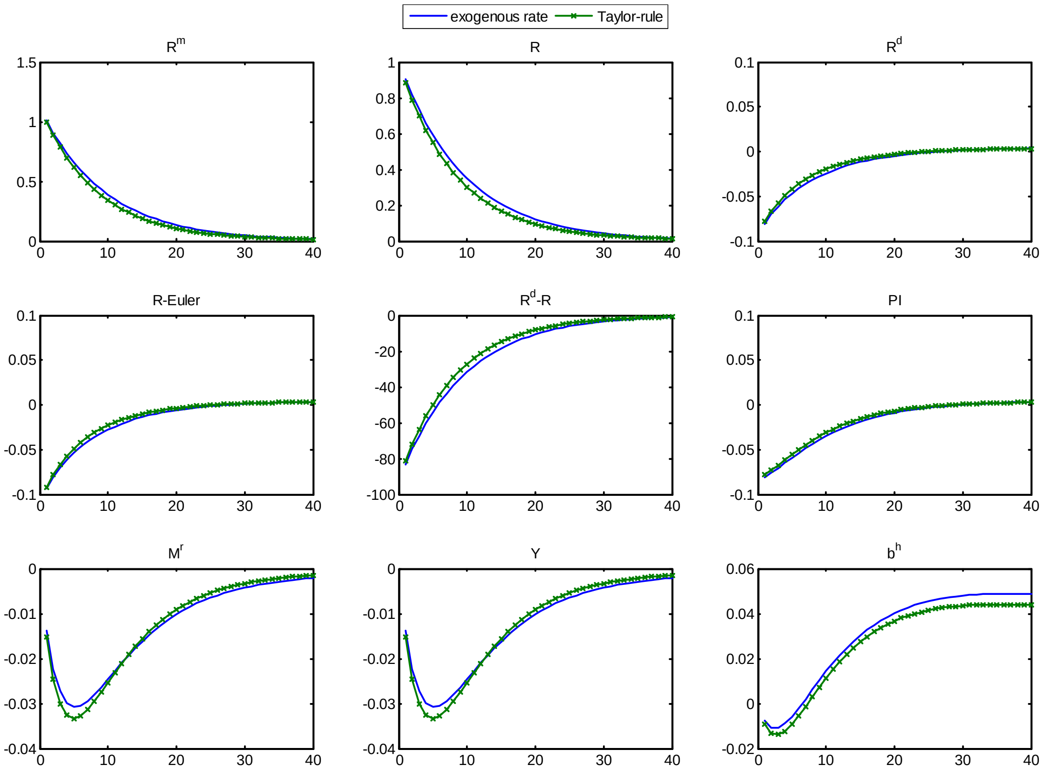
Table A1: Benchmark parameter values

β	γ	σ	$\Gamma=\pi$	mc	ϕ	α	s	Ω	ρ	ρ_π	$var(\varepsilon^\rho)$	$var(\varepsilon^a)$
0.984	2	2	1.0108	0.833	0.8	0.66	0.012	0.5	0.9	1.5	0.0001	0.0001

9 Additional figures



Federal funds and interbank rates



Responses (in % dev. from st.st.) to an interest rate shock for $\Omega = 0.1$