Debt Portfolios*

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Abstract

We provide a model with endogenous portfolios of secured and unsecured household debt. Secured debt is collateralized by owner-occupied housing whereas unsecured debt can be discharged according to bankruptcy regulations. We show that the calibrated model matches important quantitative characteristics of observed wealth and debt portfolios for prime-age consumers in the U.S. We then establish the quantitative result that home equity does not serve as informal collateral for unsecured debt since, as in the data, unsecured debtors hold small amounts of home equity in equilibrium. Thus, observed variations in homestead exemptions, which are an important part of U.S. bankruptcy regulation, have a small effect on the quantity and price of unsecured debt.

Keywords: household debt portfolios, housing, collateral, bankruptcy, commitment, income risk.

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1 Introduction

Household debt is sizeable and has increased substantially in the last decades in the U.S., the UK and most other European countries. The aggregate debt level hides substantial differences between debt types and their composition on the balance sheets of individual households. Most household debt is secured by housing collateral whereas some debt is unsecured and can be written off in bankruptcy procedures in the U.S., the UK and some, but not all, European countries.[1] Interestingly, portfolios of these debt types differ substantially across households (see Section 2). In this paper we present a model which allows for heterogeneity across households and generates such debt portfolios endogenously.

The key new feature in our model is housing which allows for a meaningful distinction between secured and unsecured debt and thus permits us to analyze debt portfolios. We obtain heterogeneity in debt portfolios by modeling consumer choices over the life cycle, assuming uncertain labor income and incomplete markets. Consumers then cannot fully insure the labor-income risk and hold different portfolios depending on their age and histories of shocks. Micro-founded heterogeneous-agent models with these characteristics have been pioneered by Deaton (1991), Aiyagari (1994) and Carroll (1997) and have attracted substantial attention in recent years.

We find that our calibrated model matches well important cross-sectional characteristics of wealth and debt portfolios in the U.S. Against this background, we establish the following quantitative results. Unsecured debtors hold small amounts of home equity in the equilibrium of the calibrated model, as in the data. Thus, home equity does not provide informal collateral for unsecured debt and hence does not mitigate the limited commitment problem resulting from the bankruptcy option. This result has important implications for bankruptcy regulation. Equilibrium pricing of unsecured debt and the associated portfolio choices imply that homestead exemptions, which are an important part of U.S. bankruptcy regulation, have a quantitatively small effect. Regulation could affect the quantity and price of unsecured debt only if the size of the exemptions is very small (less than a tenth of annual labor earnings). This finding is in contrast to results in Pavan (2008), who did not explicitly

[1] See Dynan and Kohn (2007), Tudela and Young (2005), Jentzsch and San José Riestra (2006) and their references for descriptive facts on consumer credit in these countries.

distinguish between secured and unsecured debt, but is consistent with the inconclusive empirical evidence on the effect of homestead exemptions on debt and bankruptcy incidence (see, for example, the survey in White, 2006).

Our paper relates to recent research by Athreya (2002), Chatterjee, Corbae, Nakajima and Ríos-Rull (2007) and Livshits, MacGee and Tertilt (2007) who have extended the classic heterogeneous-agent models to analyze unsecured debt. Importantly, these models assume that consumers only have access to unsecured debt. In this paper we relax this assumption and allow for an endogenous debt portfolio: consumers can take on secured debt such as mortgages, which are collateralized by housing, *and* unsecured debt like credit-card debt. To the best of our knowledge only Athreya (2006) attempts to distinguish secured and unsecured debt but does not model housing wealth. In his model the amount of collateral is exogenous whereas consumers in our model endogenously accumulate housing collateral which also generates utility. This modeling of durables like housing is closest to Fernández-Villaverde and Krueger (forthcoming), Kiyotaki, Michaelides and Nikolov (2011) and Yang (2009) who, however, do not allow for equilibrium bankruptcy and unsecured debt.[2]

The main contribution of our paper is that we explicitly model debt *portfolios*. The advantages of analyzing housing wealth, secured and unsecured debt simultaneously are at least threefold. The first advantage is that the model has an additional margin of substitution in the debt portfolio, between secured and unsecured debt. That margin adds realism by capturing the two types of debt which are relevant in consumer finance. The margin also gives households the choice to provide informal collateral for unsecured debt by holding home equity above the amount which is exempt in bankruptcy procedures. This is particularly important when one investigates the supply-side effects of the limited commitment introduced by the bankruptcy option. We will show how the joint analysis of housing wealth, secured and unsecured debt in our quantitative framework generates new insights for the effect of homestead exemptions on the equilibrium price and quantity of unsecured debt.

The second advantage is more realism in a key aspect of the analysis since most of consumers' total debt holdings in the data are secured by housing collateral. Thus, a quantitative model of household debt needs to explain not only the rather small unsecured debt

[2] See also Yao and Zhang (2005) for an analysis of housing and portfolio choice and Mankart and Rodano (2009) for an analysis of bankruptcy and entrepreneurship.

position but the whole debt portfolio. In our model the optimal choices for these portfolios are constrained by endogenous debt limits for unsecured and secured debt. This is also important for the predictions of the model concerning consumer bankruptcy because only unsecured debt can be discharged by filing for bankruptcy.

The third advantage is that the explicit modeling of housing introduces an endogenous bankruptcy cost which has been neglected in previous quantitative research. Upon seizure of owner-occupied housing, the home equity above the amount exempt in bankruptcy procedures is used to satisfy unsecured creditors' claims. Since adjusting the housing stock is costly, the cost of bankruptcy depends on the size of a consumer's housing wealth and secured debt.

The rest of this paper is structured as follows. In Section 2 we present empirical facts which are instructive for our analysis. We present the model in Section 3, followed by its numerical solution and calibration in Section 4. In Section 5 we analyze the role of home equity as informal collateral and the implications for bankruptcy regulation. We then conclude in Section 6.

2 Empirical facts

In this section we summarize the empirical facts on wealth and debt portfolios which are relevant for the quantitative application of our model. We then briefly review the key features of U.S. consumer bankruptcy regulation which the model shall capture.

2.1 Data

We use the Survey of Consumer Finances (SCF) to present facts on wealth and debt portfolios of U.S. consumers. The SCF has been widely used as it provides the most accurate information on consumer finances in the U.S. (see Kennickell, 2003, and the references therein). We focus on the SCF 2004 to calibrate our model since, for the purposes of our analysis, it is representative of the three waves of the SCF in the 2000s. We prefer the SCF 2004 to the most recent SCF 2007 for the calibration since the 2007 SCF wave contains data from interviews between May 2007 and March 2008 that are likely to be influenced by the start of the financial crisis.

We largely follow Budría Rodríguez, Díaz-Giménez, Quadrini and Ríos-Rull (2002) and Díaz-Giménez, Quadrini and Ríos-Rull (1997) in constructing measures for wealth and labor earnings in the U.S. We account for differences in household size using the equivalence scale reported in Fernández-Villaverde and Krueger (2007), Table 1, last column, with a weight of 1 for the first person in the household, 0.34 for the second person and approximately 0.3 for each additional member of the household. To make the empirical data comparable with the data generated by the model, we normalize all variables by average net labor earnings in the whole sample.[3] More precisely, we use SCF data on gross labor earnings and the NBER tax simulator described in Feenberg and Coutts (1993) to construct a measure for disposable labor earnings after taxes and transfers for each household. Arguably, after-tax rather than pre-tax earnings matter for households' consumption and portfolio decisions since some uninsurable labor earnings risk may be eliminated by redistributive taxes and transfers. More detailed information on the definition of the variables and the sample is contained in the data appendix.

We focus on empirical facts for prime-age households with a head between ages 26 and 55. As Livshits et al. (2007), our model abstracts from death before age 76 and therefore this is a good approximation of the data only up to a certain age. Life tables for the U.S. show that 90% of those born alive are still alive at age 55 and then have an average life expectancy of another 25 years (see the National Center of Health Statistics at http://www.cdc.gov/nchs/products/life_tables.htm). Allowing for a positive probability of death at all stages of the life cycle would unnecessarily increase the computational burden further and since debt portfolios are most relevant early in the life cycle, as we will see below, this simplification of our analysis seems not very restrictive.[4]

Since the questions in the SCF survey refer to income in the previous year and agents have made their consumption and portfolio choices conditional on this income, we interpret the SCF asset data as end-of-period information at the time when the survey is carried out.

[3] When computing the statistics in the data, we use the sampling weights provided in the SCF.

[4] Allowing for a positive probability of death and assuming accidental bequests, for example, would add a fixed-point problem in our numerical solution. This would be very costly given the substantial computational burden of our model.

	2000s	*2004*	*2004: ≤ 90th net worth pctile*
	(1)	(2)	(3)
Wealth			
Housing wealth (primary residence)	3.38	3.48	2.50
+ Net-financial assets	4.07	3.69	-0.01
= Total net worth (fraction of average net lab. earnings)	7.45	7.17	2.49
Financial assets			
Financial assets	4.714	4.407	0.773
+ Secured debt	-0.610	-0.688	-0.746
+ Unsecured debt	-0.031	-0.033	-0.037
= Net-financial assets (fraction of average net lab. earnings)	4.073	3.686	-0.010
Debt			
Secured debt (in % of total secured + unsecured debt above)	95.16	95.42	95.27
Home ownership (in % of sample size)	66.91	67.43	64.40
Bankrupt in previous year (in % of sample size)	1.42	1.53	1.67

Table 1: Wealth and debt portfolios of households with a head between ages 26 and 55. Sample means of cross-sections in the 2000s (2001-2007) and 2004, respectively. Means in column (3) are computed for households up to the 90th percentile of the net worth distribution. Source: Authors' calculation based on the SCF. Notes: Quantities are normalized by average net labor earnings of the whole sample in the respective sample year.

2.2 Wealth and debt portfolios

Table 1 displays the means of wealth and debt for prime-age households denominated in average net labor earnings in the respective survey year. Column (1) displays the average of the means across the SCF surveys in the 2000s (the SCF 2001, 2004 and 2007) whereas column (2) shows these averages only for the SCF 2004. The sample means are very similar in both columns of Table 1. We thus focus on the SCF 2004, the last SCF wave before the financial crisis, when we calibrate our model which is not designed for analyzing such an extreme event.

For the calibration we further restrict the sample to consumers up to the 90th percentile of the net worth distribution. The reason is that, as is well known, standard incomplete-market models cannot match the substantial wealth holdings in the top decile of the wealth distribution (see, for example, Hintermaier and Koeniger, 2011, and the references therein). If such a model were calibrated to match average wealth in the whole sample, the model would predict that consumers up to the 90th percentile hold too much wealth compared with the data. Such a prediction bias is particularly undesirable given that the focus of this paper is on debt portfolios. Table, column (3) shows that while the wealth holdings are obviously smaller for consumers below the 90th percentile of the net worth distribution, most of the other statistics are quite similar. The main difference is the much smaller financial asset position compared with column (2). Such assets are held mostly by consumers in the top decile of the net worth distribution.

Table 1, column (3) shows that average wealth of prime-age households below the 90th percentile of the net worth distribution consists mostly of owner-occupied housing wealth (for the primary residence). 64% of households own a home which can be used as collateral to secure debt in real-life debt contracts. Secured debt accounts for 95% of the debt.[5] As to the incidence of debt, 46% of prime-age households below the 90th percentile of the net worth distribution hold some debt, 39% hold secured debt and 13% hold unsecured debt. Unsecured debt can be written off in bankruptcy procedures and 1.6% of the households have made use of that option in the year previous to the survey date.

Since an important dimension of heterogeneity in our life-cycle model is age, Figures 1 and 2 illustrate how wealth and debt portfolios change with age in the SCF cross-section up to the 90th percentile of the net worth distribution.[6] Figure 1 shows that young consumers start their life with very little equity, borrowing against their housing collateral. Figure 2 shows that most of the debt is secured and unsecured debt is mainly taken on by young consumers. By their mid-thirties consumers, on average, reduce their debt and accumulate significant amounts of other equity (see Fernández-Villaverde and Krueger, forthcoming, who document similar patterns of financial assets and durables in the SCF 1995).

The data in the SCF also reveal some important patterns of difference between consumers with and without unsecured debt, which we would like to capture with our model. Households with unsecured debt are younger and have smaller labor earnings than the sample mean. Moreover, they own less housing wealth while holding more secured debt than the rest of

[5] Constructing model counterparts for secured and unsecured debt in the data is not trivial. We refer to the data appendix for details.

[6] We explain below how we compare the data cross-sections with the life-cycle profiles generated by the model. To obtain the age cross-sections displayed in the figures, we divide the range between ages 23 and 76 into 18 three-year age groups as in the model. We then compute the averages for each of these age groups.

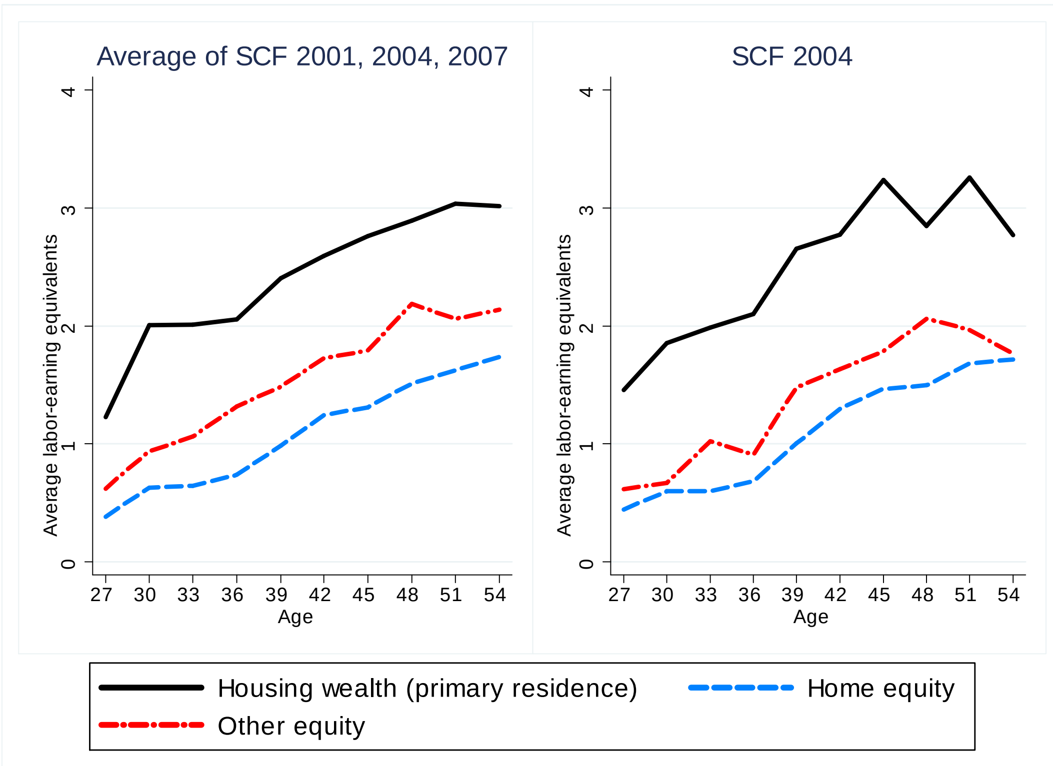

Figure 1: Average portfolios of consumers between ages 26 and 55 up to 90th percentile of the net worth distribution in the 2000s. Source: Authors' calculations based on the Survey of Consumer Finances (SCF). See the data appendix for variable definitions. Notes: The unit is the average of net labor earnings in the whole SCF sample. The axis labeling for age uses the midpoint of the age interval in the respective three-year age group. The graph plots the average for each age group.

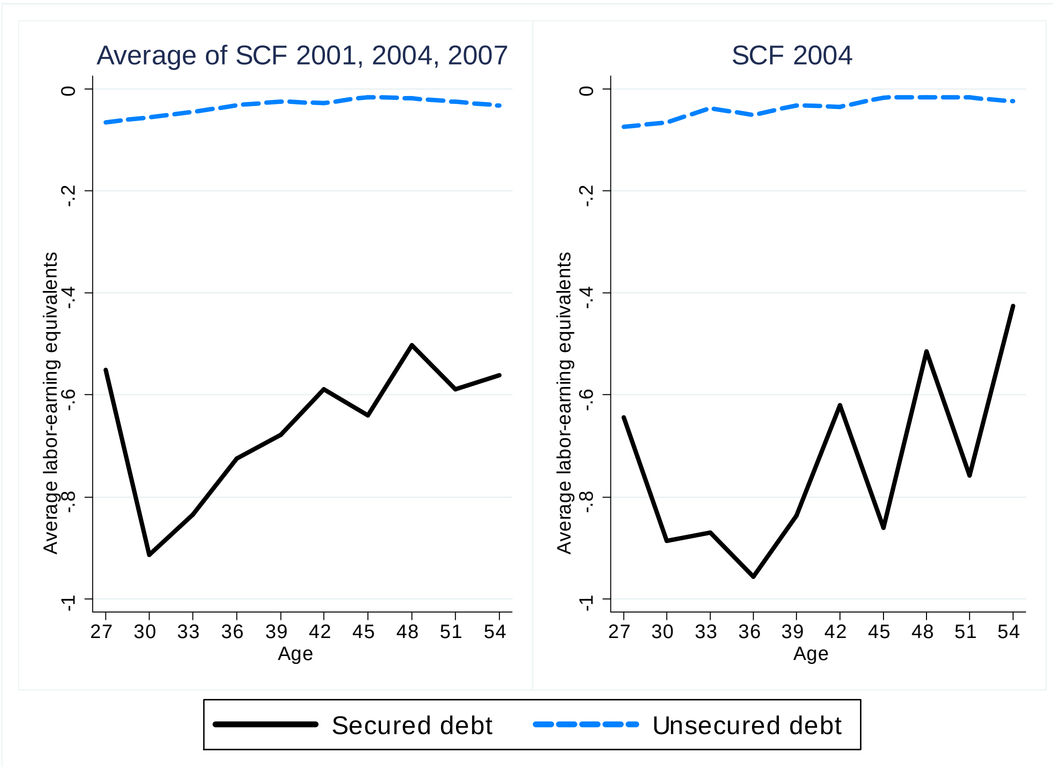

Figure 2: Average debt portfolio of consumers between ages 26 and 55 up to 90th percentile of the net worth distribution in the 2000s. Source: Authors' calculations based on the Survey of Consumer Finances (SCF). See the data appendix for variable definitions. Notes: The unit is the average of net labor earnings in the whole SCF sample. The axis labeling for age uses the midpoint of the age interval in the respective three-year age group. The graph plots the average for each age group.

the sample. Thus consumers with unsecured debt have less home equity than the rest of the sample.

2.3 U.S. consumer bankruptcy

The regulation for consumer bankruptcy in the U.S., relevant for the SCF 2004, is the Federal Bankruptcy Act of 1978. This act contains two Chapters for non-farming households. Consumers can choose to file for personal bankruptcy under either Chapter 7 or Chapter 13. The main features of these two Chapters, which are important for our analysis, can be summarized as follows (see Sullivan et al., 1999, for further details).

Under Chapter 7 of the bankruptcy act, the debtor can write off his unsecured debts but must surrender all his assets except for specified exempt amounts. Most of the bankruptcy exemptions are specified as homestead exemptions (see, for example, Grant and Koeniger, 2009). Secured debt is senior and has to be honored, however, so that bankruptcy exemptions only apply to the home equity which remains after servicing secured credit claims.[7]

Under Chapter 13, the debtor agrees to a repayment schedule for part or all of the debt and retains his assets. The repayment plan usually is specified for three years but can take up to five years. Importantly, the debtor cannot repay less under Chapter 13 than what creditors would get paid under Chapter 7. Hence, we focus on Chapter 7 in our model since it places a lower bound on the unsecured-debt claims of the creditors. This is not a strong restriction since most consumers who file for bankruptcy do so under Chapter 7 (70%) and many of the repayment plans initiated under Chapter 13 fail and are later converted into Chapter 7. If consumers file for bankruptcy under Chapter 7, they are not allowed to file for bankruptcy again within the next six years (see Sullivan et al., 1999).

The main reason for consumer bankruptcy, identified by Sullivan et al. (2000), is earnings risk. Two thirds of the bankrupt consumers mention job related problems like wage cuts or unemployment. A fifth of bankrupt consumers reports health problems (multiple responses were permitted) where in 60% of these cases the implied income losses due to missed work-

[7] Our model abstracts from house price risk and negative home equity so that we do not discuss the regulation on mortgage foreclosures and bankruptcy. Data on charge-off and delinquency rates by the Federal Reserve at http://www.federalreserve.gov/releases/chargeoff/ show that real-estate loans have been essentially secure before 2007 with charge-off and delinquency rates of less than a tenth of those of other consumer loans.

days, demotion or lost jobs are mentioned as the reason for bankruptcy. Therefore we focus on labor earnings shocks as a reason for bankruptcy in this paper.[8] Having presented the key relevant facts, we are now ready to set up the model.

3 The model

We build on the life-cycle model of unsecured debt by Livshits et al. (2007). We assume that the economy is populated by a large number of consumers, indexed by i, who live for $J = 18$ periods, where each period j has a length of three years. Life begins at age 23 and the first 14 periods (until age 65) are working periods in which people receive income shocks. In the last four periods consumers are in retirement and face no uncertainty. Life ends after age 76. Below we drop the index i for different consumers to simplify notation unless the distinction between consumers is particularly important.

Preferences. Consumers maximize expected lifetime utility. Utility is derived from a consumption basket $\Psi(c_j, f_j)$ which is non-separable in non-durable consumption c_j and the service flow f_j from housing. We assume that one unit of the housing stock, whether rented or owned, provides ϕ units of this service flow. For the quantitative application of the model we assume recursive utility since we want to have flexibility in the calibration concerning the strength of the intratemporal and intertemporal consumption smoothing motive. The utility is specified as

$$U_j = \left[\Psi(c_j, f_j)^{1-\sigma} + \beta \left(E\left[U_{j+1}^{1-\gamma}\right]\right)^{\frac{1-\sigma}{1-\gamma}}\right]^{\frac{1}{1-\sigma}},$$

with expectation operator E, discount factor β, risk aversion $\gamma \geq 0$, intertemporal elasticity of substitution $1/\sigma \geq 0$ and

$$\Psi(c_j, f_j) = (c_j)^{\theta} \left(\phi f_j + \underline{f}\right)^{1-\theta}.$$

[8] We abstract from medical expense shocks to contain the computational burden of our model with housing as an additional endogenous state variable. See Chatterjee et al. (2007) or Livshits et al. (2007) for models with health expense shocks.

The parameter $\phi = 1$ if the consumer owns housing and $\phi = \phi^r$ with $0 \leq \phi^r \leq 1$ if the consumer rents. As is common in the literature, the parameter ϕ^r allows us to capture foregone utility of renters, for example due to hold-up problems which are left unmodelled. We call ϕ^r rental efficiency.

The specification of preferences implies that the demand for housing service flows may be zero since consumers then rely on the constant autonomous level of the service flow $\underline{f} > 0$, which is assumed to be small and positive. Furthermore, consumers prefer an early resolution of uncertainty if $\gamma > 1/\sigma$, as will be the case in our calibration.

Our parametric assumptions about preferences encompass many of the previous numerical applications which we are aware of. The preferences would simplify to the standard CRRA case if $\gamma = \sigma$. Moreover, the Cobb-Douglas consumption basket is in line with empirical evidence on the substitutability between housing and non-durables (see Davis and Ortalo-Magné, 2011, and Fernández-Villaverde and Krueger, forthcoming, for further discussion and references).

Labor earnings. The log of labor earnings of consumer i at age j is given by

$$\ln y_{ij} = \phi_j + z_{ij},$$

where ϕ_j is the deterministic labor endowment of the household with age $23 + 3(j - 1)$ at the beginning of period j. The endowment ϕ_j is hump-shaped over the life cycle and z_{ij} is a persistent income shock.

Assets and market arrangements. The stock of housing that creates the service flow of utility can be either rented or owned. We denote the owned housing wealth by h. Owned housing can be used as collateral to secure debt but can only be adjusted at a cost. Alternatively, housing can be rented at a price determined by a no-arbitrage condition. The price of renting one unit of housing equals the user cost $r^a + \delta$, where r^a is the risk-free interest rate on financial assets and δ is the rate at which housing depreciates.[9]

[9] As in Gervais (2002), we implicitly assume that financial institutions take on deposits and purchase housing which they rent out. Thus, the interest rate on deposits r^a enters the user cost .

Consumers hold portfolios of secured debt $a^s \leq 0$, unsecured debt $a^u < 0$, risk-free financial assets $a^u \geq 0$ and owned housing wealth h. Secured debt is backed by owner-occupied housing as collateral and bears an interest rate r^s. Risk-free financial assets $a^u \geq 0$ earn interest r^a. We assume that there is a borrowing spread, $r^s > r^a$, due to a cost of financial intermediation. We further assume that the cost of intermediation is larger for unsecured debt so that the interest rate for unsecured debt is at least $\underline{r^u} > r^s > r^a$. As we discuss further when we calibrate the model, this is a common assumption which is realistic.

Unsecured debt $a^u < 0$ is not backed by collateral and we allow consumers to discharge unsecured debt in bankruptcy procedures. Since creditors price the possibility of bankruptcy, the interest rate on unsecured debt consists of the base rate $\underline{r^u}$ and an endogenous risk premium. We present the pricing of unsecured debt by financial intermediaries in detail below.

Adjustment costs. Whereas financial assets a^s and a^u can be adjusted costlessly by the consumer, we assume that the adjustment of owned housing h is costly. These costs can be thought of as fees for real estate agents and generate realistic lumpy investment patterns for housing. Moreover, it sharpens the distinction between owner-occupied housing and non-durables in our model as adjustment costs are one key difference between these two types of goods. Similar to Díaz and Luengo-Prado (2008), we specify the costs as

$$\alpha(h^*_{j+1}, h_j) = \begin{cases} c_f^+ h_j & \text{if } h^*_{j+1} > h_j \\ 0 & \text{if } (1-\delta)h_j \leq h^*_{j+1} \leq h_j \\ c_f^- h_j & \text{if } h^*_{j+1} < (1-\delta)h_j \end{cases},$$

where the adjustment cost is allowed to be asymmetric in the direction of adjustment. We attach asterisks to the portfolio choices to distinguish them from realizations after bankruptcy choice. Note that in our specification of the adjustment cost function there is an interval of choices h^*_{j+1} for which no adjustment costs need to be paid. The end points of this interval are the levels of housing if it depreciates and if it is fully maintained.[10]

[10] The numerical solution of the model will require discretization of choices. The above specification is compatible with the existence of choices on the discretized grid for which no adjustment costs occur.

State variables and the timing of choices. Each consumer enters the period with the pair of endogenous state variables, net-financial assets $a_j \equiv a_j^s + a_j^u$ and owned housing h_j. Two exogenous state variables are also relevant to the decision problem. These consist of the state of current income y_j and of a moving indicator m_j. This exogenous moving indicator determines if a consumer has to rent or if he can choose between either renting or owning the home. As in Díaz and Luengo-Prado (2008), a plausible calibration of exogenous variation in this moving indicator helps to capture realistic patterns of home ownership over the life cycle.

Given the endogenous and exogenous states the consumer chooses consumption and the asset portfolio. The asset returns accrue and the consumer enjoys utility before the new realizations of the exogenous state variables, y_{j+1} and m_{j+1}, are drawn. The consumer then decides whether to declare bankruptcy. The law of motion of the pair of endogenous state variables, a_{j+1} and h_{j+1} that enter the decision problem next period, depends on the bankruptcy choice. We now characterize the constraints for the consumer choices and the bankruptcy procedure in more detail before we formulate the recursive problem.

Collateral constraint. The amount of secured debt of the consumer is bounded by the collateral constraint. Only owned housing net of adjustment costs can be used as collateral to secure debt so that we specify the collateral constraint as

$$a_{j+1}^{s*} \geq -\min(\mu, 1 - c_f^-) h_{j+1}^* \ , \tag{1}$$

where μ is the exogenous maximum loan-to-value ratio imposed by the financial regulator. If $\mu < 1 - c_f^-$, the access to secured debt is more constrained than necessary to guarantee repayment in the presence of adjustment costs.

Budget constraint. If the consumer is hit by a moving shock, $m_j = 1$, the consumer cannot own a home and thus cannot hold secured debt so that $a_{j+1}^{s*} = h_{j+1}^* = 0$. The budget constraint for a renter is

$$q_j^u(\nu_{j+1}^*, y_j, B_j) a_{j+1}^{u*} + (r^a + \delta) q^h f_j + \alpha(0, h_j) + c_j \leq \ a_j + h_j + y_j \ , \tag{2}$$

where the price

$$q_j^u(\nu_{j+1}^*, y_j, B_j) = \begin{cases} (1+r^a)^{-1} & \text{if } a_{j+1}^{u*} \geq 0 \\ \left(1+r_j^u(\nu_{j+1}^*, y_j, B_j)\right)^{-1} & \text{if } a_{j+1}^{u*} < 0 \end{cases} .$$

If the consumer holds unsecured debt, $a_{j+1}^{u*} < 0$, the price depends on the portfolio $\nu_{j+1}^* \equiv (a_{j+1}^{s*}, a_{j+1}^{u*}, h_{j+1}^*)$, the exogenous state of current income y_j and the bankruptcy flag B_j which equals 1 if the consumer has declared bankruptcy in the previous period and 0 otherwise. Since the moving shock is idiosyncratic and, unlike the income shock, not persistent, the price $q_j^u(\nu_{j+1}^*, y_j, B_j)$ does not depend explicitly on the exogenous state m_j. The moving indicator m_j affects prices, however, through the restrictions which it imposes on the portfolio choices ν_{j+1}^*. We discuss the pricing of unsecured debt further below.

If $m_j = 1$, the consumer can spend resources $a_j + h_j + y_j$ on non-durable consumption c_j, the financial asset a_{j+1}^{u*} and housing rental f_j, where $q^h = 1/(1-\delta)$. Since the consumer has to divest the housing stock, adjustment costs are $\alpha(0, h_j)$.

If $m_j = 0$, the consumer can choose between renting and owning. If the consumer rents, the budget constraint is as in (2). If the consumer owns, the budget constraint is:

$$q_j^s a_{j+1}^{s*} + q_j^u(\nu_{j+1}^*, y_j, B_j) a_{j+1}^{u*} + q^h h_{j+1}^* + \alpha(h_{j+1}^*, h_j) + c_j \leq \ a_j + h_j + y_j \ , \tag{3}$$

where the expenditure on housing is $q^h h_{j+1}^*$. In this case the owned housing wealth h_{j+1}^* creates a service flow $f_j = h_{j+1}^*$ and can be used as collateral for secured debt $a_{j+1}^{s*} \leq 0$ with a price $q_j^s = 1/(1+r^s)$.

Bankruptcy. At the time of bankruptcy filing the consumer is obliged by law to reveal his financial status to the bankruptcy judge. In particular, the judge knows the amount of the owned housing wealth h_{j+1}^* and the composition of financial debt a_{j+1}^{s*} and a_{j+1}^{u*}. If $a_{j+1}^{u*} \geq 0$, the consumer has no unsecured debt which can be written off and bankruptcy is not an option. The interesting case is when $a_{j+1}^{u*} < 0$. Since *secured* debt a_{j+1}^{s*} has priority and needs to be paid irrespective of specified home exemption levels, the bankruptcy judge first computes the amount of the owned housing wealth that remains after repaying all secured debt. Note that for renters $h_{j+1}^* = a_{j+1}^{s*} = 0$. The housing wealth which remains for repaying unsecured debt is

$$h^{\text{left for unsecured}} = (1 - c_f^-)h_{j+1}^* + a_{j+1}^{s*} \ .$$

The judge then determines the maximum amount which could be divested from the remaining housing wealth, given the exemption level $h^\dagger$ specified in the bankruptcy regulation. That amount is

$$\iota^{\text{max net divestment}} = -\max\left\{h^{\text{left for unsecured}} - h^\dagger, 0\right\} \ .$$

The housing wealth used to repay unsecured debt is then equal to that maximum amount or less if the outstanding amount of unsecured debt is smaller:

$$h^{\text{to unsecured}} = \min\left\{-\iota^{\text{max net divestment}}, -a_{j+1}^{u*}\right\} \ .$$

As specified in the U.S. bankruptcy law, the judge only sells the home if some of the home equity can be used to repay unsecured debt. Thus, the housing wealth which remains for the consumer after the bankruptcy procedure is

$$h^B = \begin{cases} h^{\text{left for unsecured}} - h^{\text{to unsecured}} & \text{if } \iota^{\text{max net divestment}} < 0 \\ h_{j+1}^* & \text{if } \iota^{\text{max net divestment}} = 0 \end{cases} ,$$

and net-financial assets are

$$a^B = \begin{cases} 0 & \text{if } \iota^{\text{max net divestment}} < 0 \\ a_{j+1}^{s*} & \text{if } \iota^{\text{max net divestment}} = 0 \end{cases}$$

where the consumer starts afresh without unsecured debt.

The evolution of the assets can thus be summarized as

$$h_{j+1} = \begin{cases} h_{j+1}^* & \text{if no bankruptcy} \\ h^B(a_{j+1}^{s*}, a_{j+1}^{u*}, h_{j+1}^*) & \text{if bankruptcy} \end{cases} , \tag{4}$$

$$a_{j+1} = \begin{cases} a_{j+1}^* \equiv a_{j+1}^{s*} + a_{j+1}^{u*} & \text{if no bankruptcy} \\ a^B(a_{j+1}^{s*}, a_{j+1}^{u*}, h_{j+1}^*) & \text{if bankruptcy} \end{cases} . \tag{5}$$

The pricing of unsecured debt. The price of unsecured debt is determined by perfectly competitive financial intermediaries which observe current income y_j, the portfolio $\nu^*_{j+1} \equiv (a^{s*}_{j+1}, a^{u*}_{j+1}, h^*_{j+1})$, the bankruptcy flag B_j and the age j of the consumer. As mentioned above, the price $q^u_j(\nu^*_{j+1}, y_j, B_j)$ does not depend explicitly on the exogenous state m_j since the moving shock is idiosyncratic and not persistent as the income shock.

The intermediaries price unsecured debt forming expectations about the draws of future income, the moving shock and the implied probability of bankruptcy. There is no cross-subsidization across consumers so that consumers with different portfolios, age or income state may receive a different interest quote.

Defining the probability of bankruptcy as $\pi_j(\nu^*_{j+1}, y_j, B_j)$, the zero-profit condition implies that the price for unsecured debt is given by

$$\begin{aligned} q^u_j(\nu^*_{j+1}, y_j, B_j) = &\left(1 - \pi_j(\nu^*_{j+1}, y_j, B_j)\right) \overline{q^u} \\ &+ \pi_j(\nu^*_{j+1}, y_j, B_j)\overline{q^u} \frac{h^{\text{to unsecured}}(\nu^*_{j+1})}{\left|a^{u*}_{j+1}\right|}, \end{aligned} \tag{6}$$

where $\overline{q^u} = 1/(1+\underline{r}^u)$. If the probability of bankruptcy is zero, $\pi_j(\nu^*_{j+1}, y_j, B_j) = 0$, or if no unsecured debt is discharged when a consumer files, $h^{\text{to unsecured}}(\nu^*_{j+1}) = \left|a^{u*}_{j+1}\right|$, then the risk premium on unsecured debt is zero: $q^u_j(\nu^*_{j+1}, y_j, B_j) = \overline{q^u}$. The probability of bankruptcy is zero, for example, if the consumer has declared bankruptcy in the previous period so that $B_j = 1$.

The recursive formulation with optimal default. The decision problems of a consumer depend on the moving shock m_j and on the bankruptcy decision in the previous period. Being hit by a moving shock ($m_j = 1$) implies the restriction on portfolio choice $h^*_{j+1} = a^{s*}_{j+1} = 0$. Having declared bankruptcy in the previous period restricts the bankruptcy choice. This is consistent with the U.S. bankruptcy law which forbids consumers to file for bankruptcy within six years after a previous bankruptcy procedure. Since a period has a length of three years in our model, we assume that no bankruptcy can be declared for one period.

If the consumer is not hit by a moving shock, $m_j = 0$, he can choose whether to own or to rent housing. The value function

$$V_j(a_j, h_j, y_j) = \max\left[V_j^r(a_j, h_j, y_j), V_j^o(a_j, h_j, y_j)\right]$$

denotes the envelope of the value function when renting, $V_j^r(a_j, h_j, y_j)$, and the value function when owning the home, $V_j^o(a_j, h_j, y_j)$. The value function

$$V_j^B(a_j, h_j, y_j) = \max\left[V_j^{r,B}(a_j, h_j, y_j), V_j^{o,B}(a_j, h_j, y_j)\right]$$

denotes the envelope of the value functions for renting $V_j^{r,B}(a_j, h_j, y_j)$ and owning $V_j^{o,B}(a_j, h_j, y_j)$ if the consumer has declared bankruptcy in the previous period. If the consumer is hit by a moving shock, $m_j = 1$, he cannot own the home and only the value functions $V_j^r(a_j, h_j, y_j)$ and $V_j^{r,B}(a_j, h_j, y_j)$ determine his optimal choices. We now specify how the value functions $V_j^r(a_j, h_j, y_j)$, $V_j^{r,B}(a_j, h_j, y_j)$, $V_j^o(a_j, h_j, y_j)$ and $V_j^{o,B}(a_j, h_j, y_j)$ are obtained.

Let ω_j denote the probability of a moving shock $m_j = 1$ at age j. The value of renting is V_j^r if no bankruptcy has been declared in the previous period:

$$\begin{aligned}
&V_j^r(a_j, h_j, y_j) = \qquad (7)\\
&\max_{a_{j+1}^{u*}, f_j}\Bigg\{\Psi(\underbrace{a_j + h_j + y_j - q_j^u(\nu_{j+1}^*, y_j, 0)a_{j+1}^{u*} - (r^a + \delta)q^h f_j - \alpha(0, h_j)}_{c_j}, f_j)^{1-\sigma}\\
&\quad + \beta\Bigg[\omega_{j+1}E\left(\max_{B_{j+1}}[V_{j+1}^r(a_{j+1}^*, 0, y_{j+1}), (1-\psi)V_{j+1}^{r,B}(0, 0, y_{j+1})]^{1-\gamma}\right)\\
&\qquad +(1-\omega_{j+1})E\left(\max_{B_{j+1}}[V_{j+1}(a_{j+1}^*, 0, y_{j+1}), (1-\psi)V_{j+1}^{B}(0, 0, y_{j+1})]^{1-\gamma}\right)\Bigg]^{\frac{1-\sigma}{1-\gamma}}\Bigg\}^{\frac{1}{1-\sigma}},
\end{aligned}$$

subject to (2),

where E is the expectation operator and $0 \leq \psi \leq 1$ is an exogenous utility cost of bankruptcy. This can be interpreted as psychological pain or stigma (see Athreya, 2004).[11]

[11] In Athreya (2004) a penalty $\psi \geq 0$ is subtracted from the value function. With recursive preferences we let $0 \leq \psi \leq 1$ enter multiplicatively to ensure that the maximum value, computed in the expectation in (7), is positive.

The value of renting is $V_j^{r,B}$ if bankruptcy has been declared in the previous period:

$$V_j^{r,B}(a_j,h_j,y_j) = \tag{8}$$
$$\max_{a_{j+1}^{u*},f_j}\Bigg\{\Psi(\underbrace{a_j+h_j+y_j-q_j^u(\nu_{j+1}^*,y_j,1)a_{j+1}^{u*}-(r^a+\delta)q^h f_j-\alpha(0,h_j)}_{c_j},f_j)^{1-\sigma}$$
$$+\beta\left[\omega_{j+1}E\left(V_{j+1}^r(a_{j+1}^*,0,y_{j+1})^{1-\gamma}\right)\right.$$
$$\left.+(1-\omega_{j+1})E\left(V_{j+1}(a_{j+1}^*,0,y_{j+1})^{1-\gamma}\right)\right]^{\frac{1-\sigma}{1-\gamma}}\Bigg\}^{\frac{1}{1-\sigma}},$$

subject to (2).

Note that this type of consumer no longer has the option to declare bankruptcy in the current period.

Recalling that $f_j = h_{j+1}^*$ for owners, the value of owning is V_j^o if no bankruptcy has been declared in the previous period:

$$V_j^o(a_j,h_j,y_j) = \tag{9}$$
$$\max_{a_{j+1}^{s*},a_{j+1}^{u*},h_{j+1}^*}\Bigg\{\Psi(\underbrace{a_j+y_j+h_j-q_j^s a_{j+1}^{s*}-q_j^u(\nu_{j+1}^*,y_j,0)a_{j+1}^{u*}-q^h h_{j+1}^*-\ \alpha(h_{j+1}^*,h_j)}_{c_j},h_{j+1}^*)^{1-\sigma}$$
$$+\beta\left[\omega_{j+1}E\left(\max_{B_{j+1}}[V_{j+1}^r(a_{j+1}^*,h_{j+1}^*,y_{j+1}),(1-\psi)V_{j+1}^{r,B}(a^B,h^B,y_{j+1})]^{1-\gamma}\right)\right.$$
$$\left.+(1-\omega_{j+1})E\left(\max_{B_{j+1}}[V_{j+1}(a_{j+1}^*,h_{j+1}^*,y_{j+1}),(1-\psi)V_{j+1}^B(a^B,h^B,y_{j+1})]^{1-\gamma}\right)\right]^{\frac{1-\sigma}{1-\gamma}}\Bigg\}^{\frac{1}{1-\sigma}},$$

subject to (1) and (3).

The value of owning is $V_j^{o,B}$ if bankruptcy has been declared in the previous period:

$$V_j^{o,B}(a_j, h_j, y_j) = \tag{10}$$

$$\max_{a^{s*}_{j+1}, a^{u*}_{j+1}, h^*_{j+1}} \Bigg\{ \Psi(\underbrace{a_j + y_j + h_j - q^s_j a^{s*}_{j+1} - q^u_j(\nu^*_{j+1}, y_j, 1) a^{u*}_{j+1} - q^h h^*_{j+1} - \alpha(h^*_{j+1}, h_j)}_{c_j}, h^*_{j+1})^{1-\sigma}$$

$$+ \beta \left[\omega_{j+1} E \left(V^r_{j+1}(a^*_{j+1}, h^*_{j+1}, y_{j+1})^{1-\gamma}\right)\right.$$

$$\left. +(1-\omega_{j+1}) E \left(V_{j+1}(a^*_{j+1}, h^*_{j+1}, y_{j+1})^{1-\gamma}\right)\right]^{\frac{1-\sigma}{1-\gamma}} \Bigg\}^{\frac{1}{1-\sigma}},$$

subject to (1) and (3).

Note that the decision problem of the owner contains the collateral constraint (1).

Equations (7) and (9) show that the costs of bankruptcy are different for owners and renters. Both renters and owners are excluded from the option to declare bankruptcy and face an exogenous utility cost of bankruptcy ψ. In addition, owners have to pay adjustment costs for forced home sales in the bankruptcy procedure, in situations where the housing stock after repaying secured debt exceeds the bankruptcy exemption, $h^{\text{left for unsecured}} > h^{\dagger}$. The magnitude of the endogenous bankruptcy cost depends on the size of the owned house and is relevant for our later analysis of home equity as informal collateral and repayment commitment.

Equations (8) and (10) illustrate that we do not assume that consumers are excluded from credit markets after bankruptcy. This assumption is often imposed in models with unsecured debt to make bankruptcy costly enough. Since we have endogenous bankruptcy costs related to owned housing and the exclusion from the option to declare bankruptcy, we do not need this assumption which is at odds with empirical evidence on consumer borrowing after bankruptcy procedures.

Equilibrium definition. A recursive competitive equilibrium is characterized by the policy functions for non-durable consumption, the portfolio choices and optimal default so that for given prices $\{r^a, r^s\}$ of risk-free assets and secured debt:

(i) the envelope of value functions $V_j(a_j, h_j, y_j)$ and $V_j^B(a_j, h_j, y_j)$ attains its maximal value if $m_j = 0$, and the envelope of value functions $V_j^r(a_j, h_j, y_j)$ and $V_j^{r,B}(a_j, h_j, y_j)$ attains its maximal value if $m_j = 1$.

(ii) the pricing scheme for unsecured debt $q_j^u(\nu_{j+1}^*, y_j, B_j)$ satisfies the zero-profit condition (6), with default probabilities $\pi_j(\nu_{j+1}^*, y_j, B_j)$ being determined by optimal default.

Having presented the model and its recursive formulation we now solve the model numerically and calibrate it to match wealth and debt portfolios in the U.S.

4 Calibration and numerical results

The discrete choice in the bankruptcy decision and the presence of non-convex adjustment costs imply that we cannot use numerical algorithms for the constrained portfolio choice problem which rely on the differentiability of the value function as in Hintermaier and Koeniger (2010). Thus, we discretize portfolio choices and specify an equi-spaced grid for $a^s \in [-7.5; 0]$ and $a^u \in [-3; 0]$, with a distance between gridpoints of 0.22 in terms of annual average labor earnings. For $a^u \in [0; 90]$ we choose the same grid fineness at 0 but let the distance between gridpoints increase linearly for larger values of a^u in order to economize on computation time. This results in 35 and 54 gridpoints for a^s and a^u, respectively. Choosing equi-spaced grids for debt $a^s < 0$ and $a^u < 0$ ensures that consumers remain on the grid of possible values for the endogenous state variable $a = a^s + a^u$ with 88 gridpoints where $a \in [-7.5; 90]$. We then specify the grid for the second endogenous state variable $h \in [0; 45]$ with 148 gridpoints where the grid for h is chosen to include the bankruptcy exemption value $h^\dagger$ and the values of h implied by the grid for secured debt a^s at the collateral constraint (1).[12] We check that, for the specified grid, among the feasible housing choices there are choices for which no adjustment costs are incurred. We make sure that widening the bounds of the grid further does not affect the results. Finally, the benchmark calibration allows for 5 Markov states

[12] The grid for the rental choices is specified identically. We have checked that the results for the benchmark calibration are robust if we increase the number of gridpoints to 71 gridpoints for a^s, 113 gridpoints for a^u, with a distance between gridpoints of 0.11 in terms of annual average labor earnings, resulting in 183 gridpoints for a and 184 gridpoints for h. Due to the curse of dimensioniality, resulting both from a larger state space and choice set, computing time increased roughly by factor 10.

of the stochastic component of labor earnings, while we have checked the robustness of our benchmark results for 11 Markov states.

Since solving the model takes 13 hours on a PC of the current computing vintage using Fortran code, we do not perform a full grid search over the whole parameter space in our calibration. We solve the model in some regions of the parameter space, which we consider plausible, and then use the interpolation method of Kriging to check whether we can improve the model predictions in other parts of the parameter space.

Kriging is a geostatistical technique which, under certain assumptions, computes the best linear unbiased prediction for target statistics via interpolation on the parameter space and is closely related to regression analysis.[13] Kriging has been used substantially in geostatistical sciences and has not been applied much in Economics, to the best of our knowledge. Since solving our model on the whole parameter space is prohibitively costly (as is drilling for oil everywhere on the planet in the geostatistical applications), the method is naturally applied in calibration exercises in order to predict model statistics at parameter combinations for which the model has not been explicitly solved and simulated.

4.1 Numerical algorithm

We start with the last period J. In that period a consumer cannot declare bankruptcy and optimally chooses consumption and the asset portfolio. Note that adjustment costs and the possibility to collateralize owned housing imply that for some parameter values it is not optimal for the consumer to sell all assets before death. We then compute the available resources, with and without filing for bankruptcy in the previous period, on the state space $A \times H \times Y$ and calculate the value functions V_{J-1}, V^B_{J-1}, V^r_{J-1} and $V^{r,B}_{J-1}$.[14] The functions allow us to determine those realizations for next period income and moving shocks for which a consumer with a given portfolio declares bankruptcy, i.e., $(1-\psi)V^B_{J-1} > V_{J-1}$ or $(1-\psi)V^{r,B}_{J-1} > V^r_{J-1}$. We then compute the price of unsecured debt for all income states and *feasible* choices before we solve the maximization problem of the consumer to determine the

[13]See the description of the Matlab Kriging toolbox by Lophaven, Nielsen and Søndergaard (2002) for further references, which is available at http://www2.imm.dtu.dk/~hbn/dace/.

[14]Since the amount of owned housing after bankruptcy would in general fall off the discretized grid, we convexify the value function using a weighted average of the value function at the two neighboring gridpoints, with weights that depend on the distance to the point which is off the grid.

optimal choices. We continue with analogous computations for the previous period $J-2$ and so on until the beginning of life.

We use the model solution to simulate a population of 100,000 consumers whose initial exogenous and endogenous states at the beginning of life are determined in the following way. The stochastic income component is randomly drawn from the stationary income distribution and the initial conditions for owned housing and net-financial assets are drawn from the sample distribution of consumers with ages 23-25 in the SCF, applying the sampling weights provided in the SCF.[15]

4.2 Comparing the simulation output with SCF data

We make the output of the life-cycle model comparable across all cohorts which are observed in a specific SCF cross-section. This is achieved by reversing the correction for average income growth used for the calibration of the deterministic life-cycle earnings profile described in the appendix. We divide by the growth factor $1.01^{(age-base\ age)}$ where *base age* is the reference age for which no adjustment is necessary. In our model with income growth, this ensures that the unit of output of the life-cycle model is shrunk for cohorts which are relatively older at the time of survey. We then apply the relevant cohort weights from the SCF to replicate the age structure of the population surveyed. Synthesizing surveys from the simulated model this way guarantees that the implied statistics are directly comparable to those from the observed SCF data.

4.3 Calibration

4.3.1 Income before and after retirement

We now explain how we calibrate the income process using the SCF 2004.[16] As is standard in the literature (see, for example, Yang, 2009, or Kaplan and Violante, 2010), consumers in our model are exposed to earnings shocks before retirement. We thus calibrate a stochastic

[15] We draw from more than 100,000 observations since we discard those few observations for consumers whose draw for the initial endowments would imply an empty budget set in the equilibrium of the model.

[16] We do not to use estimates for income processes based on PSID panel data because the PSID sample generates too little inequality in net worth. As mentioned above, the earnings dispersion observed in the SCF is larger than in other surveys that do not attempt to provide accurate data on the wealth distribution.

income process for the period before retirement in the life cycle and calibrate the individual-specific retirement benefits based on the U.S. social security system for the retirement period.

The log of earnings y_{ij} of individual i at age j before retirement is additively separable in a deterministic age polynomial ϕ_j and an idiosyncratic income shock z_{ij} so that

$$\ln y_{ij} = \phi_j + z_{ij} \ , \tag{11}$$

where the shock z_{ij} follows an AR(1) process

$$z_{ij} = \rho z_{i,j-1} + \varepsilon_{ij}. \tag{12}$$

After retirement there are no income shocks and each individual receives retirement benefits from social security. We refer to the appendix for further details about the calibration of the income process.

4.3.2 Mobility shocks

The calibration of the age-dependent mobility shocks uses the moving rates from the Current Population Survey 2005 reported in Díaz and Luengo-Prado (2008), Table 2. The moving rate for our triennial periods is constructed with the one-year moving rate and the annualized five-year moving rate. Since some of the renting activity is endogenous in our model, the moving shocks should capture only the exogenous component. Thus, we downward adjust the moving rates from the CPS using survey answers on the reason to move for exogenous "non-housing reasons", as in Díaz and Luengo-Prado (2008). The resulting moving rate captures moves due to unexpected changes in the family structure or natural disasters and is 50% of the total rate. The probability of moving within a 3-year period falls with age, from 0.32 at age 23 to 0.05 at age 71.

4.3.3 Benchmark parameters.

Table 2 displays the parameter values which we calibrate for our numerical solution. For the technology parameters we assume that homes depreciate at an annual rate of $\delta = 0.02$.

	Parameters	
Preferences	β	0.956
	θ	0.76
	γ	2.0
	$1/\sigma$	0.55
	ϕ^r	0.95
	$\underline{f}$	0.01
Technology	δ	0.02
	$c_f^+, c_f^-,$	0.025
	μ	0.95
Bankruptcy	$h^\dagger$	0.405
	ψ	0.04
Interest rates	r^a	0.04
	r^s	0.05
	$\underline{r^u}$	0.055

Table 2: Benchmark parameters for the calibrated numerical solution. Notes: The unit for $h^\dagger$ is the average of annual net labor earnings. Annualized parameters.

The adjustment costs are specified symmetrically for upward and downward adjustments and are assumed to equal 2.5% of the stock, consistent with typical fees charged by real-estate brokers in the U.S. (Díaz and Luengo-Prado, 2010). These adjustment costs *per se* would imply that the consumer can use at most 97.5% of the housing stock to secure debt. Given the information of the Federal Deposit Insurance Corporation (FDIC) on real estate lending standards and supervisory loan-to-value limits, we calibrate access to secured consumer credit as slightly more restrictive with a loan-to-value ratio of 95%, $\mu = 0.95$.[17]

The parameters for the bankruptcy procedure are set as follows. We set the value of the exempt home equity amounts equal to two fifths of average annual labor earnings which shall approximate the homestead exemption in the U.S. although there is significant variation across U.S. states (Athreya, 2006). As we will discuss further below, the size of the exemption has little effect on our results in strong contrast to Athreya (2006) or Pavan (2008) who do not analyze housing wealth, secured and unsecured debt jointly.

As in Livshits et al. (2007), we assume a small-open economy and set the annual risk-free lending rate to 4%. We assume a small transaction cost for debt so that the secured

[17] Information about supervisory loan-to-value limits is available at http://www.fdic.gov/regulations/laws/rules/2000-8700.html

borrowing rate is 5% and the unsecured borrowing rate without the risk premium $\underline{r^u}$ is 5.5%. As we will see below, positive bankruptcy incidence in equilibrium implies an additional endogenous risk premium for unsecured debt. Allowing for interest spreads in the model is supported by empirical evidence in Davis, Kubler and Willen (2006), historical interest-rate data of the Federal Reserve (Table H.15) and is similar to assumptions in Athreya (2006) and Livshits et al. (2007).[18]

For the preference parameters, we set $\underline{f} = 0.01$, a small and quantitatively negligible value, which allows consumers to consume no housing. We calibrate the remaining preference parameters β, θ, σ, γ, ϕ^r and ψ to match the average statistics for the wealth and debt portfolio and the home ownership rate for households up to the 90th percentile of the net worth distribution. The reason is that, as mentioned in Section 2.2, standard incomplete-market models cannot match the substantial wealth holdings in the top decile of the wealth distribution. Calibrating such a model to match average wealth in the whole sample would worsen the fit of the model for consumers up to the 90th percentile. These consumers would hold more wealth than observed the data, a prediction bias which is undesirable given that the focus of this paper is on debt portfolios.

Table 3 shows that for $\beta = 0.956$, $\theta = 0.76$, $1/\sigma = 0.55$, $\gamma = 2$, $\phi^r = 0.95$ and $\psi = 0.04$ the model matches the data targets well. The parameter values for patience, the weight of non-durable consumption in the consumption index, the intertemporal elasticity of substitution and risk aversion are within the range of commonly calibrated values. Rental efficiency $\phi^r = 0.95$ helps to attain a realistic home ownership rate and the utility bankruptcy penalty $\psi = 0.04$ allows us to generate empirically observed unsecured debt holdings for renters. For $\psi = 0$, unsecured debt would be prohibitively expensive for renters. Given that the preferences are homogeneous of degree 1, $\psi = 0.04$ can interpreted as a 4% smaller consumption basket.

Concerning the incidence of bankruptcy reported in Table 3, we adjust the data target for bankruptcy reported in the SCF 2004 downward to 1.1% ($1.67 \times 2/3$) since job-related shocks trigger bankruptcy in our model and two thirds of the bankruptcies are job related (Sullivan et al., 2000). The calibrated model predicts a smaller bankruptcy incidence of 0.4%, more similar to the data target for bankruptcy of 0.5% in Chatterjee et al. (2007).

[18] The Table H.15 is available at http://www.federalreserve.gov/releases/h15/

Variable	*SCF 2004* (1)	*Model* (2)
Housing wealth (as fraction of net lab. earnings)	2.50	2.57
Net-financial assets (as fraction of net lab. earnings)	-0.01	-0.01
Secured debt (as fraction of net lab. earnings)	-0.75	-0.64
Unsecured debt (as fraction of net lab. earnings)	-0.04	-0.03
Financial assets (as fraction of net lab. earnings)	0.77	0.66
Home ownership (% of sample)	64.4	66.7
Job-related bankruptcy (% of sample)*	1.1	0.4

Table 3: Averages of the targets in the data and the model. Source: Authors' calculations based on the SCF and the model. Notes: *Bankruptcy incidence of 1.67 % in the data is multiplied by 2/3 for the fraction of job-related bankruptcies as reported in Sullivan et al. (2000). The unit is the average of net labor earnings.

This positive incidence of bankruptcy in equilibrium implies an average risk premium for unsecured debt of 2.1 percentage points. In addition to the results reported in Table 3, the model matches the incidence of debt rather well: 56% of consumers hold debt compared with 46% in the data, 46% hold secured debt compared with 39% in the data and 14% hold unsecured debt compared with 13% in the data. Finally, our model reproduces the empirical facts that consumers with unsecured debt are younger, have smaller labor earnings than the sample mean, own smaller but non-negligible amounts of housing wealth than the rest of the sample with substantial amounts of secured debt written against this collateral.

4.4 Life-cycle profiles and cross-sectional age profiles

After describing the calibration of the model and its ability to match our target statistics in the data, we now present the implications of our calibration for the cross-sectional age profiles which are observed in the SCF. These profiles have not been targeted explicitly by our calibration and thus give us a further indication of the model fit of the data. We first present the life-cyle profiles of the model for the whole sample before computing the cross-sectional age profiles for our sample of interest: prime-age consumers up to the 90th percentile of the net worth distribution.

Figure 3 displays the average life-cycle profiles of interest for a simulated population

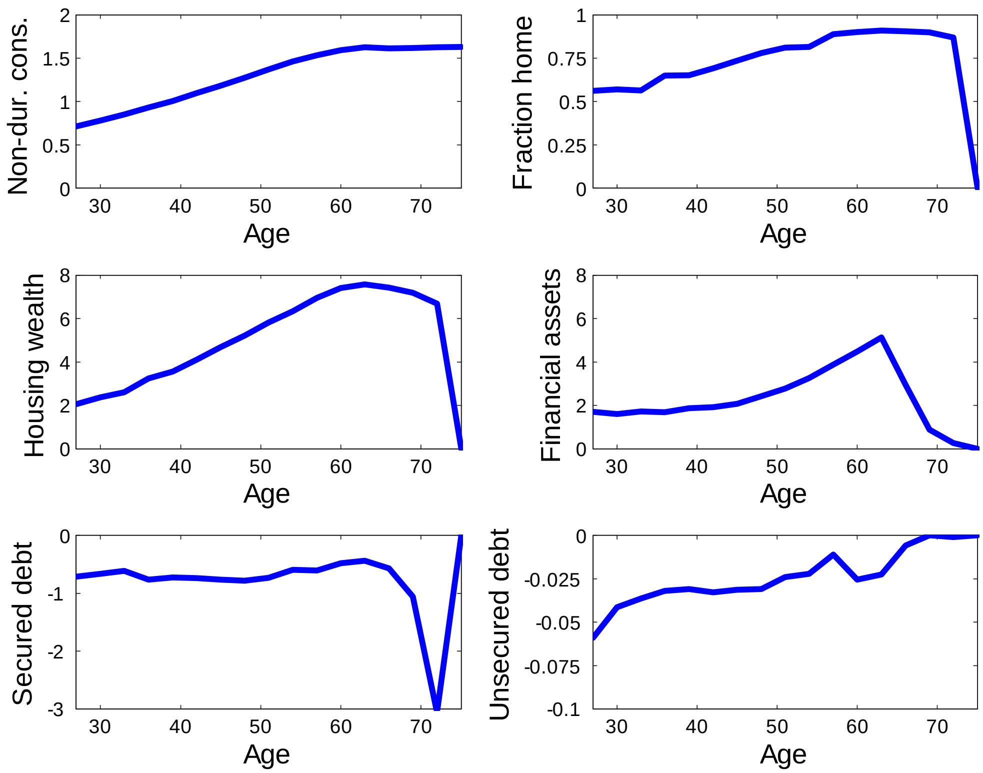

Figure 3: Life-cycle profiles predicted by the model. Source: Authors' calculations based on the model. Note: The unit is the average of net labor earnings.

of 100,000 consumers between ages 26 and 76.[19] Non-durable consumption increases over the life cycle together with average earnings and housing wealth in our incomplete-markets life-cycle model. Financial assets display the familiar tent shape over the life cycle whereas housing wealth is hump-shaped. The home ownership rate steadily increases over the life-cycle before consumers sell their owned housing wealth at the end of life. Unsecured debt is largest (in absolute terms) for young consumers and then decreases with age.[20] Secured debt first increases (in absolute terms) with age, then decreases before retirement, and increases again during the retirement period. As expected, consumers substantially reduce their home equity and financial assets during retirement. Home equity, that is housing wealth net of secured debt, drops by a large amount in the penultimate period when much of the financial assets have been depleted already.

These rather pronounced patterns towards the end of the retirement period clearly result from the assumption of a finite life. In particular, the patterns of asset decumulation depend on the calibrated values for adjustment costs and rental efficiency. Although the focus of this paper is not on explaining debt and wealth portfolios during retirement, we calibrate a more gradual reduction of home ownership in the retirement period to check that the model predictions for prime-age consumers are robust. Since this calibration requires different parameters for rental efficiency or adjustment costs during retirement compared with the rest of life, we prefer our simpler calibration.

After presenting the life-cycle profiles, we are now interested in how the cross-sectional age profiles predicted by the model compare with the observed SCF cross-sections for our sample of interest. Figure 4 displays the average cross-sectional age profiles for consumers between ages 26 and 55 up to the 90th percentile of the net worth distribution in the model (the solid graphs) and SCF data (the dashed graphs).[21] In order to compare the *life-cycle*

[19] At age 23 consumers start with a random draw from the SCF-data distribution of housing wealth and net-financial assets and a random draw for the income shock.

[20] The non-monotonic behavior of unsecured debt at age 60 is related to the last income shock before retirement which is "permanent". The last shock determines retirement income (see the appendix on the calibration of the income process) and some consumers find it attractive to insure against this shock by holding unsecured debt which can be written off if bankruptcy is declared. Indeed, we observe a spike in the bankruptcy incidence in the period of the last income shock (1% of consumers declare bankruptcy in the last period before retirement). The bankruptcy incidence then falls to zero during the retirement period when there is no more income uncertainty.

[21] Note that in the model, as for the average SCF statistics in Table 1, conditioning on consumers up to the 90th percentile mostly reduces the amount of financial assets.

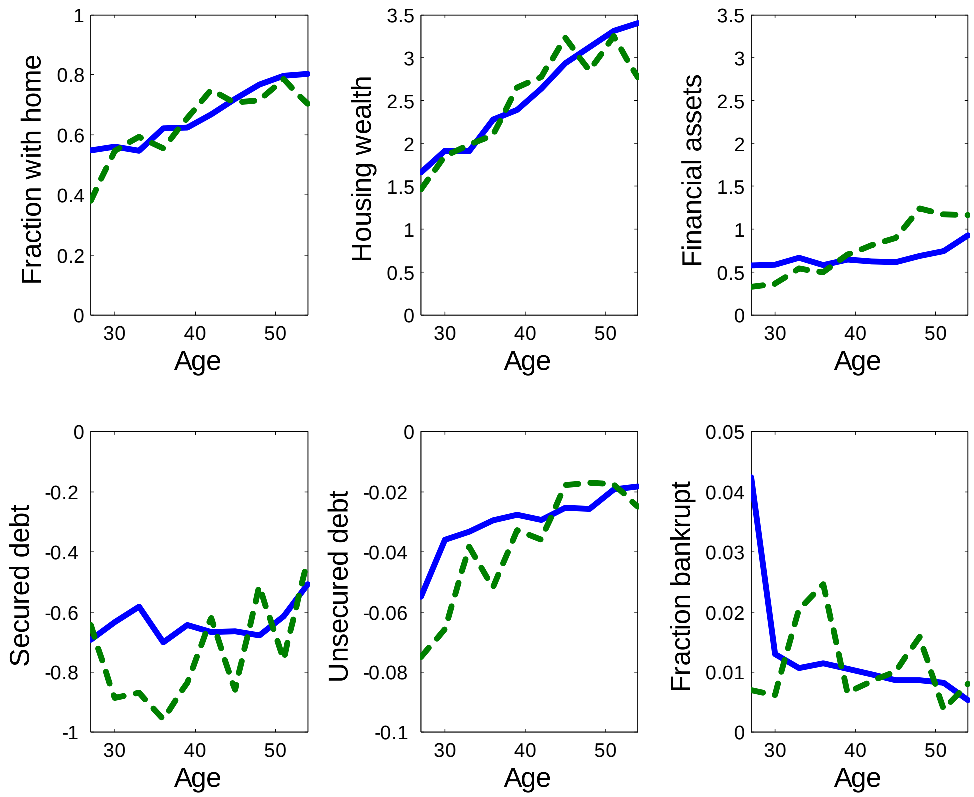

Figure 4: Cross-sectional age profiles predicted by the model (solid graph) and the data (dashed graph) for prime-age consumers with ages 26–55 up to the 90th percentile of the net worth distribution. Source: Authors' calculations based on the model and the Survey of Consumer Finances (SCF) 2004. Notes: The unit is the average of net labor earnings. The bankruptcy incidence in the data is multiplied by 2/3 for the fraction of job-related bankruptcies as reported in Sullivan et al. (2000). See the data appendix for variable definitions.

model output with the SCF *cross-section* data, we have adjusted the simulation output of the model accounting for cohort effects resulting from income growth (see Section 4.2). Figure 4 shows that the cross-sectional age profiles predicted by the model match the SCF data profiles remarkably well: home ownership rates, owner-occupied housing wealth and financial assets increase between ages 26 and 55, whereas secured and unsecured debt decrease between ages 26 and 55 in absolute terms. Noting the different scale for secured and unsecured in Figure 4, the model predicts that debt is mostly secured as in the data. Moreover, we find that about a third of young consumers with housing wealth at the beginning of life is at the collateral constraint. For the relevant parameters which produce a quantitative fit of the facts, consumers close to, or at, the collateral constraint with little housing equity hold the more expensive unsecured debt. Most consumers repay their unsecured debt and the bankruptcy incidence falls between ages 26 and 55 both in the model and the data. The model predicts somewhat more bankruptcies for young consumers than the SCF data which may not be of major concern, however, given that the cross-sectional age profile for bankruptcy is quite noisily measured in the data.

4.5 Sensitivity analysis

Given the good fit of the model with the SCF data, we perform three experiments in order to better understand the model. We compute special cases of the model in which consumers do not have access to secured debt, unsecured debt and the bankruptcy option, respectively. This will help to understand the importance of the debt instruments and the bankruptcy option in the model for intratemporal and intertemporal consumption smoothing. Moreover, the experiments provide intuition for the role of home equity as informal collateral which is of major interest in this paper.

Preferences in our model imply consumption smoothing across states and across time. We compute the variance of the logarithm of the consumption basket (including rented or owned housing wealth) as an indicator for intratemporal consumption smoothing and average total debt (the sum of secured and unsecured debt) as an indicator for the extent of intertemporal consumption smoothing. Figure 5 displays the results for these indicators for the benchmark calibration and our experiments.

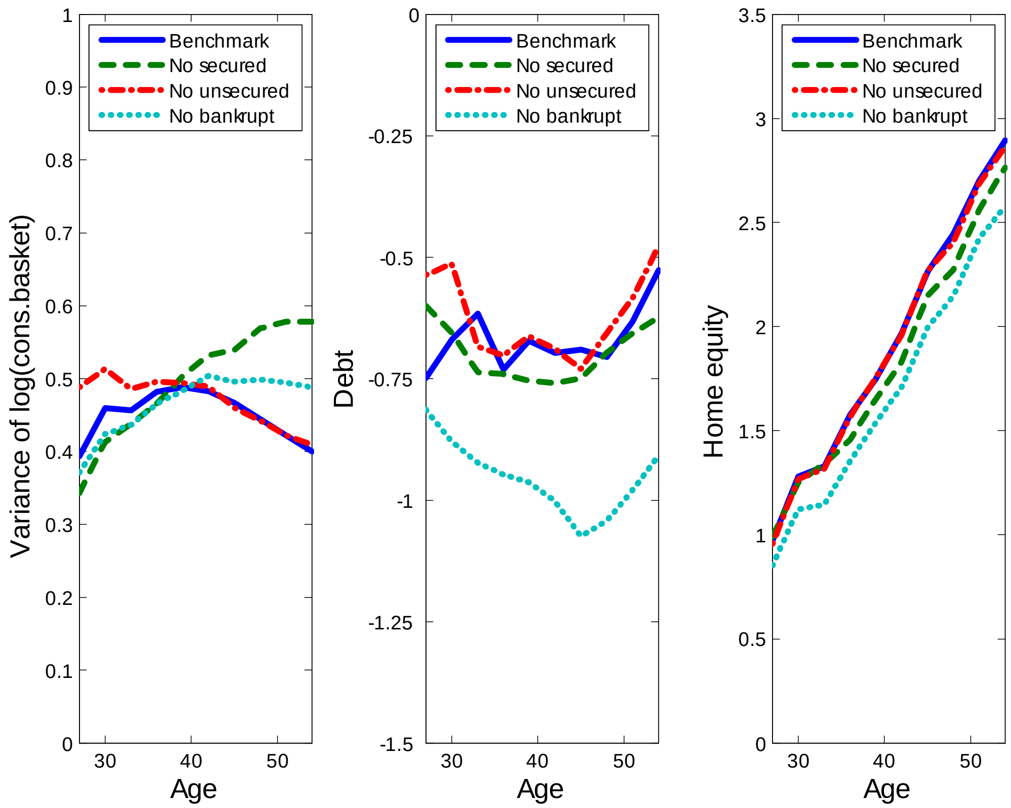

Figure 5: The variance of the log consumption basket, average debt and home equity for the benchmark and three experiments. Source: Authors' calculations based on the model. Notes: The unit is the average of net labor earnings. Debt is the sum of secured and unsecured debt. Home equity is housing wealth net of secured debt.

In our benchmark calibration the variance of the log consumption basket, plotted as the solid graph, has a concave shape for consumers between ages 26 and 55, as in the model without housing and secured debt in Livshits et al. (2007), Figure 3B. If agents do not have access to unsecured debt, the variance of the log consumption basket, plotted as the dash-dotted graph, increases by up to 0.1 for young consumers compared with the benchmark calibration. This is a sizeable increase of the variance by 25%. After age 40, the variance is very similar to the benchmark. As illustrated in the graph for total debt, young consumers take on less total debt (in absolute terms) to smooth consumption if they do not have access to unsecured debt and can only borrow against housing collateral. Unsecured debt is thus important for young consumers to smooth consumption across states and time.

If the consumer has no bankruptcy option, total debt (plotted as the dotted graph in Figure 5) increases substantially in absolute terms due to an increase in unsecured debt.[22] We find that full repayment commitment reduces the risk premium on unsecured debt and thus allows consumers to smooth consumption more cheaply intertemporally. The effect on intratemporal consumption smoothing is unclear, however, since bankruptcy allows to smooth consumption after the realization of bad income states. We find that the variance of the log consumption basket decreases for young consumers and increases after age 40 compared with the benchmark case, similar to results reported in Livshits et al. (2007), Figure 3B. Interestingly, Figure 5 also shows that home equity decreases if there is no bankruptcy option and thus full repayment commitment: both housing wealth and secured debt decrease in absolute terms but housing wealth decreases more strongly. At first glance, one might be tempted to read this finding as pointing to a potential role of home equity as informal collateral. The next section demonstrates, however, that home equity does not provide informal collateral in our quantitative model.

If the consumer has no access to secured debt, average owned housing wealth falls nearly by the same amount so that average home equity is very close to the benchmark case. In Figure 5 this is illustrated by the dashed and the solid graph. Since the home ownership rate falls from 67% to 39%, this home equity is held by a much smaller fraction of consumers so

[22] The case with no bankruptcy option is implemented by setting $\psi = 1$. This implies that the value is zero if bankruptcy is declared, which is strictly smaller than the strictly positive value under repayment. Thus, the max-operator in equations (7) and (9) is redundant and the problem is equivalent to a problem without a bankruptcy option.

that the home equity per owner is larger if there is no access to secured debt. Lack of access to secured debt implies that housing wealth equals home equity. This makes bankruptcy less attractive for owners who have financed their housing purchase with debt, since home equity is much larger than the exempt level in bankruptcy procedures for most owners. Indeed, consumers no longer declare bankruptcy in equilibrium in the model case without secured debt. Although all debt is unsecured debt in this case, banks are willing to lend unsecured debt so that average total debt is very similar to the benchmark case. Moreover, the age profile of the variance of the log-consumption basket is qualitatively similar to the case without a bankruptcy option, where the variance in the case without secured debt is quantitatively larger after age 40. In Figure 5 this illustrated by the dashed and the dotted graph.

5 Commitment by home equity?

Compared with the existing literature a new feature of our model is the joint analysis of housing wealth, secured and unsecured debt since we do not fully consolidate the household balance sheet by computing a single measure of net worth. As we will see, the joint analysis affords new insights for the quantitative importance of committing by using home equity as informal collateral.

Home equity provides informal collateral for unsecured debt if home equity, net of adjustment cost, is above the exemption level $h^{\dagger} = 0.405$. In this case unsecured creditors receive some repayment since the bankruptcy judge will sell housing wealth in order to repay some of the unsecured debt. Thus, home equity above the exemption level $h^{\dagger}$ makes bankruptcy less attractive for consumers, also because of the wasteful adjustment costs, and allows them cheaper access to unsecured debt since banks realize that home equity above the exemption level $h^{\dagger}$ provides informal collateral.

To gauge the quantitative importance of home equity as informal collateral in our model, Figure 6 plots the price for unsecured debt in our benchmark calibration as a function of unsecured debt for different values of housing wealth. As an example, we plot graphs for young consumers at ages 26–28 in the fourth income state. Since consumers who hold

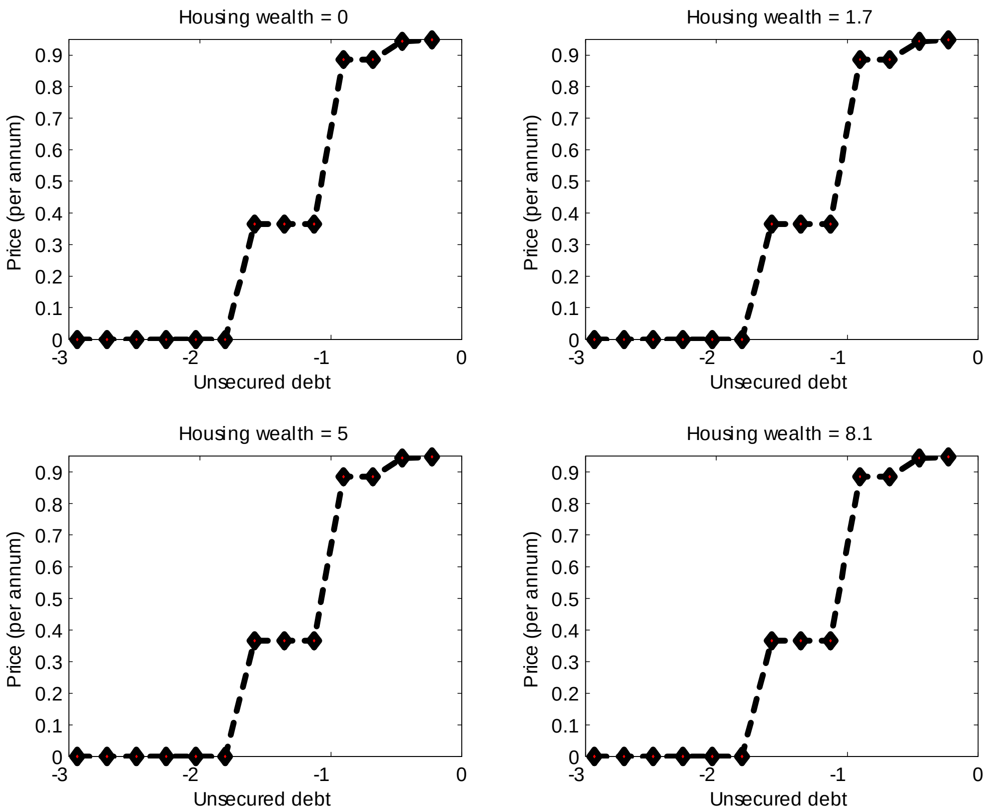

Figure 6: The price of unsecured debt as a function of unsecured debt for different housing wealth. Source: Authors' calculations based on the model. Notes: Prices are per annum for consumers who are at the collateral constraint in the fourth income state at ages 26–28. The unit of unsecured debt is the average of annual net labor earnings.

unsecured debt are at, or close to, the collateral constraint in our calibration, the graphs in Figure 6 are plotted for portfolio choices of housing wealth h^*_{j+1} and secured debt a^{s*}_{j+1} at the collateral constraint. This allows us to reduce the dimension of the graphs since $a^{s*}_{j+1} = -\mu h^*_{j+1}$.

Figure 6 shows that the price for unsecured debt falls with the volume of unsecured debt held by a consumer. Not surprisingly, this is due to the risk premium for unsecured debt which increases with the amount of unsecured debt. In fact, the price for unsecured debt is zero at large levels of unsecured debt, thus pinning down the borrowing limit endogenously. What is remarkable in Figure 6 is that the pricing function remains the same across very different levels of housing wealth.

The intuition for this quantitative result is the following. Consumers with unsecured debt at the collateral constraint hold very little home equity, net of adjustment costs. Given that $\mu = 0.95 < 1 - c_f^- = 0.975$ in our calibration, home equity net of adjustment costs equals $0.025 * h^*_{j+1}$. For this equity to be larger than the exempt amount $h^\dagger = 0.405$, in which case housing wealth would provide informal collateral, housing wealth needs to be larger than $0.405/0.025 = 16.2$. In the equilibrium of our benchmark calibration, however, consumers with unsecured debt hold housing wealth which is smaller than 2.4. Hence, home equity is not large enough in our calibration to reduce the exposure for banks and increase repayment commitment in equilibrium.

Our finding that housing wealth does not provide much informal collateral for unsecured debt is quantitative. In general, consumers with unsecured debt do not necessarily need to be at, or close to, the collateral constraint. Consumers with higher risk aversion, for example, would value the state contingency of unsecured debt more. Higher risk aversion, however, would worsen the match between the model and the data.[23] Figure 7 shows that consumers with unsecured debt in the SCF data indeed hold less home equity than other consumers, as predicted by our calibrated model.

[23] This result relates to the literature on the equity premium puzzle (Mehra and Prescott, 2008) in which implausibly high levels of risk aversion are needed, even in models with incomplete markets, to generate the observed large equity premium. A related finding in this literature is that with a large equity premium implausibly high levels of risk aversion are needed to generate realistic consumer portfolios with rather small shares of positive risky *assets* (Heaton and Lucas, 1997). In our model with *debt* portfolios, however, high risk aversion worsens the fit of the model with the data.

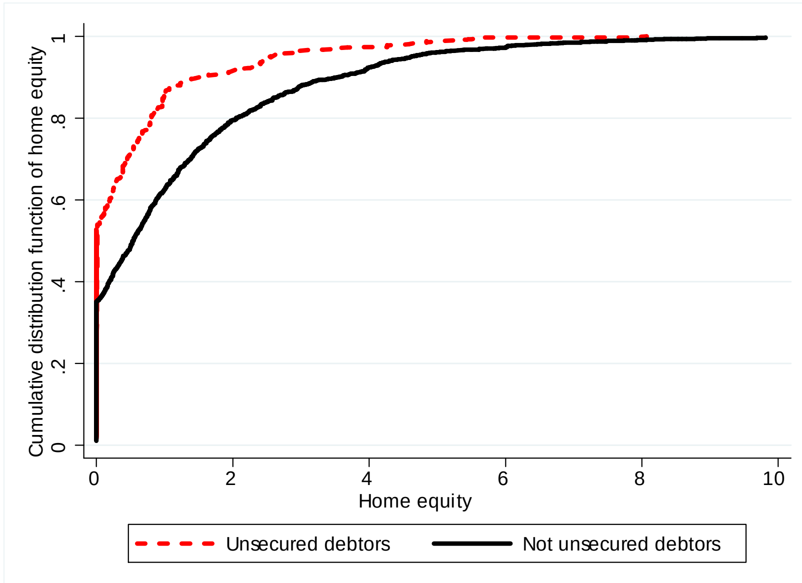

Figure 7: The cumulative distribution function of home equity for prime-age consumers with and without unsecured debt. Source: Authors' calculations based on the SCF 2004. Notes: The functions are plotted for home equity in the interval $[0; 10]$. The unit of home equity is the average of annual net labor earnings.

5.1 Implications for bankruptcy regulation

The small quantitative role of home equity as informal collateral and repayment commitment in the equilibrium of our calibrated model has interesting implications for the relevance of exemption levels that are specified in U.S. bankruptcy regulation. Previous research by Pavan (2008) has argued that home equity provides informal collateral for unsecured credit so that home equity exemptions, for example, make the supply of unsecured credit more costly. A key difference with our paper is that we explicitly model debt portfolios of unsecured and secured debt and that consumers derive utility from the stock of housing wealth rather than home equity.

Table 4 shows that the amount of home equity, that is exempt in bankruptcy procedures, does not affect the equilibrium much in our calibrated model, unless the exemption is very small: the results reported in columns (2) to (7) are quantitatively very similar. The results in column (1) show, however, that it matters for the equilibrium whether there exists a positive bankruptcy exemption. A very small positive exemption level suffices to reduce the repayment commitment substantially, and to affect the price schedules for unsecured debt

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Variable / Exemption level $h^{\dagger}$ =	0	0.1	0.2	Benchmark model: 0.405	1	5	10
Unsecured debt (fraction of net lab. earnings)	-0.40	-0.04	-0.03	-0.03	-0.04	-0.04	-0.04
Risk premium on interest (in percentage points)	0	1.8	2.0	2.1	2.3	2.5	2.5
Job-related bankruptcy (in percent)	0	0.4	0.4	0.4	0.5	0.5	0.5

Table 4: Equilibrium effect of the bankruptcy exemption. Source: Authors' calculations based on the model. Notes: The unit is the average of net labor earnings.

so that much less unsecured debt is held in equilibrium.

The intuition is that for $\mu < 1 - c_f^-$, as in our calibration, and $h^{\dagger} = 0$ the judge will always sell the housing wealth if bankruptcy is declared. This is because $\mu < 1 - c_f^-$ implies that home equity net of adjustment costs is always positive, even if very small for many consumers, and thus strictly larger than $h^{\dagger} = 0$. Thus, declaring bankruptcy is less attractive if $h^{\dagger} = 0$ due to the wasted adjustment costs. Moreover, the exposure of banks is smaller since they always recover a strictly positive payment if consumers declare bankruptcy.

The results in Table 4 show that an exemption as small as a tenth of annual labor earnings, $h^{\dagger} = 0.1$, almost restores the equilibrium of the benchmark in column (4). The reason is that home equity net of adjustment costs is very small for many consumers so that a small exemption of $h^{\dagger} = 0.1$ eliminates the informal collateral. This increases the exposure of banks since they do not recover any resources from these consumers in bankruptcy procedures. Importantly, it also reduces the repayment commitment implied by the wasteful adjustment costs since, unlike in column (1), the judge no longer always sells housing wealth in the bankruptcy procedures.

The finding that the value of positive bankruptcy exemptions matters little for the equilibrium is consistent with the rather inconclusive existing empirical evidence on the effect of home equity exemptions in bankruptcy procedures on debt, wealth accumulation and bankruptcy incidence in the U.S. (see for example White, 2006, Section 7.4, and references therein). Data on homestead exemptions in the U.S. compiled by Athreya (2006), Table 1, show that only four of the U.S. states have homestead exemptions below a tenth of annual labor earnings. As Table 4 shows, exemption levels above a tenth of annual labor earnings do not affect the equilibrium, as illustrated in columns (2) to (7).

6 Conclusion

We introduced a model in which consumers hold portfolios of secured and unsecured debt. The model quantitatively explains important characteristics of wealth and debt portfolios for prime-age consumers in the U.S. We find that consumers with unsecured debt hold small amounts of home equity in the calibrated model and in SCF data, so that home equity does not provide much informal collateral for unsecured debt. As a consequence, observed variations in homestead exemptions, which are an important part of U.S. bankruptcy regulation, matter very little for access to unsecured credit and the debt portfolio.

In future research it would be interesting to relax some of the assumptions made to contain the computational burden in this paper. Allowing for health risk or longevity risk may improve the predictions of the model for asset holdings during retirement. Introducing aggregate price risk for housing wealth into the model, albeit challenging, would allow to investigate the relevance of our analysis for more recent macroeconomic episodes.

Appendices

Data appendix

This data appendix describes how we construct data counterparts for the wealth and debt portfolio as well as labor earnings in the model, using data from the Survey of Consumer Finances (SCF). We construct all variables for the full SCF sample and then apply the sample-selection criteria mentioned below.

Gross labor income is the sum of wage and salary income. As in Budría Rodríguez et al. (2002) we add a fraction of the business income where this fraction is the average share of labor income in total income in the SCF. *Disposable labor income* is computed using the NBER tax simulator. We use the programs by Kevin Moore provided on http://www.nber.org/~taxsim/ to construct disposable labor earnings for each household in the respective SCF wave. Following the standardized instructions on the NBER website, we feed the following required SCF data into the NBER tax simulator: the U.S. state (where available, otherwise we use the average of the state tax payments across states), marital status, number of dependents, taxpayers above age 65 and dependent children in the household, wage income, dividend income, interest and other property income, pensions and gross social security benefits, non-taxable transfer income, rents paid, property tax, other itemized deductions, unemployment benefits, mortgage interest paid, short and long-term capital gains or losses. We then divide the resulting federal and state income tax payments as well as federal insurance contributions of each household by the household's gross total income in the SCF. This yields the implicit average tax rate for each household. The mean of that average tax rate for consumers in the working age 23-64 in the SCF 2004 is 21.5%. Finally, we use the average tax rate of each household in the respective SCF wave to compute household disposable labor income as (1 - household average tax rate) * household gross labor income (including taxable transfers) and then add non-taxable transfers.

When constructing data counterparts for the wealth and debt portfolio of each household in the model, it is useful to refer to the following stylized household balance sheet:

Household balance sheet

Assets	Liabilities
Housing wealth (primary residence)	Gross debt secured by housing
	Home equity
Gross financial assets	Gross unsecured debt
	Other equity

Housing wealth is defined as the sum of the value of owner-occupied home that is the primary residence.

Gross secured debt is defined as the sum of mortgage and housing debt written against the primary residence.

The difference between the value of housing wealth and gross secured debt is the *home equity* held by the household.

Gross financial assets are defined as the sum of assets besides the housing wealth defined above. This is the sum of money in checking accounts, savings accounts, money-market accounts, money-market mutual funds, call accounts in brokerages, certificates of deposit, bonds, account-type pension plans, thrift accounts, the current value of life insurance, savings bonds, other managed funds, other financial assets, stocks and mutual funds, owned non-financial business assets, residential and non-residential property that is not included in housing wealth, vehicles, jewelry, antiques, or other small durable items.

Gross unsecured debt is defined as all debt besides the gross secured debt defined above.

The difference between the gross financial assets and gross unsecured debt is the *other equity* held by each household.

Net worth is then defined as the sum of durable equity and other equity.

We still need to define the data counterparts for unsecured debt, secured debt and financial assets in the model. These counterparts are not equal to the gross positions since many households in the data hold debt and financial assets at the same time which cannot occur in the model. In order match the SCF data to the model, we consolidate the data at the household level so that households indeed either hold debt or financial assets. We proceed in the following way:

Unsecured debt is zero for households with nonnegative other equity and equals other equity if other equity is negative. *Secured debt* for households whose other equity is negative is set equal to their gross secured debt and their *financial assets* are set to zero.

For households who hold positive amounts of other equity we then consolidate these positions with gross secured debt to obtain the corresponding measures as follows.

Secured debt is zero for households whose sum of gross secured debt and positive amounts of other equity is positive. Otherwise secured debt equals gross secured debt net of positive amounts of other equity.

Financial assets are zero for households whose sum of gross secured debt and positive amounts of other equity is negative. Otherwise financial assets equal positive amounts of other equity net of gross secured debt.

Net financial assets are the sum of *financial assets*, *secured debt* and *unsecured debt.*

It remains to describe how we classify households as bankrupt.

Bankruptcy: We classify a household as bankrupt in the SCF 2004 if the household head or husband/wife/partner have filed for bankruptcy in the last year.

Sample selection criteria: We drop observations if gross labor income is negative or not sufficient information is available to compute net labor earnings with the NBER tax simulator (17 observations in the SCF 2004 are deleted), net worth is smaller than -1.2 in terms of the population average of disposable labor income in the respective year (additional 19 observations of the SCF 2004 are deleted), and unsecured debt is larger than 10 in terms of the sample average of disposable labor income (an additional observation in the SCF 2004 is deleted). These sample selection criteria contain the effect of outliers on the sample means, where some of the outliers seem to be related to entrepreneurial activity in which we are not interested in this paper. The resulting sample size is 4,483. The sample size of prime-age households between age 26 and 55 is 2,577.

Calibration of the income process

Income before retirement.

In order to construct a measure for earnings risk before retirement, we recover ϕ_j from the SCF data for consumers between ages 24 and 65, which corresponds to income realizations in the model between ages 23 and 64 since households are asked about income in the previous year. We regress the log of earnings on a quartic age polynomial which approximates the age-earnings patterns in the data well (Murphy and Welch, 1990). We then use the standard deviation of the residuals in the regression to calibrate the distribution of earnings shocks z_{ij}. We assume that the shocks are drawn from a log-normal distribution, where in our calibration to the SCF 2004, $z_{2004} \sim \mathcal{N}(0, 0.603)$. Although a formal test rejects log-normality due to some skewness, log-normality is a rather good parametric approximation of the data. The assumption of log-normality is attractive because it is convenient when we approximate the AR(1) income process by a Markov chain.

We calibrate the annual autocorrelation of log-earnings shocks as $\rho = 0.95$ which implies a variance for the innovations ε_{ij} of 0.059. We have checked the robustness of our results for $\rho = 0.97$ which implies a lower variance for the innovations of 0.036. These values for the autocorrelation and the variance of the innovations are within the range of values commonly used in the literature (see for example Kopecky and Suen, 2010). We approximate the AR(1) process for z_{ij} in (12) by a Markov chain with 5 income states to contain the computational burden, using the so-called Rouwenhorst method. As pointed out by Kopecky and Suen (2010) this method performs particularly well for highly persistent processes.

Since the SCF surveys are repeated cross-sections and we do not observe the full life-cycle income of most cohorts in the period for which SCF surveys are available, we convert the cross-sectional age-earnings patterns into deterministic life-cycle profiles accounting for growth in life-cycle income. As further explained below, we compute the growth rate of average net labor earnings by constructing a pseudo panel using all comparable SCF waves since 1983. We use that panel to regress log-labor earnings on a quartic age polynomial and a linear time trend. We find that this parsimonious specification explains the data well. Most importantly for our purposes we find that annual earnings growth is 1%. The estimation results also support our assumption that cohort effects are not important, beyond the linear time trend of earnings, when constructing the life-cycle profiles with cross-sectional data. Statistically we cannot reject the hypothesis at the 1% significance level that the coefficients of cohort dummies are zero in the regression of log-labor earnings on a quartic age polynomial and a linear time trend.

Given these results, we use average labor earnings as income unit which grow at an annual rate of 1%. This deterministic growth is taken into account by adjusting the cross-sectional age-earnings patterns with a growth factor $1.01^{(age-base\ age)}$. The *base age* is the reference age which will allow us to make income units comparable across cohorts in a specific year.

By considering deterministic income growth over the life cycle we attribute only part of the cross-sectional variation in earnings to idiosyncratic labor income risk. Compared with studies based on other surveys that do not include as many wealth-rich consumers as the SCF, our variances of idiosyncratic income are higher. For example, in our calibration the variance of log-earnings is roughly 0.1 above those reported in Krueger and Perri (2006), Figure 4.

Income after retirement.

After retirement, consumers receive individual-specific retirement benefits b_i. These benefits are approximated based on the U.S. social security legislation (see http://www.ssa.gov). Retirement benefits in the U.S. depend on the 35 highest annual earnings before retirement. In terms of the recursive formulation of the model this would imply that, until retirement, the history of labor earnings would enter the model as a state variable. Clearly this would make the numerical solution of the model, let alone estimation, extremely costly. We thus follow Yang (2009) and determine retirement benefits conditional on the last income before retirement. More precisely, we proceed in the following steps.

Firstly, we transform the net labor earnings y_{ij} of the model into gross labor earnings $\widetilde{y}_{ij}$ using the average tax rate of 0.215 for the sample of households with a head between ages 24 and 65 in the SCF 2004 (including FICA taxes).

Secondly, we take into account that, for the computation of retirement benefits in the U.S., age-j earnings of individual i are scaled by average earnings growth that has occurred between age j and the last period before retirement $T^r - 1$. We thus multiply gross labor earnings $\widetilde{y}_{ij}$ in periods $j < T^r$ by the factor $1.01^{(T^r-1-j)}$ to obtain indexed gross labor earnings.

Thirdly, we compute the average indexed gross labor earnings $\overline{y}(z_{i,T^r-1})$ over the last 35 years before retirement $[T^r - 35, T^r - 1]$ for a consumer who has a realization of the stochastic component of labor earnings z_{i,T^r-1} and gross earnings $\widetilde{y}_{i,T^r-1}$ in the last year before retirement. Clearly, there are many different histories of earnings which lead to $\widetilde{y}_{i,T^r-1}$. We assign probabilities to these histories using the reverse transition probability $R(z_{ij}, z_{i,j-1})$. This corresponds to the probability that $z_{i,j-1}$ is the predecessor of z_{ij}. Applying Bayes' rule we can compute this probability as

$$R(z_{ij}, z_{i,j-1}) = f(z_{i,j-1}) \frac{P(z_{i,j-1}, z_{ij})}{f(z_{ij})},$$

where P is the standard "forward" transition probability and $f(\cdot)$ is the unconditional probability obtained from the stationary distribution.

Fourthly, we set the social-security income cap to \$87,000 and the first and the second bendpoint to \$606 and \$3,653, respectively, as specified in the U.S. social security legislation for 2003 (since labor earnings in the SCF 2004 are recorded for the previous year). We then convert this cap and these bendpoints into model units, dividing by the average equivalized net labor earnings of \$30,866 in the SCF 2004, and adjust them for average earnings growth over the life cycle as specified in the U.S. social security legislation.

Finally, we apply the formula as documented on the website

http://www.ssa.gov/OACT/COLA/piaformula.html to compute retirement benefits as

$$b(z_{i,T^r-1}) = \begin{cases} 0.9\ \overline{y} & \text{if } \overline{y} < bp_1 \\ 0.9\ bp_1 + 0.32\ (\overline{y} - bp_1) & \text{if } bp_1 \leq \overline{y} < bp_2 \\ 0.9\ bp_1 + 0.32\ (bp_2 - bp_1) + 0.15\ (\overline{y} - bp_1) & \text{if } bp_1 \leq \overline{y} < cap \\ 0.9\ bp_1 + 0.32\ (bp_2 - bp_1) + 0.15\ (cap - bp_1) & \text{if } \overline{y} \geq cap\ , \end{cases}$$

where $\overline{y} = \overline{y}(z_{i,T^r-1})$ and bp_1 and bp_2 denote the two bendpoints.

Our calibration of retirement benefits implies that the replacement ratio of benefits over gross income is 51% for the median income in the last period before retirement. This replacement rate is similar to the rates reported in Biggs and Springstead (2008).

Pseudo-panel estimation to compute average earnings growth

The SCF is a triennial survey and comparable data exist for the period from 1983 to 2007. As is common practice, we do not use the 1986 survey since it was only a limited reinterview survey with respondents to the 1983 SCF. This leaves us with eight repeated cross-sectional surveys in 1983, 1989, 1992, 1995, 1998, 2001, 2004 and 2007.

We construct a pseudo panel for three-year age cohorts, computing cohort averages for log-labor earnings and the terms of the quartic age polynomial. Table 5 displays the number of observations for each of the cohort-year cells in the unbalanced pseudo panel for working-age households with a head between ages 24 and 65. Recall that this corresponds to earnings between model ages 23 and 64 since households in the SCF are asked about their earnings in the previous year. Cell averages are computed with at least 90 observations and well above 100 observations for most cohort-year cells. See the seminal paper by Browning, Deaton and Irish (1985) for further background on pseudo panels.

We augment the income process before retirement, presented in the calibration section, with a linear time trend to capture time effects and use the pseudo panel to estimate the log-linear specification. Note that this specification derives from the structural assumptions about the income process. Whereas the log-linear regression of labor earnings thus has a

Cohort number	*Age in 1983*	*1983*	*1989*	*1992*	*1995*	*1998*	*2001*	*2004*	*2007*
1	3–5	–	–	–	–	–	–	147	147
2	6–8	–	–	–	–	–	140	162	173
3	9–11	–	–	–	–	171	197	164	188
4	12–14	–	–	–	157	175	193	211	210
5	15–17	–	–	143	210	193	190	249	242
6	18–20	–	91	165	225	209	236	266	287
7	21–23	–	117	209	226	261	296	296	261
8	24–26	237	133	249	257	286	330	306	338
9	27–29	277	204	237	270	290	347	334	306
10	30–33	251	176	241	290	275	323	340	313
11	34–36	262	208	249	310	297	269	303	277
12	37–39	232	219	255	249	299	292	291	270
13	40–42	238	185	218	269	234	235	279	241
14	43–45	214	177	225	188	218	192	181	–
15	46–48	205	171	186	199	178	175	–	–
16	48–50	196	180	154	203	168	–	–	–
17	51–53	211	165	185	189	–	–	–	–
18	54–56	198	165	206	–	–	–	–	–
19	57–59	197	162	–	–	–	–	–	–
Sums		2,718	2,353	2,922	3,242	3,254	3,415	3,529	3,253

Table 5: Number of households in each earnings cohort per year. Source: Authors' calculations based on the SCF.

structural interpretation, a similar regression with wealth as dependent variable has not. In fact such a regression would be misspecified for our model. Moreover, we want to calibrate our model to the most recent data in the 2000s so that we use the pseudo panel regressions only to compute the annual growth rate of earnings. We then use this growth rate to map between the age cross sections in the last available SCF survey 2004 before the financial crisis and the life-cycle profiles of labor earnings and wealth in the model.

Table 6 displays the results of the regressions. In our preferred specification in column (1), we estimate an annual growth rate of labor earnings of 1%. In column (2) we replace the linear time trend with time dummies. Comparing the adjusted R^2 statistic in columns (1) and (2) reveals that the fit of the data remains good with the more parsimonious specification in column (1). As is well known, column (3) shows that the data variation could also be explained with cohort dummies. Because of linear dependence, we cannot simultaneously use age, year and cohort dummies as regressors. If we restrict the age effects to a quartic

	Dependent variable: log-labor earnings		
Regressors	(1)	(2)	(3)
age	0.511 (0.203)*	0.515 (0.198)**	0.590 (0.198)**
age^2	-0.018 (0.007)*	-0.018 (0.007)**	-0.019 (0.007)**
age^3	0.00028(0.00011)*	0.00028(0.00011)**	0.00031(0.00011)**
age^4	$-1.7/10^6$ $(6.2/10^7)$**	$-1.7/10^6$ $(6.1/10^7)$**	$-1.8/10^6$ $(6.1/10^7)$**
linear time trend	0.010 (0.001)**	–	–
constant	4.04 (2.06)	4.00 (2.01)*	3.32 (2.01)
time dummies	No	Yes**	No
cohort dummies	No	No	Yes**
Adj. R^2	0.818	0.827	0.833
Observations	109	109	109

Table 6: Regressions for log-labor earnings of cohorts between ages 23 and 64. Notes: Standard errors in brackets. **: 1% significance level, *: 5% significance level. Source: Authors' calculations based on the SCF.

polynomial and the time effect to a linear trend, as in the specification in column (1), we cannot reject the hypothesis that the coefficients of cohort dummies are jointly zero when cohort dummies are added to that specification. The test statistic of the likelihood ratio test is $\chi^2(17) = 28.90$ with a p-value of 0.050. The lack of strong significance gives some support to the assumption in the paper that cohort effects are captured with the linear time trend.

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