Effective Programming Practices for Economists

2. Math Refresher

Hans-Martin von Gaudecker

Department of Economics, Universität Bonn

Math refresher

- Skim through this webpage:
  - `http://www.jfsowa.com/logic/math.htm`
  - Sections 1, 2, 4, 8

Math refresher

- Section 1: Sets, Bags and Sequences
- Section 2: Functions
- Section 4: Graphs
- Section 8: Propositional Logic

Sets, Bags, and Sequences

- **Definition:** A *set* is an arbitrary collection of elements without duplicates
- Examples:
  - $\{1, \quad 2, \quad 3\}$
  - $\{2, \quad 3, \quad 1\}$
  - $\{x \mid x \text{ is an integer and } 0 < x < 4\}$
  - $\{x \mid x \text{ is a positive integer, } x \text{ divides 6, and } x \neq 6\}$
  - $\{x \mid x = 1, \text{ or } x = 2, \text{ or } x = 3\}$

Sets, Bags, and Sequences

- Union: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- Intersection: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- Complement: $-A = \{x \mid x \in U \text{ and not } x \in A\}$
- Difference: $A - B = \{x \mid x \in A \text{ and not } x \in B\}$
- Subset: $A \subset B,$ if $x \in A,$ then $x \in B$

Sets, Bags, and Sequences

- Idempotency: $A \cap A$ is identical to A and $A \cup A$ is identical to A
- Commutativity: $A \cap B$ is identical to $B \cap A$ and $A \cup B$ is identical to $B \cup A$
- Associativity: $A \cap (B \cap C)$ is identical to $(A \cap B) \cap C$, and $A \cup (B \cup C)$ is identical to $(A \cup B) \cup C$
- Distributivity: $A \cap (B \cup C)$ is identical to $(A \cap B) \cup (A \cap C)$, and $A \cup (B \cap C)$ is identical to $(A \cup B) \cap (A \cup C)$

Sets, Bags, and Sequences

- **Definition:** A *bag* is a collection of elements with possible duplicates

- **Definition:** A *sequence* is a collection of ordered elements

Functions

- **Definition:** A function is a rule for mapping the elements of one set to elments of another set
  - $f : A \to B$
  - A: domain
  - B: range
- Computer science: $x \in A$ called **input** and $f(x) \in B$ is called **output**

Functions

- Properties:
  - *onto*, every element of its range is the image of some element of its domain
  - *one-to-one*, no two elements of its domain are mapped into the same element of its range
  - *isomorphic*, function that is both one-to-one and onto

Functions

- Examples:
  - Z: set of all integers
  - N: set of all non-negative integers
  - E: set of even integers
  - O: set of odd integers

Functions

- $square : Z \to N$
  - $square(x) = x^2$
- $abs : Z \to N$
  - $abs(n) = \begin{cases} +x & \text{if } x \geq 0 \\ -x & \text{if } x \leq 0 \end{cases}$
- $increment : E \to O$
  - $increment(x) = x + 1$

Graphs

- $G = (N, E)$
- Examples:
  - $N = \{x_0, x_1, x_2, x_3\},\ E = \{\{x_0, x_2\}, \{x_2, x_1\}\}$
  - $N = \{x_0, x_1, x_2, x_3\},\ E = \{(x_0, x_2), (x_2, x_1), (x_1, x_2)\}$

Graphs

- Directed
- Cycles
- Directed acyclic
- Paths
- Directed paths
- Skeleton
- Tree
- Chain
- Connected
- Complete

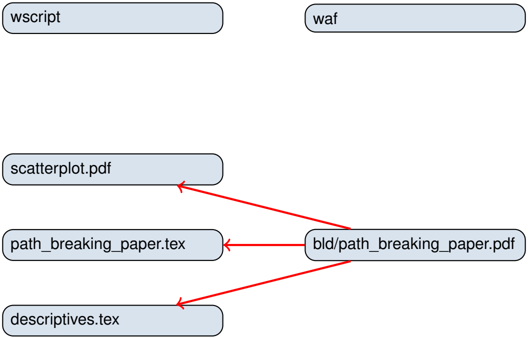
Nodes

- Adjacent
- Connected
- Disconnected
- Parents
- Children
- Family
- Root
- Sink

Edges

- Undirected
- Directed
- Bidirected
- Loops

Graphs


wscript
waf
scatterplot.pdf
path_breaking_paper.tex
bld/path_breaking_paper.pdf
descriptives.tex

Propositional Logic

- *Proposition p:* The Sun is shining
- *Proposition q:* It is raining

Propositional Logic

- $p \wedge q$, conjunction
  - The sun is shining, and it is raining
- $p \vee q$, disjunction
  - The sun is shining, or it is raining
- $\sim p$, negation
  - The sun is not shining

Propositional Logic

- $p \supset q$, material implications
  - If the sun is shining, then it is raining
- $p \equiv q$, equivalence
  - The sun is shining if and only if it is raining

Truth Table

Input						
				Output		
p	q	$p \wedge q$	$p \vee q$	$\sim p$	$p \supset q$	$p \equiv q$
T	T	T	T	F	T	T
T	F	F	T	F	F	F
F	T	F	T	T	T	F
F	F	F	F	T	T	T

Acknowledgements

- This course is designed after and borrows a lot from the `Software Carpentry` course designed by `Greg Wilson` for scientists and engineers.
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